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A Note on Market Size Effects and Labor Adjustment in a Two-Region Model

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ABSTRACT

In this note, firstly the static market size effect is analyzed in a simple two-region model and next the dynamic extension to it is explored. In the static framework with increasing returns, love of variety, and transportation costs, it is predicted that other things being equal, the larger the relative market size of one region becomes, the more population and firms tend to concentrate into the same region. This market size effect is considered to be one of the dominant forces to cause maldistribution of factors of production and uneven regional growth in a national economy. In order to develop a growth model reflecting these economic geographical aspects, we extend the static model by introducing heterogeneity among regions and individuals and the dynamic labor entry process. Due to the market size effect and introduced heterogeneity, generally a balanced state between regions is unlikely to happen. From these considerations, one implication is derived: such investments in human capital as improve upon individual labor efficiency may have an effect to accelerate regional divergence.

1. Introduction

This note is an attempt to construct a model which reflects the important aspects of macroeconomic growth: regional concentration and divergence, and uneven growth processes among regions. Therefore, subjects discussed in what follows are primarily related to the main problem in the fields of regional economics and economic geography, i. e., exploration of economic logic behind the spatially uneven distribution pattern of productive factors, which is observed casually in most national economies. In particular, as one of the dominant economic forces to cause regional concentration and divergence, we focus on the market size effect which was first formulated in the context of international trade theory (for example, see Helpman and Krugman (1985)) and then introduced into discussions of the problems of regional economics by Krugman (1991). Our intention is to make a first step to the dynamic analysis of a multi-region growth model with the market size effect.

Concerning the market size effect which is considered to be one of the main forces to work in the economic geographical problem mentioned above, Krugman

(1991) proposed an analytical framework based upon his own approach to international trade theory (Krugman(1980) and Helpman and Krugman referred to above). According to his static two-region model analysis, under preferences with love of variety, increasing returns in production technology, and costs incurred in transportation of products across regions, a difference in the market size between regions may have an effect to strengthen regional divergence. That is, the larger region is likely to experience more concentration, while the smaller is going to face outflow of resources and contraction of activities. To be more specific, Krugman's analysis predicts that nominal wages, consumer price indices, and the variety of goods produced all tend to be determined in favor of residents in a region with the larger market size.

In this note, we examine a dynamic version of the static two-region model, allowing for the entry process of overlapping generations. Applying the mechanism of sectoral adjustment explored by Matsuyama (1992), we combine the market size effect with movements of the regional total labor supply. In one point, our formulation is more complicated than Matsuyama's. The reason is that the equilibrium sequence of relative wage between regions must be determined endogenously in the former, while the sequence of relative wage between sectors is exogenously given in the latter. As will become clear later, this complication and the unstable nature due to the market size effect make global analysis difficult in our model. More elaborated and tractable modelling is needed to derive definite propositions and predictions. Nevertheless, it seems still meaningful to report the present stage of this study, though it may be far from complete, for the reason explained below at some length.

It is often pointed out that the size of domestic demand is one of the key factors in economic development and growth processes. Of course, this may be subject to a theoretical objection that domestic demand should not constrain growth of a national economy if the world trade is free and costless. Nevertheless, extensive empirical studies made so far seem to indicate that a large part of GDP growth was attributed to expansion of domestic demand in many countries. For example, Murphy, Shleifer, and Vishny (1989) argued that in countries with populations over 20 million, expansion of domestic demand accounts for 72-74 percent of the increase in domestic industrial output, referring to the data in Chenery, Robinson and Syrquin (1986).

However, in the study of economic growth, another important fact that we should note is the maldistribution of demands and markets over domestic regions in a country. Through the growth process, domestic demand is distributed not uniformly but unevenly over a national economy. That is, domestic markets do not so much exist or grow spontaneously in every region as are formed in some specific regions as the result of industrialization and concentration. Indeed, to

see the case of Japan will make that point clear. According to Yoshikawa (1992), the rapid growth during 1960s in Japan was, to a large extent, attributed to increases in domestic demand for consumer durables. Such expansion of domestic demand was mainly due to the increase in the number of households in urban regions, which was the result of migration from rural and agricultural regions. That is, a 'virtuous circle' worked there: migration gave the source of labor supply to meet increasing labor demand and at the same time it created large markets for consumer durables in urban regions. Thus, in the past experience, macroeconomic growth and regional divergence were head and tail of the same coin.

