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<th>R &amp; D Rivalry, the Role of the Market Size and Social Welfare</th>
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Our model allows firms with different technologies to co-exist by assuming the amount of R & D investment differs among firms since the development of new technology requires a large amount of R & D investment which is a fixed cost. The technology which firms choose, and consequently the type of market competition that occurs, thus crucially depends upon the size of the market due to the existence of these fixed costs. Using a game theoretical approach, we are also able to estimate the welfare effects of a subsidization policy which promotes R & D investment given different market structures.

1. Introduction

It is widely believed that firms' R & D activities are crucial for their survival. In the theory of industrial organization, R & D investment either creates new products (product innovation), reduces the cost of producing existing products (process innovation), or both [see Tirole (1989)]. Creating new products raises social welfare because consumers can now enjoy new products and producers can increase profits, while process innovation makes research intensive firms relatively stronger and intensifies competition among firms and consumers gain in the form of lower prices. Consequentially, a subsidization policy which promotes R & D investment that encourages firms to enter the market when they do not have sufficient incentives to do so and/or process innovation may be socially desirable. These topics, however, are usually treated separately. This paper focuses on the role market size has on the state of competition in the market, and discusses the possibility of an effective subsidization policy. As is often observed, firms with different technologies often co-exist in the market [see Cohen and Krepper (1992)]. Our model allows firms with different technologies to co-exist by assuming the amount of R & D investment differs among firms since the development of new technology requires a large amount of R & D investment, which is mostly a fixed cost. Accordingly, the technology which firms choose, and hence the type of market competition that occurs, crucially depends upon the size of the market, due to the existence of fixed costs.

In the existing literature of related topics, most authors a priori assume that firms share the same technology and that research intensive technology is
superior to the traditional one. That is, R & D activities are considered to be a race in which the winner is awarded an exogenous prize and the losers get nothing [for example, see Loury (1979) and Delbono and Denicolo (1991)]. Contrary to this, we will assume that the fruits of R & D activities are equally shared among firms. As this paper allows for the possibility that firms with old technology can co-exist with firms with new technology, several different market patterns will emerge, depending upon market size, the effect of R & D activities, and the amount of R & D investment. When both market size and reductions in marginal cost due to R & D activities are relatively small, firms with old technology only will survive. Even given a small market size, however, if the reductions in marginal costs are large, firms with new technology can enter the market. This raises important welfare implications. Since firms employing new technology press down the commodity price and increase consumers’ surplus, social welfare can be raised by subsidizing them, even if the market is a small one.

Using a game-theoretic approach, [for example see Besley and Suzumura (1992) and Rowthorn (1992)], this paper investigates the relationship between market size and reductions in marginal costs which are the result of R & D activities. For analytical simplicity, we will consider the duopolistic situation where each firm has equal opportunity to enter the market, either by using traditional technology or by developing research intensive technology, which increases the firm's fixed costs. We will assume that the fixed costs of the firm employing research intensive technology are twice those of the firm employing traditional technology. Accordingly, the duopolists have two moves. In the first move of the game, they choose their technology, either the “traditional” one or “research intensive” one. The important point here is that the realization of R & D efforts is equally known among firms with certainty. In the second move of the game, they determine how much they will produce given their technology choice. Employing the concept of Nash equilibrium, we can predict the exact patterns of competition that will occur in a market and examine the benefits for both consumers and producers. This allows us to suggest the appropriate circumstances in which a subsidization policy can be used to raise social welfare.

This paper is organized as follows: Section 2 states the basic framework of our model. Section 3 deals with firms’ strategies and provides the pay-off matrix for firms. Section 4 discusses the Nash equilibria and derives the pattern of the market competition. Section 5 considers price changes under a given market size and examines the changes in social welfare that occur. Finally, concluding remarks are given in section 6.
2. The Model

We will assume there are two firms in the market, each producing a homogeneous good and that they share the same technical knowledge. In order to produce the product, they have two technology choices. They can employ either the traditional technology or research intensive technology. Without loss of generality, the former case can be expressed by the cost function,

$$C = Q + F,$$

where $Q$ and $F$ are output and fixed costs respectively.\(^2\) Marginal cost is normalized. In the latter case, we assume the cost function takes the following form.

$$C = aQ + \beta F;$$

where $0 < a < 1 < \beta$. Thus from eq.(2), while R & D investment incurs increased fixed costs compared to the traditional (or less research intensive) technology, marginal costs are reduced [see Flaherty (1980)].

