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Durability and the Optimal Level of Voluntary Export Restraints

Hiroshi Ono

The purpose of this paper is to extend Harris' analysis of VER to include durable goods. We employ a Bulow-Mann-Waldman two-period model which also allows for a used durable goods market, and which also plays a critical role when the foreign firm chooses its output levels before and after the introduction of VER. Contrary to Harris' non-durable goods case when durability is high, a foreign monopolist may increase his profits. As durability decreases, the foreign monopolist's profits also decrease. Furthermore, our model predicts that the foreign firm always increases production before the introduction of a VER.

JEL classification: F1

Key words: VER, durable goods, used goods market, two-period analysis, Stackelberg duopoly game

1. Introduction

Voluntary export restraints (hereafter VER) is one trade policy which has attracted a great deal of attention recently. For instance, Japanese auto-makers voluntarily restrict their exports to the US market, and there are several empirical works which assess the social welfare effects of these VER, mainly on the US economy (see Collynse and Dunaway (1987), Crandall (1987), and Feenstra (1985)). Yano (1989) examined expectations on introduction of VER in detail, while Flam (1994) extended the bilateral duopoly framework to a multilateral version. A particularly interesting idea was provided by Harris (1985), who considered US automakers as price leaders and Japanese auto-makers as price followers within the Stackelberg framework. He has shown that US auto-makers, now price leaders, even reduced their supply, but safely raised auto prices, because the supply of Japanese autos was constant. This interesting paper, however, does not show how the level of the VER is determined. By introducing one of the important aspects of automobiles, i.e., durability, we supplement the Harris analysis. If the number of Japanese autos supplied is limited, US consumers anticipate price hikes of automobiles and purchase them before the VER takes effect.

Discussion on durable goods itself has a long list of contributions. One of the most famous results is the so-called "Coase conjecture," which suggests that a monopolistic supplier faces a time inconsistency (see Coase(1972)). Bulow (1982),
Mann (1992), and Waldman (1993, 1996), among others, developed the two-period model for durable goods, which we employ in this article. In the traditional two-period model, the monopolist supplies durable goods of the same quality in each period. Durable goods supplied in period 1, however, depreciate at a certain constant rate at the beginning of period 2. Therefore, those consumers who have purchased durable goods in period 1 face two choices in period 2—either to continue to keep the used durable good, or to purchase a new one. When they purchase new ones, then the same number of used durable goods is supplied in the secondhand market. In this sense, both the price of newly produced durable goods and the price of used durable goods affect consumer choice. In addition, those consumers who do not purchase durable goods in period 1 may purchase used durable goods, depending on the price of used durable goods and their reservation prices.

If we introduce a VER into this framework, the problem seems easily solved. Under the VER, the monopolist now announces the number of durable goods to be supplied in period 2 at the beginning of period 1 and must keep to his commitment. Even if the monopolist voluntarily restricts his supply in period 2, his commitment must be binding. Therefore, the monopolist now must choose the total supply of durable goods at the beginning of period 1. The monopolist perfectly anticipates the impact of the supply of durable goods in period 2 on the price of newly produced durable goods and on the price of used durable goods. Since the monopolist now foresees future price changes, the monopolist will reduce the amount supplied in period 2 and take into account price increases in both new durable goods and used durable goods. Therefore, the switch from free trade to the VER implies, for the monopolist, a change in the optimization problem from the traditional two-period analysis to an overall profit maximization analysis. Moreover, when the durable goods market is oligopolistic, the situation may drastically change. Suppose that two firms, one home and the other foreign, compete in a Cournot fashion under free trade. Under the VER, the foreign firm is forced to announce his supply of durable goods in period 2 at the beginning of period 1. The domestic firm, however, follows the traditional two-period analysis. This asymmetry in behaviour by the domestic and foreign firms creates, depending on durability, two counteracting forces. On the one hand, the foreign firm takes into account both current and future price changes when he chooses period 2's output level in period 1. As expected in the monopoly case, the foreign firm tends to reduce his supply in period 2, compared with supply in period 2 in the free trade case. On the other hand, since the foreign firm chooses his supply of period 2's output at the beginning of period 1, knowing the domestic firm's reaction in period 2, the foreign firm now becomes a leader and the domestic firm a follower. If the property of durability is not very strong, then the foreign firm produces more as a leader within the Stackelberg framework than in the Cournot one. Therefore, the relative level of durability be-
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comes a critical factor in explaining the optimal level of output under the VER.