In the standard practices in growth theory, however, a national economy is looked upon as a point with no spatial extension like a 'mass point' in mechanics and the process of migration and market formation within domestic regions is set aside. Combination of growth theory and economic geography is necessary for the more detailed analysis of development and growth processes.

The rest of this note is organized as follows. In section 2, the market size effect will be formulated in a static two-region model. Then, in section 3, the basic model will be extended to a dynamic one with a continuum of overlapping generations by introduction of some kind of heterogeneity among regions and individuals, and dynamic labor entry process. In the final section, we will consider some implications possible to derive from the present study. It is concerned with effects of human capital investment on regional divergence.

2. The Basic Two-Region Model and the Market Size Effect

In this section, the market size effects on nominal wage and consumer's utility are analyzed. To do so, following Krugman's framework¹, we set up a two-region model with the representative consumer and monopolistically competing firms that produce differentiated consumption goods. The main assumptions concerning the regional economies are that two regions are homogeneous in a physical sense and that it takes costs of the iceberg form to transport goods from one region to the other. The latter means that when the price of a good produced in one region is p , the price of that good which holds in the other region after transportation becomes p/τ ($0 < \tau < 1$).

The consumer who resides in region i supplies one unit of labor inelastically and earns a nominal wage w_i ($i=1, 2$) established in the regional labor market. Denote the available total variety of goods by N and let each type of goods be indexed continuously by $z \in [0, N]$. Using the Dixit-Stiglitz (1977) type of utility function with a constant elasticity of substitution between any pair of goods, the behavior of consumers belonging to region i is formulated as follows (we tentatively ignore the transportation cost here. After demand functions are

derived, the assumed iceberg form cost will be readily introduced below) :

$$(1) \quad \max. U_i = \left\{ \int_0^N [c(z)]^{1-\frac{1}{\varepsilon}} dz \right\}^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1$$

$$(2) \quad \text{sub. to } \int_0^N p(z) c(z) dz \leq w_i,$$

where $c(z)$ is consumption demand for the good of type z , $p(z)$ is a price of the good of type z , and ε is the elasticity of substitution which is assumed to be greater than unity so that a monopolistically competitive equilibrium should exist. Solving this maximization problem, we have the demand function for good z

$$(3) \quad c(z) = \frac{[p(z)]^{-\varepsilon}}{\int_0^N [p(z)]^{1-\varepsilon} dz} w_i.$$

From (3), the elasticity of demand for each type of goods with respect to its own price is known to be ε .

Concerning firm's technology, we assume the common cost conditions: any firm incurs the fixed cost F and the marginal cost a denominated by labor unit. And they are assumed to be identical values among all firms irrespectively of the type of goods produced. That is to say, we have the following cost function in terms of labor unit :

$$(4) \quad l_z = ax(z) + F, \quad \forall z \in [0, N]$$

Then, the profit maximization condition is written to

$$(5) \quad p(z) \left(1 - \frac{1}{\varepsilon} \right) = aw_i,$$

where i indicates the index of region in which the firm is located. Pricing formula (5) implies that every good produced in the same region takes on an identical price with each other irrespectively of its type.

