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Unbalanced Growth with Saturation of Demand

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This paper examines the consequences, in a two-sector version of Pasinetti's pure labour model with unbalanced productivity, of the introduction of equations of demand which show saturation in time.

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JEL Classification: E11; E24; O41

1. Introduction

This paper examines the consequences for the short and long run dynamics of production and employment of the hypothesis of saturating demand functions in a model of unbalanced productivity.

I have chosen the framework of Pasinetti's pure labour production model, in a simplified two-sector form. In Notarangelo (1997) I have attempted to show that, together with the hypothesis of unbalanced productivity, this version of the model can be considered an enlarged model of unbalanced growth as presented by Baumol (1967). The enlargement regards the explicit introduction of the demand functions (in an exponential form), and the consideration of the conditions for macroeconomic equilibrium. These modifications allow us to arrive at much the same conclusions as those drawn by Baumol, but they also serve to enrich them further. For example, the hypothesis of constant output demonstrated an employment shift towards the less progressive sector, as foreseen by Baumol, but we were also able to notice that an equilibrium path of growth requires that the employed population grows at a variable rate, and this can, for specific values of the parameters, imply a period of unavoidable unemployment. When the labour force grows at the equilibrium rate, we found that demand was the only factor determining the growth of output. Because the growth of employment, in order to preserve equilibrium, cannot be exogenous, Baumol's conclusion regarding the slowing down of the growth of output holds only if the system has already exhausted its expansion possibilities, and no more labour force is available. It therefore turns out that the model is unable to account for

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the very common situation of slow growth in the presence of unemployment.

I propose to overcome this shortcoming by introducing demand functions with saturation, in the form of logistic functions. As is well known, this kind of curve depicts the growth of the variable as asymptotically approaching the maximum carrying capacity of the system. In our case, we can think that, as time passes, the need for one commodity spreads among the population, and demand for it increases. Gradually, consumers' demand reaches its maximum level, the total demand growth slows down, and eventually approaches saturation, because, as Marshall (1920) reminded us, " There is an endless variety of wants, but there is a limit to each separate want ". Pasinetti himself (1993) suggested, but did not explicitly adopt, the introduction of demand with saturation : he adopted instead the device of considering the growth rate of demand r_i as variable.

2. General Model with Saturation of Demand

Let us consider a two-sector economy in which the productivity of the first sector is stagnant, while the other increases over time with positive rate ρ_2, so that the productive technology of the system is given by

$$l_1(t)=\frac{L_1(t)}{Q_1(t)}=l_1(0), \tag{1}$$

$$l_2(t)=\frac{L_2(t)}{Q_2(t)}=l_2(0)e^{-\rho_2 t}, \tag{2}$$

where $L_i(t)$ and $Q_i(t)$ $(i=1, 2)$ are the quantities of labour and of production. Moreover, we assume that the total population N grows at an exogenous rate g and that the total employed labour force $L=L_1+L_2$ is not coincident with population, but is a variable fraction of it :

$$L(t)=\mu(t)N(t)=\mu(t)N(0)e^{gt}. \tag{3}$$

We point out that μ can be influenced not only through a variation of the number of persons in the labour market, but also by a variation in the ratio of working to total time for each worker. This consideration has great importance for policy making. The real output of each of the two sectors is determined by the entire population's demand for each good

$$Q_i(t)=c_i(t)N(t) \qquad i=1, 2, \tag{4}$$

where $c_i(t)$ is the per capita demand for good i. This can be considered as a Keynesian equation of effective demand.

From equations (1), (2), (3) and (4), we can derive the condition for macroeconomic real equilibrium

$$\mu(t)=l_1(t)c_1(t)+l_2(t)c_2(t). \tag{5}$$

This equation represents the labour force required for the production of the demanded quantities. Observation of the time evolution of μ allows us to determine the dynamics of the labour requirements of the system, and, through this, the dynamics of the potential unemployment. We remind ourselves that $1-\mu$ represents the maximum possible unemployment rate and not its actual value. In fact, variations in the participation rate appear mixed with variations of the unemployment rate in every change of μ. We can consider it to be the equilibrium condition for the labour market, but it is also the equation that binds the real variables at the microeconomic level with the macroeconomic real variables, i. e. the general level of employment (Pasinetti (1993)).