The market demand function is assumed to take a linear form.

$$P(D) = A - D,$$

where $P$ and $D$ are the price and quantity demanded of the good respectively. $A$ is a constant.

Assuming that the market is large enough only for the monopolist with the cost function given by eq. (2), then the profit of the monopolist is,

$$\pi = P(Q)Q - [aQ + \beta F].$$

Differentiating with respect to $Q$, then the first-order condition for profit maximization is,

$$A - 2Q - a = 0.$$

Thus from eq.(4) we know output and profits are given by

$$Q(a) = \mu(a) \text{ and } \pi(a) = [\mu(a)^2 - \beta F];$$

where $\mu(a) = (A - a)/2$.

When the monopolist uses the traditional technology, then

$$Q(1) = \mu(1) \text{ and } \pi(1) = [\mu(1)^2 - F].$$

Following Rowthorn, we define $S = \mu(1)^2/F$, where $S$ is an index of whether or not the market is large enough to sustain a monopolist with traditional technology. Accordingly, the profit of a monopolist with traditional technology is: $\pi =$
By defining $\lambda = (\mu(a)/\mu(a) - 1) > 0$, we can measure the sustainability of the monopolist employing new technology with reference to $S$. That is,

$$\pi(a) = (\lambda + 1)^2 S - \beta F.$$ 

Since $\lambda > 0$ and $\beta > 1$, the monopolist can survive in the market only if $S > \beta/((\lambda + 1)^2)$.

3. Firms' Strategies

For analytical simplicity, we will consider the case where $\beta = 2$. That is, when firms engage in R & D activities, their fixed costs are twice as much as when they employ traditional technology. We will consider the duopolistic case. Firms have two moves in the game. In the first move of the game, they simultaneously determine which technology they will employ, the traditional one or the research intensive one. They have three choices: (1) They do not choose either technology and do not enter the market; (2) they can employ traditional technology represented by the cost function of eq. (1); and (3) they can choose the new technology given by the cost function of eq. (2).

In the second move of the game, they simultaneously decide how much they will produce. When one of the duopolists does not enter the market, the other duopolist becomes a monopolist and produces either $Q(1) = \mu(1)$ if they use the traditional technology, or $Q(a) = \mu(a)$ if they employ the research intensive technology. The profits of the firms for the above two cases are $\pi(1) = (S - 1)F$ and $\pi(a) = [(\lambda + 1)^2 S - 2]F$ respectively.

When both duopolists employ research intensive technology, the first order condition for profit maximization is

$$\frac{\partial \pi_i}{\partial Q_i} = A - Q_i - 2Q_i - a = 0;$$

where $Q_i (i = 1, 2)$ stands for firm $i$'s output. Since we assumed symmetry, i.e. $Q_i = Q$, then $Q_i = 2\mu(a)/3 = 2\mu(1)(\lambda + 1)$ and each firm's payoff is $\pi_i = \left\{ 4 \over 9 \right\} (\lambda + 1)^2 S - 2 F$. Of course, when $\lambda = 0$, the above equations yield the output and payoff of the firm employing the traditional technology and this can thus be considered to be a special case.

When firm $j$ employs the traditional technology and firm $i$ uses research intensive technology, then the first-order condition for maximizing profits by firm $j$ is

$$\frac{\partial \pi_j}{\partial Q_j} = A - Q_j - 2Q_j - 1 = 0.$$ 

Solving both eqs. (5) and (6) simultaneously, we can obtain,
Table 1 summarizes the pay-off matrix for firms 1 and 2, where $N$, $T$, and $R$ are the three choices the firm faces. That is, $N$, $T$, and $R$ are when the firm chooses not to enter the market, to enter the market but employ the traditional technology, and to enter the market and invest in $R$ & $D$ activities respectively.