Using this result, now we can derive consumer demand functions under the presence of transportation costs. Define a variable for $i, j=1, 2$, as follows :

$$(6) \quad d_{ij} = \begin{cases} 1, & \text{if } i=j, \\ \frac{1}{\tau}, & \text{if } i \neq j. \end{cases}$$

By substituting this into (3), consumption demand for a good produced in region j by the consumer residing in region i is written to

$$(7) \quad c_{ij}(z) = \frac{\left(\frac{p_j}{d_{ij} \tau} \right)^{-\varepsilon}}{n_1 \left(\frac{p_1}{d_{i1} \tau} \right)^{1-\varepsilon} + n_2 \left(\frac{p_2}{d_{i2} \tau} \right)^{1-\varepsilon}} w_i, \quad \text{for } i, j=1, 2,$$

where n_i is the variety of goods produced in region i or equivalently the number of firms located in region i . (7) means that the consumer demands for goods produced in the same region are equal with each other. That is, $c_{i1}(z) = c_{i1} \forall z \in [0, n_1]$ and $c_{i2}(z) = c_{i2} \forall z \in (n_1, N]$ where $N - n_1 = n_2$ and, c_{i1} and c_{i2} are the quantities demanded which are determined by (7). Note that introduction of transportation costs of the iceberg form does not change the value of demand elasticity faced by monopolistically competing firms.

In the long-run, free entry drives the profit to zero and its condition implies

$$(8) \quad x(z) = \frac{(\varepsilon - 1)F}{a}, \quad \forall z \in [0, N].$$

Therefore, in the long-run, quantities produced are equal among all firms irrespective of the type of product and of location. Furthermore, substituting (8) into the local labor market equilibrium condition

$$(9) \quad L_i = n_i(ax + F),$$

we obtain the number of firms locating in each region:

$$(10) \quad n_i = \frac{L_i}{\varepsilon F}, \quad i = 1, 2.$$

This implies the proportional relationship between the number of firms and the market size:

$$(11) \quad \frac{n_1}{n_2} = \frac{L_1}{L_2}.$$

Next, in order to derive the main result of this section, the market size effect, we consider the income identities which must hold between two regions.

$$(12a) \quad w_1 L_1 = \frac{r_{11}}{1 + r_{11}} w_1 L_1 + \frac{r_{21}}{1 + r_{21}} w_2 L_2,$$

$$(12b) \quad w_2 L_2 = \frac{1}{1 + r_{11}} w_1 L_1 + \frac{1}{1 + r_{21}} w_2 L_2,$$

where r_{11} is the ratio of expenditures on goods produced in region 1 to those on goods produced in region 2 by region 1 residents, namely,

$$(13) \quad r_{11} = \frac{n_1 \hat{p}_1 c_{11}}{n_2 \left(\frac{\hat{p}_2}{\tau}\right) c_{12}} = \left(\frac{n_1}{n_2}\right) \left(\frac{\hat{p}_1 \tau}{\hat{p}_2}\right) \left(\frac{w_1 \tau}{w_2}\right)^{-\varepsilon} = \left(\frac{L_1}{L_2}\right) \left(\frac{w_1 \tau}{w_2}\right)^{1-\varepsilon},$$

and similarly, r_{21} is the ratio of expenditures on region 1 products to those on region 2 products by region 2 residents

$$(14) \quad r_{21} = \frac{n_1 \left(\frac{p_1}{\tau} \right) c_{21}}{n_2 p_2 c_{22}} = \left(\frac{L_1}{L_2} \right) \left(\frac{w_1}{w_2 \tau} \right)^{1-\varepsilon}.$$

Note that (12a) and (12b) reduce to the following one identity:

$$(15) \quad \frac{1}{1+r_{11}} w_1 L_1 = \frac{r_{22}}{1+r_{22}} w_2 L_2.$$

By denoting the relative nominal wage by $\omega \equiv w_1/w_2$ and the population ratio, or the relative market size, by $\nu \equiv L_1/L_2$, respectively, (15) is rewritten to

$$(16) \quad \nu = \frac{\omega^\varepsilon - \tau^{\varepsilon-1}}{\omega(\omega^{-\varepsilon} - \tau^{\varepsilon-1})}.$$

By totally differentiating (16) in ω and ν and evaluating the result at $\nu = \omega = 1$, we have

$$(17) \quad \left. \frac{d\omega}{d\nu} \right|_{\nu=1} = -\frac{1 - \tau^{\varepsilon-1}}{(2\varepsilon-1) + \tau^{\varepsilon-1}} > 0,$$

where the last inequality follows because $0 < \tau < 1$ and $\varepsilon > 1$. This result, though it is confined to the neighborhood of $\nu = \omega = 1$, means the market size effect on the relative nominal wage, i. e., other things being equal, the larger the market size becomes in one region, the higher relative nominal wage is established there.