We can also define a corresponding price system by assuming that wages are equal in the two sectors, and grow at a rate corresponding to the increase of productivity in the dynamic sector,

$$w(t)=w(0)e^{\rho_2 t}. \tag{6}$$

The above assumption (i. e., that ρ_2 is the growth rate of the unitary wage) means that we can adopt the price of the dynamic sector as the *numêraire* for the determination of all other relative prices. From this definition of wages we can derive then the equations for natural prices[1]

$$p_1(t)=w(t)l_1(t)=w(0)l_1(0)e^{\rho_2 t}, \tag{7}$$

$$p_2(t)=w(t)l_2(t)=w(0)l_2(0). \tag{8}$$

The condition for the equilibrium of the price system is that per capita income and total expenditures are equal :

$$\mu(t)w(t)=p_1(t)c_1(t)+p_2(t)c_2(t). \tag{9}$$

Since each system yields solutions without the need to consider the other one, it is clear that the two systems are completely independent.

We now introduce into the above specified model the particular hypothesis that per capita demand for each good follows a logistic growth path designed by

$$c_i(t)=c_i(0)\frac{2e^{2r_i t}}{1+e^{2r_i t}}, \qquad i=1,2, \qquad (r_i\geq 0) \tag{10}$$

with saturation level at $2c_i(0)$, to which it tends asymptotically. This assumption differs from Notarangelo (1997), where I consider the exponential equations $c_i(t)=c_i(0)e^{rt}$. Both these functions belong to the two-parameter family of logistic functions[2] given by

1) For an explanation of natural prices, see Pasinetti (1993) or Bortis (1993).

2) Note incidentally that (11) is the solution of the differential equation $\dot{c}=rc(\gamma-(\gamma-1)c)$.

$$c(t)=c(0)\frac{\gamma e^{\gamma rt}}{1+(\gamma-1)\,e^{\gamma rt}} \qquad (\gamma \geq 1). \tag{11}$$

By choosing $\gamma=2$ we have equation (10), while choosing $\gamma=1$ we have the exponential case. The characteristic of this family is that the behaviour at first order for small t does not depend on the parameter γ: i. e., for every γ we have $\dot{c}(0)/c(0)=r$. We can therefore expect that, for small values of t, our analysis will yield identical results, allowing us to connect neatly the two stages of the analysis.

3. Hypothesis of Constant Relative Output

Let us propose the hypothesis that the ratio $Q_1(t)/Q_2(t)$ of the output of the two sectors, or equivalently the ratio $c_1(t)/c_2(t)$ of the per capita demand of the two commodities, is constant over time. This obviously implies $r_1=r_2=r$ and yields, for both per capita demand and per capita output, a growth rate $2r/(1+e^{2rt})$, which is the same at the aggregate and sectoral level. We see that the growth of per capita output tends to zero as time passes (see Fig. 1): this is a direct consequence of the hypothesis of saturation of demand, but it is important to point out that this result is independent of any hypothesis about the production side or the supply of labour force. The case which I examined in Notarangelo (1997), with demand function of the exponential form, yielded the same result only under the specific hypothesis of unbalanced productivity and of zero growth of the employed labour force.

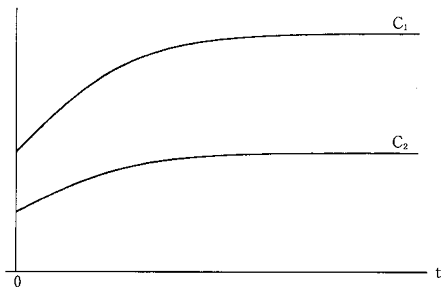


Figure 1. Sectoral Demand for $c_1(0)>c_2(0)$ and $r_1=r_2$.

We can also observe that, in the case of constant relative output, the ratio of the employment of the two sectors follows the same law as in the case of demand without saturation. In fact, from (1), (2) and (10), we obtain

$$\frac{L_1(t)}{L_2(t)}=Ke^{\rho_2 t}, \tag{12}$$

where

$$K\equiv\frac{l_1(0)c_1(0)}{l_2(0)c_2(0)}\ , \tag{13}$$

a result which shows that also in presence of saturation of demand, if the ratio of the demands for the two goods is constant in physical terms, the less progressive sector will tend to absorb the greater part of the labour force.