Table 1. Pay-off Matrix

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<tr>
<td>$N$</td>
<td>$[0, 0]$</td>
<td>$[0, S-1]$</td>
<td>$[0, (1+\lambda)S-\beta]$</td>
</tr>
<tr>
<td>$T$</td>
<td>$[S-1, 0]$</td>
<td>$\left[\frac{4}{9}(S-1), \frac{4}{9}S-1\right]$</td>
<td>$\left[\frac{4}{9}(1+\lambda)S-1, \frac{4}{9}(2\lambda+1)S-\beta\right]$</td>
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<tr>
<td>$R$</td>
<td>$\left[(\lambda+1)S-\beta, 0\right]$</td>
<td>$\left[\frac{4}{9}(2\lambda+1)S-\beta, \frac{4}{9}(1-\lambda)S-1\right]$</td>
<td>$\left[\frac{4}{9}(\lambda+1)S-\beta, \frac{4}{9}(\lambda+1)S-\beta\right]$</td>
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* The true value of pay-offs should be multiplied by $F$ for each argument.

4. Nash Equilibria

Consider the Nash equilibria of the game specified by Table 1. Referring to the pay-offs in the table, we can draw Figure 1 for the parametric regions of

![Figure 1](image-url)
Nash equilibria based on the values of $\lambda$ and $S$. While there are nine pairs of strategies for firms 1 and 2, only six pairs will be examined because of our symmetric assumption. Arithmetic numbers in Figure 1 indicate the number of firms in the market, while the subscripts $T$ and $R$ indicate whether a firm employs traditional technology or research intensive technology respectively.

Suppose that the reductions in marginal cost due to $R & D$ investment are small. Since fixed costs increase when $R & D$ activities are undertaken, a firm making use of traditional technology has a comparative cost advantage when the size of the market is small. When $R & D$ investment drastically reduces marginal cost and mitigates the burden of the fixed cost, however, a firm making use of research intensive technology possesses a comparative cost advantage. In the following, for example, the expression $(T, R)$ indicates that there are two firms in the market; one employs the traditional technology while the other employs research intensive technology. Of course, $(T, 0)$ indicates that there is a monopolist with traditional technology in the market.

Let us sum up some of our main results.

1. When $R & D$ investment induces only minor marginal cost reductions ($\lambda < 0.1478$), an increase in market size indicates a gradual evolution of market competitiveness in the market. In order for the firm with traditional technology to survive in the market, $S$ must be larger than unity. When $S$ is greater than $9/4$ and market size increases, duopolists can co-exist and pass through the stages $(T, T)$, secondly $(T, R)$, and finally $(R, R)$.

2. When $R & D$ investment is moderately effective in reducing marginal costs ($0.207 < \lambda < 0.414$), then the market moves from the monopolistic situation, $(T, 0)$, to $(R, 0)$, and then to the duopolistic situation, $(R, R)$. The stages $(T, T)$ and $(T, R)$ both disappear.

3. When $R & D$ investment is effective enough ($\lambda > \sqrt{2} - 1 = 0.414$), then market evolution only allows the existence of firm(s) making use of research intensive technology; first with the monopolist appearing, $(R, 0)$, and then the duopolistic stage, $(R, R)$, occuring.

5. Price Changes and Welfare Effects

1. Suppose $S < 1$, so that the market size is not large enough to allow a firm with traditional technology to behave as a monopolist. Even in this case, however, as a result of the increase in effectiveness of $R & D$ investment, the value of $\lambda$ decreases and the firm making use of research intensive technology can make a profit. Since the appearance of the monopolist means a new product is supplied, the monopolist creates consumers' as well as producers' surplus. Therefore, there may be an incentive for the government to subsidize the monopolist to enter the market. Further improvement in cost reduction may allow two
firms with research intensive technology to coexist in the market. Figure 2
shows the price changes when the economy moves into Regions 0 to 1, to 2RR.
In Region 0, both consumers' and producers' surpluses are assumed to be zero.

