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# Corporate Financial Policy and the Cost of Capital

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## 1. Introduction

We consider the effects of personal and corporate taxation on the firm's financial structure and its investment decision. Corporate financial policy and the theory of investment have been investigated by Modigliani-Miller [2, 3], Stiglitz [4], and King [1] etc.. Analysis in this area consists of two separate issues. (1) Given the firm's investment-profit sequence and the structure of taxation, what is an optimal method of financing among alternative methods? (2) Given the firm's method of financing and the structure of taxation, what are the optimal level of investment and the cost of capital? The question is how the two problems are related. For example, we may ask how an optimal financial policy is related to the cost of capital. I will show one relation between problems (1) and (2) by using King's model. One feature in this paper is that I use the same set-up for both corporate financial and investment policies. For example, in King the set-up in corporate financial policy is slightly different from that in the investment theory, since King is interested in another issue.

## 2. Methods of Financing and Share Valuation

Consider the firm that owns capital stock and whose technology is representable by the production function $Y_t = F(K_t, L_t)$, where $Y_t$ is a single output, and $K_t$ ($L_t$) is capital stock (the labour input, respectively). We assume that $L_t$ is fixed, i.e. $L_t = L$, and that $F(\cdot)$ is differentiable, monotone and strictly concave. Let the firm's profit be $\Pi(Y_t) = p(Y_t)\, Y_t - wL$. Let $\phi_1$ and $\phi_2$ be the rate of corporation tax on $(\Pi_t - G_t)$ and $G_t$ respectively, where $G_t$ is total gross dividends. Define the total tax liability of the firm $T_t$ by

$$(1) \qquad T_t = \phi_1(\Pi_t - G_t) + \phi_2 G_t + mG_t,\ \phi_1 \in I,\ \phi_2 \in R,\ m \in I$$

where $m$ is the rate of income tax, and $I \equiv [0, 1]$. Let $\Omega \equiv I^3 \times R$ be the set of taxes with a generic element $\tau \equiv (m, z, \phi_1, \phi_2)$, where $z$ is the rate of tax on capital gains. Let the level of retained profits $R$ be

$$(2) \qquad R_t = \Pi_t - D_t - T_t + X_t$$

where $D_t = (1 - m)\, G_t$ is total net dividends, and $X_t$ is the amount raised by either borrowing or issuing new shares.

Consider the three methods of financing: retention, equity finance, and debt finance. In the notations below, $r$, $s$, and $b$, stand for retention, equity finance, and debt finance, respectively. From (1) and (2) total net dividends, $D_t^j$, paid by the firm at the end of period $t$ are

$$(3) \qquad D_t^j(Z_t^j, \tau) = \theta\,[(1-\phi_1)\,\Pi_t^j - R_t^j + X_t^j] \qquad j \in \{r, s, b\}$$

at $Z_t^j \equiv (R_t^j, \Pi_t^j) \in R_+^2$ and $\tau \in \Omega$, where

$$\theta \equiv \frac{1-m}{1-\phi_1+\phi_2}.$$

Note that $\dfrac{\partial D_t^j}{\partial R_t^j} = -\theta$. Define the net dividend per share $d_t^j$ by

$$(4) \qquad d_t^j(Z_t^j, \tau) = \frac{D_t^j(Z_t^j, \tau)}{N_t^j} \qquad j \in \{r, s, b\}$$

at $(Z_t^j, \tau) \in R_+^2 \times \Omega$, where $N_t^j$ is the number of shares in period $t$ when method $j$ is used. For each $j \in \{r, s, b\}$, define $X_t^j$ and $N_t^j$ as follows.

[1] Retentions

Total net dividends, $D_t^r$, is defined by substituting $X_t^r = 0$ for all $t \geq 0$ in (3). For $d_t^r$, let $N_t^r = N$ for all $t \geq 0$ in (4).

[2] Equity Finance

In order to define $D_t^s$, set $X_0^s = \Delta N V_1^s$ and $X_t^s = 0$ for all $t \geq 1$ in (3), where $\Delta N$ is the number of new shares issued at the end of period 0, which are sold at a price $V_1^s$. Substitute $N_0^s = N$ and $N_t^s = N + \Delta N$ for all $t \geq 1$ into (4) to define $d_t^s$.