Next, let us consider the effect of a change in the relative market size on real wage, i. e., consumer's utility. Substituting demand functions (7) into utility function (1), we obtain the following indirect utility functions of people in region 1 and 2:

$$(18a) \quad V_1 = \left\{ n_1 p_1^{1-\varepsilon} + n_2 \left(\frac{p_2}{\tau} \right)^{1-\varepsilon} \right\}^{\frac{1}{\varepsilon-1}} w_1,$$

$$(18b) \quad V_2 = \left\{ n_1 \left(\frac{p_1}{\tau} \right)^{1-\varepsilon} + n_2 p_2^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}} w_2.$$

Furthermore, using (5) and (10), the right-hand sides of these two functions are rewritten as follows, respectively:

$$(19a) \quad V_1 = \omega \tau \{ \omega^{1-\varepsilon} \tau^{1-\varepsilon} L_1 + L_2 \}^{\frac{1}{\varepsilon-1}}$$

$$(19b) \quad V_2 = \{ \omega^{1-\varepsilon} \tau^{\varepsilon-1} L_1 + L_2 \}^{\frac{1}{\varepsilon-1}},$$

where a constant positive factor common to both formulae is excluded.

From inspections of (15), (19a), and (19b), the following relation is found to hold between the relative utility and the relative wage²:

$$(20) \quad \frac{V_1}{V_2} = \omega^{\frac{2\varepsilon-1}{\varepsilon-1}}.$$

This indicates that the relative utility is monotonously increasing in the relative wage and that the elasticity is greater than unity because $(2\varepsilon-1)/(\varepsilon-1) > 1$. Therefore, an increase in the relative market size L_1/L_2 tends to increase utility of residents of region 1 relative to that of region 2 and vice versa.

The market size effect which has been analyzed so far is explained intuitively as follows. Suppose that the initial equilibrium is disturbed for some reasons and that it changes to an imbalance situation such that $L_1 > L_2$, for example. Then, under full employment in each region, firms in region 1 experience reduction in the average cost due to increasing returns to scale, while firms under the same technology in region 2 face cost-up. Thus, the number of firms operating in region 1 increases and the opposite thing happens in region 2 through the entry and exit process. At the same time, the advantage yielded to firms in region 1 by the disturbance must be cancelled out by the increase in the nominal wage in region 1.

Through this mechanism, the nominal wage and the variety or number of goods produced in region 1 increase. The consumer price index faced by residents in region 1 thus changes in favor of them, because the weight of expenditures on region 1 products in their budget becomes greater and there is no transportation cost in that part. As a result, people in region 1 enjoy the higher utility level.

3. Making the Model Dynamic

In the static model of the previous section, the initial equilibrium at $L_1 = L_2$ is not a stable one; once an imbalance in the size of population between two regions has occurred for some reasons, it gives rise to a difference in utility levels favorable to residents in the region with the larger population, at least, locally around the initial equilibrium. After the static market size effect becomes clear, the next question occurring to us is how regional growth is affected by a change in the market size and subsequent movements in relative wage and utility. In this section, we consider the transition in the regional total labor supply by making the basic model of section 2 dynamic.

In doing so, the first point to be noted is that in reality, migration is an adjustment process associated with heavy costs for households. Taking account of this point, we suppose in this section that once an individual has made a decision in which region to live, he will be stuck in the same region all his life. This 'irreversibility' of choice among regions and 'putty-clay' nature of labor adjustment across regions are parallel to the assumption introduced into the model of sectoral adjustment by Matsuyama (1992) and have much plausibility also in the regional economics subject we are discussing. In other words, we go to the other extreme of the assumption of perfect labor mobility across regions, focusing on

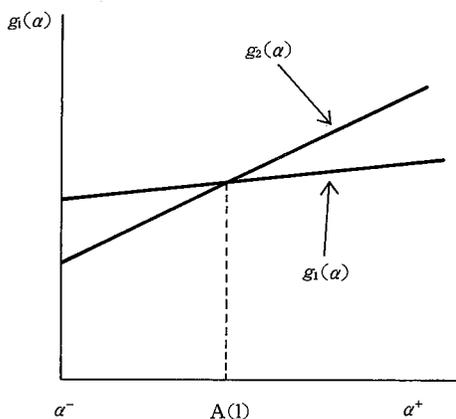
the entry side.