First consider the welfare changes of a movement from Region 1 to Region
1. The consumers' surplus in Region 1, $CS_1$, is easily computed as $CS_1 = SF$ $(\lambda+1)^2/2$, as $CS_1 = Q_1^2/2$ and $Q_1 = \mu(1)(\lambda+1)$. The producers' surplus, $PS_1$, can be read from Table 1. Thus the change in welfare induced by a movement from Regions 0 to 1 is

$$\Delta W = CS_1 + PS_1 = \frac{3}{2} SF(\lambda+1)^2 - 2F.$$ 

It should be noted that when we evaluate the welfare changes at $\lambda = \lambda_1$, and
$\Delta W = F > 0$ only when $(\lambda+1)^2 > 4/3S$. This implies that even if the monopolist
with research intensive technology exists in Region 0 but is located at $\lambda \in \{ \lambda :$

$$\frac{4}{3S} < (\lambda+1)^2 \leq \frac{2}{S} \}$$

for a given $S (<1)$, social welfare will be raised by subsidizing firms to become monopolists.4

Next, consider a change in commodity prices from $p_2$ to $p_3$ in Figure 2.
The consumers' and producers' surplus in Region 2RR are

$$CS_{2RR} = \frac{8}{9} SF(\lambda+1)^2$$
and

\[ PS_{BB} = 2\left[ \frac{4}{9}(\lambda+1)^2S - \beta \right] \]

respectively. Thus movement from Regions 1_\alpha to 2_\alpha results in changes in consumers' and producers' surpluses of,

\[ \Delta CS = \frac{7}{18}SF(\lambda+1)^2 > 0 \]

and

\[ \Delta PS = \left( \frac{1}{9}(\lambda+1)^2S + \beta \right)F < 0. \]

Therefore, the change in social welfare is,

\[ \Delta W = \Delta CS + \Delta PS = \frac{5}{18}(\lambda+1)^2SF - \beta F. \]

Thus when the monopolist is located in Region 1_\alpha, \((\lambda+1)^2 \leq 9\beta/4S\). Therefore, a subsidization policy inducing a monopolistic competitive market to become a duopolistic one is not Pareto efficient.

(2) Assuming \(1 < S < 9/4\), then given a small reduction in marginal costs due to \(R & D\) investment, a firm with traditional technology can still operate, having a comparative cost advantage relative to the firm with research intensive technology. When \(\lambda\) increases, however, a firm with research intensive technology now appears as a monopolist in the market. Further increases in \(\lambda\) force the monopolist to allow another entrant. Figure 3 shows the price changes as a result of cost reduction. Since the welfare effect of a price decline from \(p_6\) to \(p_7\) is already examined above, we only investigate the welfare effects of a price decline from \(p_4\) to \(p_5\). When the economy is located in Region 1_r, consumers' and producers' surpluses are \(CS_r = SF/2\) and \(PS_r = (S-1)F\). The consumers' and producers' surpluses in Region 1_\alpha, however, are \(CS_\alpha = SF(\lambda+1)^2/2\) and \(PS_\alpha = (\lambda+1)^2S-2\)F. Thus the changes in consumers' and producers' surpluses caused by a movement from Region 1_r to 1_\alpha are

\[ \Delta CS = \frac{SF}{2}(\lambda^2 + 2\lambda) > 0 \]

and

\[ \Delta PS = (\lambda^2 + 2\lambda)S-1)F. \]

Thus from eqs. (10) and (11), the change in social welfare is

\[ \Delta W = \left( \frac{3}{2}(\lambda^2 + 2\lambda)S - 1 \right)F. \]

When the economy is located in Region 1_r, \((\lambda+1)^2 \leq 1+1/S\). From eq. (12), \(\Delta W > 0\) if and only if \((\lambda+1)^2 > 1+2/3S\). Defining \(T = \left\{ \lambda : \frac{2}{3S} + 1 < (\lambda+1)^2 \leq \frac{1}{S} + \right\}\)
then, for $\lambda \in T$, a subsidization policy will raise social welfare. We have already shown that a price decline from $p_b$ to $p_h$ results in a social welfare loss.