[3] Debt Finance

Set $X_0^b = B$, and $X_1^b = -\{1 + (1-\phi_1)\,i\}\,B$, and $X_t^b = 0\ (t \geq 2)$ in (3) to define $D_t^b$, where $B$ is the amount borrowed which is repaid one period later, and $i$ is the market interest rate. For $d_t^b$, let $N_t^b = N$ for all $t \geq 0$ in (4).

For method $j \in \{r, s, b\}$, define the current market share value $V_0^j$ by

$$V_0^j(Z^j, \tau) = \left(\frac{1}{1-z}\right) \sum_{t=0}^{\infty} \left[\frac{d_t^j(Z_t^j, \tau)}{\alpha^{t+1}}\right]$$

at $Z^j \equiv \{Z_t^j\}_{t=0}^{\infty}$ and $\tau \in \Omega$, where $\alpha \equiv 1 + \dfrac{(1-m)\,i}{1-z}$.

## 3. The Firms' Rational Behaviors

### 3.1. Optimal Financial Policy

The firm's optimal financial policy is defined as follows. Given the sequence

$Z \equiv \{Z_t\}_{t=0}^{\infty}$ and tax $\tau \in \Omega$, the firm chooses method of financing $j \in \{r, s, b\}$ so that

$$V_0^j(Z, \tau) = \max\{V_0^r(Z, \tau), V_0^s(Z, \tau), V_0^b(Z, \tau)\}.$$

Now the effect of personal and corporate taxation on the firm's current market share value is stated in the proposition below, which is proved in the Appendix.

**PROPOSITION 1:**

For any sequence $Z \equiv \{Z_t\}_{t=0}^{\infty}$ we have

[1] $V_0^s(Z, \tau) - V_0^r(Z, \tau) \gtreqless 0 \Leftrightarrow (\theta + z) \gtreqless 1$

[2] $V_0^b(Z, \tau) - V_0^r(Z, \tau) \gtreqless 0 \Leftrightarrow (1 - m) \gtreqless (1 - \phi_1)(1 - z)$

[3] $V_0^b(Z, \tau) - V_0^s(Z, \tau) \gtreqless 0 \Leftrightarrow (1 - z) + (1 - m)i \gtreqless \theta\{1 + (1 - \phi_1)i\}$.

From Proposition 1 we can derive the following optimal corporate financial policy immediately.

**COROLLARY 1:**

For any sequence $Z \equiv \{Z_t\}_{t=0}^{\infty}$, we have

[1] Retention is optimal if and only if
$(1 - z)[\phi_1 - \min\{0, \phi_2\}] \leq (m - z)$.

[2] Equity finance is optimal if and only if
$(m - z) \leq (1 - z)(\phi_1 - \phi_2) - (1 - m)i \cdot \max\{0, \phi_2\}$.

[3] Debt finance is optimal if and only if
$\phi_1(1 - z) \geq (m - z) \geq (1 - z)(\phi_1 - \phi_2) - (1 - m)\phi_2 i$.

Next we have the following neutrality theorem from Proposition 1.

**COROLLARY 2:**

For any sequence $Z \equiv \{Z_t\}_{t=0}^{\infty}$, three methods of financing are indifferent for the firm, i.e.,

$$V_0^r(Z, \tau) = V_0^s(Z, \tau) = V_0^b(Z, \tau) \text{ if and only if } \phi_2 = 0 \text{ and } \phi_1 = \frac{m - z}{1 - z}.$$

### 3. 2. Optimal Investment Decision

Define the level of investment $R_t$ by

$$(5) \qquad K_{t+1} - (1-\delta) K_t = \frac{R_t}{q_t}$$

where $\delta$ is the proportion of capital stock which wears out in each period, and $q_t$ is the price of capital goods in period $t$. For simplicity, we assume that $\delta = 0$, and that $q_t = 1$ for all $t$. Let us consider the firm's current investment policy, that is, $R_0 \geq 0$ *and* $R_t = 0\ (t \geq 1)$. Then by (5) we have $K_1 = K_0 + R_0$, $K_t = K_1 = K$ (say) $(t \geq 1)$.