The dynamics of the employment of the two sectors is, though, a little more complicated than in our simpler case of exponential growth of demand, as now the rate of change is time dependent (see Fig.2). We have

$$\frac{\dot{L}_1(t)}{L_1(t)}=g+\frac{2r}{1+e^{2rt}}, \tag{14}$$

$$\frac{\dot{L}_2(t)}{L_2(t)}=g-\rho_2+\frac{2r}{1+e^{2rt}}, \tag{15}$$

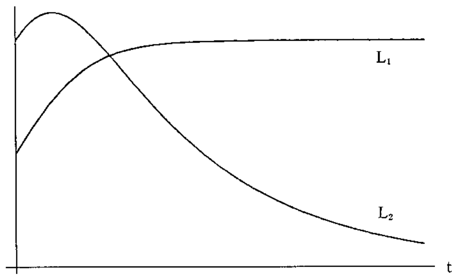


Figure 2. Sectoral Employment for $r_1=r_2>\rho_2$ and $g=0$.

The growth of the employment in the first sector tends to g in the long run, and it is always positive. On the other hand, we see that the growth rate of employment in the progressive sector tends in the long run to $g-\rho_2$, and can be negative, as a consequence of the diminishing need for labour due to technical progress. In the short run, though, its value depends also on the relative magnitude of r and ρ_2. In particular we can say that the employment in this sector will tend to rise initially if

$$g+r>\rho_2. \tag{16}$$

One can then find the growth rate of the total employment by

$$\frac{\dot{L}}{L}=\frac{Ke^{\rho_2 t}}{1+Ke^{\rho_2 t}}\frac{\dot{L}_1}{L_1}+\frac{1}{1+Ke^{\rho_2 t}}\frac{\dot{L}_2}{L_2}. \tag{17}$$

Using (14) and (15) we obtain

$$\frac{\dot{L}(t)}{L(t)}=g+\frac{2r}{1+e^{2rt}}-\frac{\rho_2}{1+Ke^{\rho_2 t}}, \tag{18}$$

and thereby

$$\frac{\dot{\mu}(t)}{\mu(t)}=\frac{\dot{L}}{L}-\frac{\dot{N}}{N}=\frac{2r}{1+e^{2rt}}-\frac{\rho_2}{1+Ke^{\rho_2 t}}. \tag{19}$$

4. Employment Rate

Since a trend towards an increase in the level of unemployment is a problem most developed countries have to deal with, it would be interesting to find which, in our model, are the conditions for positive growth of employment, and to determine the relevant variables for its level in the long run. We enquire therefore into what happens to the aggregate employment rate represented by $\mu(t)$ when the macroeconomic equilibrium is maintained under the present hypotheses of saturation of demand and constant demand in real terms. From (5), the explicit formula is given by

$$\mu(t)=l_1(0)c_1(0)\frac{2e^{2rt}}{1+e^{2rt}}+l_2(0)c_2(0)\frac{2e^{(2r-\rho_2)t}}{1+e^{2rt}}\ . \tag{20}$$

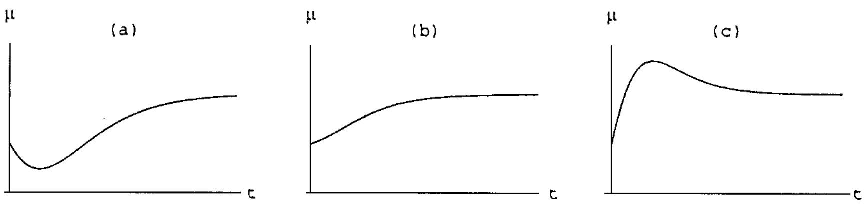
If we define $\bar{\mu}$ as the asymptotic value of μ for $t\to\infty$, we have

$$\frac{\bar{\mu}}{\mu(0)}=\frac{2K}{1+K}\ , \tag{21}$$

where K is defined by (13). This means that the final level of the employment rate is determined by the initial relative size of employment in the two sectors. We easily see that $\bar{\mu}>\mu(0)$ if and only if $K>1$.

However, we are interested also in the specific evolution of the variable in the intermediate range. In order to find it out, we consider the growth rate of μ, that is given by equation (19). The employment growth tends asymptotically to zero as t grows, but before reaching the steady state different dynamics are possible, depending on the values of r, ρ_2 and K.

To fix the ideas, suppose that $K>1$, i. e., that the initial employment in the sector with stagnant productivity is greater than that of the other sector. We can then distinguish three cases for the intermediate dynamics (see Fig. 3):



$r<\rho_2/(1+K)$ $\rho_2/(1+K)\leq r\leq\rho_2/2$ $r>\rho_2/2$

Figure 3. Employment Rate with $K>1$.