(3) Suppose $S > 9/4$. This is the most interesting case as the economy moves from Region $2_{TT}$ to $2_{TR}$ to $2_{RR}$. The price changes of the good as the value of $\lambda$ increases are shown in Figure 4. Consumers' and producers' surpluses in each region are

\begin{align*}
CS_{TT} &= \frac{8}{9} SF, \quad PS_{TT} = \left\{ \frac{8}{9} S - 2 \right\} F; \\
CS_{TR} &= \frac{2}{9} (\lambda + 2)^2 SF, \quad PS_{TR} = \left\{ \frac{4}{9} (5\lambda^2 + 2\lambda + 2) S - 3 \right\} F; \\
CS_{RR} &= \frac{8}{9} (\lambda + 1)^2 SF, \quad PS_{RR} = \left\{ \frac{8}{9} (\lambda + 1)^2 S - 4 \right\} F.
\end{align*}

When the economy moves from Region $2_{TT}$ to $2_{TR}$, the changes in consumers' and producers' surpluses are

\begin{align*}
\Delta CS &= \frac{2}{9} (\lambda^2 + 4\lambda) SF > 0
\end{align*}

and
Figure 4. \((S > 9/4)\)

\[ \begin{align*}
\rho_0 &= A - 4\mu(1)/3; \\
\rho_9 &= A - 2\mu(1)(\lambda_0 + 1)/3; \\
\rho_{10} &= A - 4\mu(1)(\lambda_0 + 1)/3; \\
\rho_{11} &= A - 4\mu(1)(\lambda_0 + 1)/3; \\
\lambda_0 &= \frac{1}{2}\left(\frac{9}{4S} + 1 - 1\right); \\
\lambda_0 &= 9/16S.
\end{align*} \]

\((17)\) \[ \Delta PS = \left\{\frac{4}{9}\lambda(5\lambda + 2)S - 1\right\}F. \]

Estimating the changes in producers' surplus at \(\lambda = \lambda_0\), then \(\Delta PS = 4\{(1 - \lambda)^2 - 1\}/9 < 0\). Accordingly, the change in social welfare is

\[(18)\] \[ \Delta W = \frac{22}{9}SF\left\{\lambda^2 + \frac{8}{11}\lambda - \frac{9}{22S}\right\}; \]

where \(\Delta W > 0\) if \(\lambda > \frac{4}{11}\left(\sqrt{1 + \frac{99}{32S}} - 1\right) \approx 0.178\). When the economy is in Region 2TR, \(\Delta W > 0\) when \(\lambda \leq \frac{1}{2}\left(\sqrt{1 + \frac{9}{10}} - 1\right) \approx 0.19\). Therefore, as long as \(0.178 < \lambda \leq 0.19\), a subsidization policy will increase social welfare.

Lastly, let's consider the change in social welfare when the economy moves from Region 2TR to 2RR. The changes in the consumers' and producers' surplus are

\[(19)\] \[ \Delta CS = \frac{1}{9}\{6\lambda^2 + 8\lambda\}SF > 0 \]

and

\[(20)\] \[ \Delta PS = \frac{4}{9}SF(2\lambda - 3\lambda^2) - F. \]

Since in Region 2TR \(\lambda \leq 9/16S\), then \(\Delta PS \leq (12\lambda^2 + 8\lambda)SF/9 < 0\). Thus the change in social welfare is
Therefore, a subsidization policy in this case is Pareto inefficient.