The previous static model is modified as follows. Let the total size of population be constant over time and normalized to be unity. And assume that population consists of a continuum of overlapping generations and each individual faces an instantaneous probability of death, p_d . That is, at every moment, a new cohort is born at rate p_d . Furthermore, removing the assumption of complete homogeneity of regions and individuals, we introduce some kind of heterogeneity which gives rise to differences in individual labor efficiency. Let each individual belonging to the same cohort be numbered in order of magnitude of parameter $\alpha > 0$ endowed to him. And assume that on the support, $[\alpha^-, \alpha^+]$, the continuous distribution function of α , $\Phi(\alpha)$, is defined and that $\Phi'(\alpha) > 0$. This parameter α represents human capital in a sense and is defined to have the following property (P) which is illustrated in Figure 1.

(P) Let $g_i(\alpha)$ denote labor efficiency when an individual endowed with α resides and works in region i . Then,

$$(21) \quad \frac{g_2(\alpha)}{g_1(\alpha)} \text{ is monotone increasing in } \alpha, \text{ and} \\ 1 \in \left[\frac{g_2(\alpha^-)}{g_1(\alpha^-)}, \frac{g_2(\alpha^+)}{g_1(\alpha^+)} \right].$$

Figure 1. An Illustration of Shapes of $g_1(\alpha)$ Functions.



This is to say that an individual with higher α has 'comparative advantage' in working in 'forward' region 2 the meaning of which is given below.

One possible justification of this assumption (P), for example, to consider that region 2 is more advanced in equipment of industrial infrastructures which are implicit in the present model than region 1 and that the value of α represents

ability to work efficiently in more modern and advanced working conditions.

At birth, an individual decides in which region to live and once he has made up his mind, he will be stuck in that region all his life. An individual born at time t will choose to live in region 2, if the discounted sum of his lifetime utility when he resides in region 2 is greater than when he resides in region 1. Noting that indirect utility functions (19a) and (19b) are homogeneous of degree one in an income variable, and that a wage income which an individual with α would earn in region i is $g_i(\alpha)w_i$, the above condition is written to the following expression in terms of the relative utility³:

$$(22) \quad \int_t^\infty g_2(\alpha) v_2(s) \exp\{-(\delta+p_d)(s-t)\} ds > \int_t^\infty g_1(\alpha) v_1(s) \exp\{-(\delta+p_d)(s-t)\} ds,$$

where δ is a constant discount rate, $v_2=1$ and $v_1=V_1/V_2$, respectively. Then, substituting (20), this inequality is rewritten to

$$(23) \quad \frac{g_2(\alpha)}{g_1(\alpha)} > Q(t),$$

where $Q(t) = (\delta+p_d) \int_t^\infty [\omega(s)]^{\frac{2\varepsilon-1}{\varepsilon-1}} \exp\{-(\delta+p_d)(s-t)\} ds$. Denote the inverse function of $Q = g_2(\alpha)/g_1(\alpha)$ by $\alpha = A(Q)$, where $A' > 0$ by the assumption (21). Among the cohort born at time t , those people such that $\alpha > A[Q(t)]$ decide to live in region 2 and others such that $\alpha \leq A[Q(t)]$ go to region 1.