We see from (19) that if

$$r>\frac{\rho_2}{1+K}\ , \tag{22}$$

we have $\dot{\mu}(0)>0$, and in this case the initial trend is towards an increase of the employment level (Fig.(3. b) and (3. c)). For small t, in fact, the growth of total demand is able to overrun the negative effect on employment by the technological change in the progressive sector. This is because for small t the growth rate of demand of both sectors r is greater than the growth rate of total productivity $\rho_2/(1+K)$, and thus the total effect on employment is positive. However, this is true for greater t only if

$$\frac{\rho_2}{1+K}<r\leq\frac{\rho_2}{2}\ , \tag{23}$$

when we have a plain monotonic growth (Fig. (3. b)). If we have, instead,

$$r>\frac{\rho_2}{2}\ , \tag{24}$$

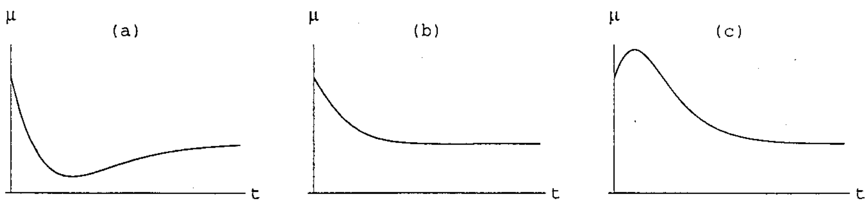
we see from equation (19) that $\dot{\mu}(t)\to 0^-$ for $t\to\infty$, which means that in this case employment grows temporarily larger than the asymptotic value $\bar{\mu}$ to which it eventually decreases (Fig. (3. c)). In fact, the condition $r>\rho_2/2$ is the mathematical condition for the existence of a maximum. For small t, condition (24) assures a positive growth of employment greater than the weaker condition (22), but, for larger t, the contribution to growth of demand, given by the term $2r/(1+e^{2rt})$ in equation (19), decreases more quickly than the negative effect of the productivity increase $\rho_2/(1+e^{\rho_2 t})$, and so there is a regression in the scale of employment. We want to point out that this result depends on the greater influence exercised on the trend by the nonprogressive sector, and is independent of the possible period of positive growth of the employment in the progressive sector, whose condition $r>\rho_2$ is more restrictive than (24).

In the remaining case

$$r<\rho_2\frac{1}{1+K}\ , \tag{25}$$

we have, correspondingly, an initial decrease of the employment rate, due to the prevailing influence of the progressive sector, until the positive growth trend determined by the nonprogressive sector prevails (Fig. (3. a)).

For $K<1$, the relation between $\mu(0)$ and $\bar{\mu}$ is inverted, as well as the relative size of $\rho_2/(1+K)$ and $\rho_2/2$ therefore we can similarly distinguish the three cases (see Fig. 4) :



$r<\rho_2/2$ $\quad$ $\rho_2/2\leq r\leq\rho_2/(1+K)$ $\quad$ $r>\rho_2/(1+K)$

Figure 4. Employment Rate with $K<1$.

5. Income and Budget Constraint

Let us examine briefly the monetary aspects of the model. First, we see that the condition for the equilibrium between total expenditure and per capita income (9) is satisfied if the economy follows the equilibrium path indicated by equation (5), as one can see by substituting the equation for prices (7) and (8). We also can see that, in equilibrium, the per capita income becomes

$$\mu(t)w(t)=w(0)c_1(0)l_1(0)\frac{2e^{(2r+\rho_2)t}}{1+e^{2rt}}+w(0)c_2(0)l_2(0)\frac{2e^{2rt}}{1+e^{2rt}}, \qquad (26)$$

whose rate of growth $2r/(1+e^{2rt})+\rho_2 Ke^{\rho_2 t}/(1+Ke^{\rho_2 t})$ is always positive and tends to ρ_2 for great t. This means that the per capita income does not decrease even in the presence of unemployment, because the loss due to the lower level of employment is fully compensated for by the increase in unitary wage, but it can follow different paths in the short run, according to the different possible values of ρ_2 and especially of r and K.

We can note, moreover, that the positivity of per capita income growth is assured for our two-sector model, but it could be different for a multisectoral model, in which the price chosen as the numéraire is not that of the most progressive sector. In this case, there could be sectors in which the loss of employment is not fully compensated for by the increase in income, and, if not compensated at the aggregate level by a greater increase in the richer sectors, the effect could be of a general loss. We remind ourselves that from the analysis of the price structure in Pasinetti (1993) emerges that the choice of the numéraire has heavy consequences for the stability of the monetary side of the system: usually it is advisable to choose a sector whose productivity rate of change is close to the average of the system in order to avoid structural inflation (or deflation).