The firm's maximization problem is defined as follows: Given methods of financing $j \in \{r, s, b\}$ and tax $\tau \in \Omega$, the firm chooses level of investment $R_0^{j\star}$ so that

$$R_0^{j\star} = \arg\max \{ V_0^j (R_0^j) \colon R_0^j \geq 0 \}$$

where

$$V_0^j = \left[ \frac{1}{1-z} \right] \sum_{t=0}^{\infty} \left[ \frac{d_t^j}{\alpha^{t+1}} \right]$$

$$(6) \qquad d_t^j = \frac{\theta}{N_t^j} \left[ (1-\phi_1) \Pi_t^j - R_t^j + X_t^j \right] \qquad (t \geq 0)$$

$$\Pi_t^j = p(Y_t^j) Y_t^j - wL \qquad (t \geq 0), \quad Y_t^j = F(K_t^j, L) \qquad (t \geq 0)$$

$$K_t^j = K_0 + R_0^j = K^j \text{ (say)} \qquad (t \geq 1)$$

and $X_t^j$ depends on $R_0^j$. Note that $V_0^j$ is a function of one variable $R_0^j$. The optimal level of capital stock for method of financing $j \in \{r, s, b\}$ is $K_t^{j\star} = R_0^{j\star} + K_0$, $(t \geq 1)$.

For three methods of financing explained in section 2, we have

**PROPOSITION 2:**

The cost of capital for each method of financing is the following:

[1] Retention

$$p\sigma(K) F_k = \frac{(1-m) i}{(1-\phi_1)(1-z)} \qquad \text{at } K = K^{\star r}.$$

[2] Equity Finance

$$p\sigma(K) F_k = \frac{(1-m) i}{\theta (1-\phi_1)} \qquad \text{at } K = K^{\star s}.$$

[3] Debt Finance

$$(7) \qquad p\sigma(K) F_k = \frac{(1-m)\{1 + (1-\phi_1) i\}}{(1-\phi_1)\{(1-z) + (1-m) i\}} i \qquad \text{at } K = K^{\star b}.$$

In particular, if retention and debt finance are indifferent for the firm, i.e., $V_0^r = V_0^b$, then for debt finance we have $p\sigma(K) F_k = i$ at $K = K^{\star b}$, where $\sigma(K)$ is

one plus the reciprocal of the price elasticity of demand, and $F_k$ is a derivative of the production function with respect to $K$.

**PROOF OF PROPOSITION 2:**

PROOF OF [1] AND [3]: Let $B = hR_0$, $0 \leq h \leq 1$. We omit $j$ in (6) shown above. For (6), let $X_0 = B$, $X_1 = -\{1 + (1 - \phi_1)\, i\}\, B$, $X_t = 0$ for all $t \geq 0$, and let $N_t = N$ for all $t \geq 0$. Since we have $B = hR_0$, $\dfrac{\partial \Pi_t}{\partial R_0} = p\sigma(K)\, F_k$ for $t \geq 1$, and $\dfrac{\partial \Pi_0}{\partial R_0} = 0$, we have $\dfrac{\partial V_0}{\partial R_0} = 0$ if and only if

$$\text{(i)} \qquad p\sigma(K)\, F_k \left[ \frac{1 - \phi_1}{\alpha - 1} \right] = (1 - h) + \frac{h}{\alpha} \left[ 1 + (1 - \phi_1)\, i \right]$$

In the case of retention, set $h = 0$ into $(i)$. In the case of debt finance, set $h = 1$ in $(i)$. Then the proof is completed from the definition of $\alpha$. In particular, if $V_0^r = V_0^b$, then from Proposition 1 we have

$$\text{(ii)} \qquad (1 - m) = (1 - \phi_1)(1 - z).$$

By substituting (ii) into (7) we have $p\sigma(K)\, F_k = i$ at $K = K^{\star b}$. Q. E. D.

PROOF OF [2]: For (6), let $X_0 = \Delta N V_1^s$, $X_t = 0$ for all $t \geq 1$, and let $N_0 = N$, $N_t = N + \Delta N$ for all $t \geq 1$. Note that

$$\text{(iii)} \qquad V_1^s = \left[ \frac{1}{1 - z} \right] \sum_{t=1}^{\infty} \left( \frac{d_t}{\alpha^t} \right) = H \frac{\Pi_1}{N + \Delta N}$$

where $H \equiv \dfrac{\theta(1 - \phi_1)}{(1 - z)(\alpha - 1)}$. Since we have $R_0 = \Delta N V_1^s$ and (iii), it follows that

$$\frac{\partial V_0^s}{\partial R_0} = 0 \Leftrightarrow \frac{\partial V_1^s}{\partial R_0} = 0 \Leftrightarrow p\sigma(K)\, F_k = \frac{1}{H} \qquad \text{at } K = K^{\star s}.$$

Here the first equivalence follows from $\dfrac{\partial d_0}{\partial R_0} = 0$, and the second one follows from $V_1^s = \dfrac{H}{N}(H\Pi_1 - R_0)$. Thus the proof is completed. Q. E. D.