Now, we are in a position to consider dynamics of the total labor supply in each region. The transition in the total labor supply in region i during the small time interval $[t, t+\Delta t]$ is expressed as follows:

$$(24) \quad L_i(t+\Delta t) = L_i(t) - L_i(t)p_d\Delta t + \bar{L}_i(Q(t))p_d\Delta t,$$

where $\bar{L}_1(Q(t)) = \int_{\alpha^-}^{A[Q(t)]} g_1(\alpha) d\Phi(\alpha)$ and $\bar{L}_2(Q(t)) = \int_{A[Q(t)]}^{\alpha^+} g_2(\alpha) d\Phi(\alpha)$.

In the right-hand side of equation (24), the second term with the negative sign represents a decrease in the total supply due to death of 'old generations' in region i and the third term represents a labor supply increase resulting from inflow of new offsprings who choose to live in that region.

From these considerations, we have the following set of differential equations which describe transitions in the total labor supply in each region:

$$(25a) \quad \dot{L}_1(t) = p_d\{\bar{L}_1(Q(t)) - L_1(t)\}$$

$$(25b) \quad \dot{L}_2(t) = p_d\{\bar{L}_2(Q(t)) - L_2(t)\},$$

where dot marks mean time derivatives.

Then, differentiating both sides of (16) which holds at any point of time and substituting (25a) and (25b), we obtain the following differential equation with respect to the relative wage :

$$(26) \quad \frac{\dot{\omega}(t)}{\omega(t)} = \left\{ \frac{1 - \tau^{\varepsilon-1}}{(2\varepsilon-1) + \tau^{\varepsilon-1}} \right\} p_d \left\{ \frac{\bar{L}_1(Q(t))}{L_1(t)} - \frac{\bar{L}_2(Q(t))}{L_2(t)} \right\},$$

where the coefficient in brackets in the right-hand side is taking on the value evaluated at $L_1(t) = L_2(t)$ and is the same as the one appearing in (17) by construction.

Complete mathematical investigations of dynamic paths is much complicated and beyond the scope of the present study because as was the case in static analysis in section 1, behaviors of variables are difficult to capture in areas distant from a balanced state also in the present dynamic model. Mathematically more tractable modelling and analysis are left as the author's future research. Instead, let us confine our attention to a balanced state beginning with the initial condition $L_1(0) = L_2(0)$ and where $\omega(t)$ and $Q(t)$ are unity over time. This balanced state exists if function $\alpha = A(Q)$ and distribution function $\Phi(\alpha)$ are such that $\bar{L}_1(1) = \bar{L}_2(1)$, i. e.,

$$(27) \quad \int_{\alpha^-}^{A(1)} g_1(\alpha) d\Phi(\alpha) = \int_{A(1)}^{\alpha^+} g_2(\alpha) d\Phi(\alpha).$$

When this relation continues to hold, the total labor supply of each region is equal with each other and augmenting at the same rate over time. And then, no imbalance occurs in the relative wage and utility between two regions.

As is seen from these considerations, in the dynamic framework with regional and individual heterogeneity, a balanced state is a very rare and unlikely event. Generally, heterogeneity in labor efficiency among the new cohort necessarily makes a difference in growth rates of the total labor supply between two regions. Once such a difference has been brought about, the relative wage and utility changes through equation (26) and this dynamic market size effect sets regional divergence in motion. $\omega(t)$ and therefore $Q(t)$ changes so that the larger regions should attract more people in the new cohort.

4. Concluding Comments: Human Capital Investment and Regional Divergence

In the preceding sections, we examined how to model regional concentration and divergence, allowing for the market size effect in a simple two-region setting. Following static analysis of the market size effect in section 2, the basic model was extended by introduction of heterogeneity among regions and individuals, and a dynamic entry process of the new cohort in section 3. However,

more mathematical elaboration is required to derive definite propositions and predictions from the dynamic model and it remains as a future research. In this final section, let us consider one implication suggested from analysis made so far. It is concerned with the effect of investment in human capital on regional divergence.

In our framework in section 3, effects of investment in human capital such as public or private investment in education and etc. are captured in two ways: the shift of distribution function $\Phi(\alpha)$ that increases the mean value of α over a cohort, for one thing, and the upward shift of the function $g_1(\alpha)$, for another.

Figure 2. An Illustration of the Case where Human Capital Investment Increases the Mean Value of α .