6. Comment

We can make several observations with regard to this simple model. First, we see that the hypothesis of unbalanced productivity brings changes in the structure of sectoral employment: in the long run, the nonprogressive sector will absorb a growing share of employment, and this structural transformation can give rise to short term unemployment, as we had already noticed in the analysis of exponential growth.

The particular hypothesis of saturation of demand determines, instead, a slowdown of employment growth in the long run, and often also a period of negative growth. This model, thus, turns out to be able to account for the slowing down of real production in presence of growing unemployment, a situation that Baumol's model with exponential growth was unable to explain. We see therefore that an increase in productivity must be associated with a growth in effective demand in order to increase the well-being of the economic agents. When this condition is not satisfied, we have the situation where the eventual positive growth of per capita income is due only to the growth of employment in the nonprogressive sector. The sector in which there is no productivity growth becomes then the only supporter of both income and employment.

Since in our model neither real production nor per capita income actually diminish, we do not have any reduction of wealth for the system as a whole, even if the level of employment required for equilibrium is lower. Nevertheless, this is true also at the level of single consumers only in the presence of an efficient system of redistribution. We can think about redistribution in different ways. For the present purpose, in fact, it is not really important to determine whether it is a natural phenomena of mutual support among relatives and neighbours, or a redistribution of wealth by a central institution, as long as its effect is to let the whole population share equally the earned income. In fact, since the receivers of income become fewer because of the increase of productivity, at least the members of the population who have lost their jobs will be worse-off with respect to their previous condition if the others do not support them.

Another possible solution would be to " redistribute " jobs instead of income. As we have already noticed, μ can vary with the proportion of persons on the labour market and also with the proportion of laboured to total time. Therefore, the reduction of the working time for an individual person and the corresponding increasing of the number of persons could have the same effect of a redistribution of income.

As an alternative to redistribution, a strong development of less progressive sectors (natural or through governmental intervention) might be helpful in confronting the structural tendency to a slow-down of growth. This choice has its price, though, first because if the support given to the nonprogressive sector has to be supplied artificially by the intervention of the government, it can easily generate a public deficit.[3] Besides, if the influence of the first sector is the most important, while we avoid growing unemployment in any stage of development, we shall have no remarkable growth trend either, as can be well understood by the comparison of case (b) and (c) in Fig.3.

More positive effects could derive from the creation of entirely new productive sectors, whose demand could compensate for the decline of the old ones. In this case the new demand could sustain the growth of both production and employment. This scheme is not without difficulties, because it requires continuous efforts for product innovation, and a continuous stimulus of new correspond-

3) For a detailed analyses of the public sector intervention in the context of this model, see Pasinetti (1993), Chapter 6.

ing needs in the consumers. According to several theories and studies, there is ground to believe that the invention of new products is not a phenomenon continuous in time, but rather, after a burst due to a major innovation, the development of new products follows an adaptive walk with exponential slowing in the rate at which new variants are found.[4] If the slowdown is not followed soon by a new burst of creativity, we can reasonably assume that the saturation trend of demand will take over, and a general slowdown of economic growth will follow. The theory to explain radical innovations is still rather scanty, and it is difficult to foresee if it will be possible to make up for the deficiency in the near future, due to the particular indeterminate nature of the problem.

7. Conclusion

In this paper, I have analyzed the dynamics of a two-sector pure labour model of the kind proposed by Pasinetti (1993). We have seen that the hypothesis of unbalanced productivity associated with that of a constant proportion of the physical output of the two sectors initiates an alteration of the structure of sectoral labour. If the system follows an equilibrium path, these structural dynamics can generate a period of short term unemployment if the trend of the progressive sector to decrease prevails over that of the other.

Introducing the hypothesis of saturation of demand, we see that employment growth tends to slow down in the long run, and that, under certain conditions it is possible to have a period of negative growth. This seems to be consistent with the actual situation faced by many countries which have experienced a slowdown of production in the presence of high unemployment. We discussed briefly the policy implication of our model, pointing out the importance of institutional intervention in the form of redistribution or of stimulus on the development of new or low-productivity sectors.

One limitation of the present model is that it suggests no explanation of the behaviour of the two variables $l_i(t)$ and $c_i(t)$. This problem is rather important, especially for the per capita demand, because we can expect that in reality it is influenced by other variables like the level of income and of prices. An attempt to transform the two exogenous variables, productivity and demand, into dependent variables is therefore particularly interesting as a future extension of the analysis, as could also be the attempt to insert the emergence of new sectors.

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