## 4. Financial Policy and the Cost of Capital

We consider the relation between optimal financial and investment policies. The question is how optimal financial policy is related to the cost of capital. For this question we must recognize that tax rules play an important role. The role of tax is analogous to that of price in a competitive market mechanism, where the marginal rate of substitution among economic agents are equal through parametric prices. From Propositions 1 and 2 we have the following proposition:

**PROPOSITION 3:**

Let $j_1, j_2 \in \{r, s, b\}$, $j_1 \neq j_2$. Then for any investment-profit sequence $Z \equiv \{Z_t\}_{t=0}^{\infty}$ we have

$$V_0^{j1}(Z) \gtreqless V_0^{j2}(Z) \Leftrightarrow C^{j1} \lesseqgtr C^{j2}$$

where $C^j$ is the cost of capital, that is, $C^j \equiv p\sigma(K) F_k$ evaluated at $K = K^{\star j}$, $j \in \{r, s, b\}$.

**PROOF OF PROPOSITION 3:**

By Propositions 1 and 2 we have

[case 1]:

$$V_0^s \gtreqless V_0^r \Leftrightarrow (\theta + z) \gtreqless 1$$

$$\Leftrightarrow C^s \equiv \frac{(1-m)\,i}{\theta(1-\phi_1)} \lesseqgtr \frac{(1-m)\,i}{(1-\phi_1)(1-z)} \equiv C^r.$$

[case 2]:

$$V_0^b \gtreqless V_0^r \Leftrightarrow (1-m) \gtreqless (1-\phi_1)(1-z)$$

$$\Leftrightarrow C^b \equiv \frac{(1-m)\{1+(1-\phi_1)\,i\}}{(1-\phi_1)\{(1-z)+(1-m)\,i\}}\,i \lesseqgtr \frac{(1-m)\,i}{(1-\phi_1)(1-z)} \equiv C^r.$$

[case 3]:

$$V_0^b \gtreqless V_0^s \Leftrightarrow (1-z)+(1-m)\,i \gtreqless \theta\{1+(1-\phi_1)\,i\}$$

$$\Leftrightarrow C^b \equiv \frac{(1-m)\{1+(1-\phi_1)\,i\}}{(1-\phi_1)\{(1-z)+(1-m)\,i\}}\,i \lesseqgtr \frac{(1-m)\,i}{\theta(1-\phi_1)} \equiv C^s.$$

Q. E. D.

From Proposition 3 we have Corollaries 1 and 2 stated below.

**COROLLARY 3:**

The firm's optimal method of financing leads to the minimal cost of capital among three methods of financing. Formally, given $\tau \in \Omega$, if for some $j \in \{r, s, b\}$

$$V_0^j(Z, \tau) = \max\{V_0^r(Z, \tau), V_0^s(Z, \tau), V_0^b(Z, \tau)\}$$

at any $Z \equiv \{Z_t\}_{t=0}^{\infty}$, then we have

$$C^j(\tau) = \min\{C^r(\tau), C^s(\tau), C^b(\tau)\}.$$

Finally, for the neutrality theorem we have

**COROLLARY 4:**

If three methods of financing are indifferent for the firm, then the cost of capital is equal in all methods of financing. Formally, for some $\tau^0 \in \Omega$, if

$$V_0^r(Z, \tau^0) = V_0^s(Z, \tau^0) = V_0^b(Z, \tau^0)$$

at any $Z \equiv \{Z_t\}_{t=0}^{\infty}$, then we have

$$C^r(\tau^0) = C^s(\tau^0) = C^b(\tau^0).$$

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## APPENDIX

PROOF OF PROPOSITION 1: The proof is immediate from the following:

[1] $V_0^s - V_0^r = \dfrac{\Delta NV_1^s}{\alpha(1-z)N}\{\theta - (1-z)\}$

[2] $V_0^b - V_0^r = \dfrac{\theta B}{\alpha^2(1-z)N}[\alpha - \{1 + (1-\phi_1)i\}]$.

[3] $V_0^b - V_0^s = \dfrac{\Delta NV_1^s}{\alpha^2(1-z)N}[\alpha(1-z) - \theta\{1 + (1-\phi_1)i\}]$

where the last equation holds from $\Delta NV_1^s = B$. Q. E. D.

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