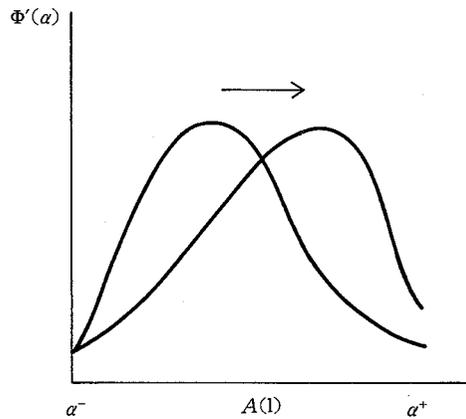
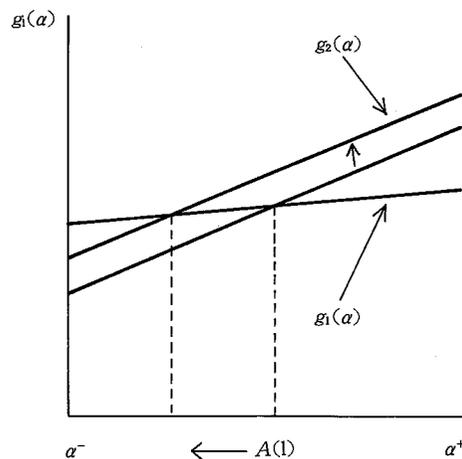


Figure 3. An Illustration of the Upward Shift of $g_2(\alpha)$ due to Human Capital Investment.



Suppose that two regional economies are in a balanced state described in section 3. As is shown in Figure 2, the mean-augmenting shift of $\Phi(a)$ violates condition (27) and a greater fraction of offsprings will choose to live in 'forward' region 2. On the other hand, if investment in education has an effect to improve labor efficiency of every individual, e.g., to shift $g_2(a)$ upward uniformly, it reduces the critical value of $A(1)$ and again violates (27). As a result, more people will go to region 2 as is seen in Figure 3.

Therefore, to 'backward' and small-size regions, investment which reinforces accumulation of human capital has only adverse effects to make their market size much smaller relative to 'forward' regions, if appropriate industrial policies and development of modern sectors which can attract people with a higher level of human capital are not carried out.

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Notes

1. Krugman's original model has two regions and two sectors, agriculture under constant returns to scale technology and manufacture subject to increasing returns to scale and monopolistic competition. As far as analysis of the market size effect is concerned, it is innocuous to abstract from agricultural sector and to consider only monopolistically competing manufacturing sector. In order to concentrate on analysis of effects only due to a difference in the market size between regions, we ignore agricultural sector in the present study.
2. After some manipulations, (15) is rewritten to

$$\left\{ \left(\frac{w_1 \tau}{w_2} \right)^{1-\varepsilon} L_1 + L_2 \right\} / \left\{ \left(\frac{w_1}{w_2 \tau} \right)^{1-\varepsilon} L_1 + L_2 \right\} = \omega^\varepsilon \tau^{1-\varepsilon}.$$

By raising both sides of this identity to power $\{1/(1-\varepsilon)\}$ and multiplying the results by $\omega\tau$, we have

$$\frac{\omega\tau(\omega^{1-\varepsilon}\tau^{1-\varepsilon}L_1+L_2)^{\frac{1}{\varepsilon-1}}}{(\omega^{1-\varepsilon}\tau^{\varepsilon-1}L_1+L_2)^{\frac{1}{\varepsilon-1}}} = \omega^{\frac{2\varepsilon-1}{\varepsilon-1}}.$$

3. An individual faces a constant instantaneous death probability p_d at every moment of time after birth. In this situation, as is well known, the duration time is described by the exponential distribution. The probability that an individual born at time t is dead by time $(t+s)$ ($s \geq 0$) is $\{1 - \exp(-p_d s)\}$. Therefore, the probability that he will survive until time $(t+s)$ is given by $\exp(-p_d s)$. In calculation of the discounted lifetime utility, we are taking account of the latter probability.

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