6. Concluding Remarks

While we only examined the duopoly case here, without loss of generality, our model can be easily extended to a symmetric case where the number of firms with different technologies are more than one and equal. While our analytical framework is very simple, we have shown a close relationship between market size and the efficiencies of \( R \& D \) investment exists. We have confirmed the intuition that for a given market size, higher efficiency levels yield relative cost advantages to firms with research intensive technology and that for a given efficiency level, the larger the size of the market, the more favorable the environment for firm(s) with research intensive technology. Accordingly several different market patterns of competition will emerge depending on the type of technologies firms choose.

Having examined the social welfare changes, our results give us two major policy implications. Firstly, a subsidization policy designed to increase market competitiveness is not necessarily desirable from a social point of view. For example, when the market size is moderately small \((S<9/4)\), then a subsidization policy motivating the existing monopolist with research intensive technology to allow another entrant in the market is Pareto inefficient. Secondly, even if the market patterns do not change, the subsidization policy encourages firms to adopt new technology. Consider the case where \(1<S<9/4\) and assume there is a monopolist using traditional technology in the market. Then a subsidization policy which causes the monopolist to shift to research intensive technology is Pareto efficient. In the case of a duopolistic competitive market, where both firms employ traditional technology, a subsidization policy, urging one of the duopolists to choose research intensive technology is also Pareto efficient. A subsidization policy that causes both firms to employ research intensive technology, however, is Pareto inefficient.

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**Notes**

1. A subsidization policy here implies that when the transition of some market state to another one produces positive changes in social welfare, the government can execute a feasible transfer payment from consumers to producers and provide an incentive for firms to supply the product. However, when the transition creates negative changes in the social welfare, a subsidization policy is not feasible, even if the transition yields a positive consumers' surplus.

2. The form of the cost function, given by eq.(1), is the same as in Rowthorn(1992).
3. The assumption that $\beta = 2$ does not affect most of our results. Some complication occurs when $4 > \beta \geq 2$; however, as the regional boundary between $1_2$ and $1_3$ is now given by 
\[ \lambda = \sqrt{\frac{1}{S}} + 1 \] and 
\[ \lambda = \frac{1}{2} \left( \frac{3}{2} \sqrt{\frac{2}{S}} - 1 \right) \]. These equations cross in the region. Furthermore, if $\beta > 4$, the boundary of Region $1_3$ is given by 
\[ \lambda = \frac{1}{2} \left( \frac{3}{2} \sqrt{\frac{2}{S}} - 1 \right) \].

4. We show here that this claim holds for $\beta > 1$. Let $T_e$ be the amount of subsidization firms receive and that is taken from consumers. Defining $T_e = \{(1 + \lambda) - (1 + \lambda)^2\}S + \varepsilon$, where $\varepsilon > 0$, then $PSR + T_e = \varepsilon > 0$. In this case the monopolist has an incentive to produce. Let $\delta = (1 + \lambda)^2 - (2\beta/3S) > 0$. Since $CS_2 - T_e = \delta - \varepsilon$, given $\varepsilon$ smaller than $\delta$, then a subsidization policy is feasible and raises the social welfare.

5. In general, $T$ is defined as 
\[ T = \left\{ \lambda : \frac{2(\beta - 1)}{3S} + 1 < (\lambda + 1)^2 \leq \frac{\beta - 1}{S} + 1 \right\} \].

6. In most cases, whenever 
\[ \frac{4}{11} \left( \frac{1}{1 + \frac{99(\beta - 1)}{2S}} - 1 \right) \lambda \leq \frac{1}{2} \left( \sqrt{1 + \frac{9(\beta - 1)}{4S}} - 1 \right) \], a subsidization policy raises social welfare. Tedium calculation also show that the above inequality always holds for $\beta - 1 > 0$.

7. Rowthorn examined the relationship between the market size and international investment by using a duopolistic model. Ono (1992) extended his model to the dual symmetric case and showed that his model gives essentially equivalent results to those derived by the symmetric case. This suggests that we can study a kind of dual symmetric system case when the number of the firms with traditional technology is not necessarily one and different from that with research intensive technology. Without complicating the analysis, however, it allows us to derive economically meaningful results within a simple framework.

References