Section 2 explains the basic notations used and the behaviour of consumers. Section 3 deals with the monopoly case, which is applicable if commodities concerned have a monopoly market. Section 4 analyzes the duopoly case, where both domestic and foreign firms produce the same durable goods. Finally, section 4 gives brief concluding remarks.

2. Basic Assumptions and Consumer Behaviour

We employ the two-period model used by Mann (1992) and Waldman (1993). Commodities concerned here are assumed durable and last for two periods. Consumers may purchase one unit of a durable good in each period. They are uniformly distributed according to their preference $R$, which ranges from zero to unity. Therefore, consumer $i$’s preference, $R_i$, represents the utility level of consumer $i$ whose preference parameter is located in $[0, 1]$. For simplicity, we assume that durable goods of the same quality are supplied in period 1 and period 2, but the quality of durable goods supplied in period 1 depreciates by $1 - \delta$ in period 2. That is, when consumer $i$ keeps durable goods purchased in period 1, his utility becomes $(1 - \delta) R_i$.

We use the following notations.

$\Pi$: the price of durable goods in period $i$

$P_u$: the price of used durable goods

$x_1$: the number of consumers who purchase a durable good in period 1

$x_2$: the number of consumers who purchase a durable good in period 2

We assume:

A1. $x_1 > x_2$.

This is the traditional case and assumed by Mann (1992). A1 implies that the supply of durable goods in the secondhand market comes solely from consumers who purchased a durable good in period 1 and replaced it for a new one in period 2. This also means that the number of consumers equivalent to $1 - x_1 - x_2$ do not purchase durable goods at all. As for the secondhand goods market, we assume:

A2. The secondhand market is competitive.

First we consider the determination of the price of secondhand durable goods in period 2. Those consumers who purchase used durable goods in period 2 must satisfy the following inequality.

$$(1 - \delta) R_i - P_u \leq 0.$$ 

The number of consumers who want to purchase a used good equals $1 - x_1 - x_2 \geq 0$. 
Therefore, \( P_U \) is determined as follows:

\[
P_U = (1 - \delta)(1 - x_1 - x_2) = D^U(x_1, x_2).
\]  

(1)

Define \( D_U^I = \frac{\partial D_U}{\partial x_1} = -(1 - \delta) < 0 \) and \( D_U^J = \frac{\partial D_U}{\partial x_2} = -(1 - \delta) < 0 \).

Next, we consider the determination of \( P_2 \). Those consumers who purchase durable goods in period 2 must satisfy the following inequality:

\[
R_i - P_2 + P_U \geq (1 - \delta)R_i.
\]

Since the critical location of \( R_j \) equals \( 1 - x_2 \), \( P_2 \) is determined as follows:

\[
P_2 = \delta R_i + P_U = 1 - (1 - \delta)x_1 - x_2 = D^2(x_1, x_2).
\]  

(2)

Define \( D_2^I = \frac{\partial D_2}{\partial x_1} = -(1 - \delta) < 0 \) and \( D_2^J = \frac{\partial D_2}{\partial x_2} = -1 < 0 \).

By denoting the discount rate as \( q \), those consumers who purchase durable goods in period 1 must satisfy the following inequality:

\[
R_i - P_1 + q(1 - \delta)R_i \geq q[(1 - \delta)R_i - P_U].
\]

Since \( R_i \) equals \( 1 - x_1 \) in this case, the price of durable goods in period 1 is given by equation (3).

\[
P_1 = 1 - x_1 + qP_U = [1 + q(1 - \delta)](1 - x_1) - q(1 - \delta)x_2 \]

\[
= D^1(x_1, x_2).
\]  

(3)

Define \( D_1^I = \frac{\partial D^1}{\partial x_1} = -[1 + q(1 - \delta)] < 0 \) and \( D_1^J = \frac{\partial D^1}{\partial x_2} = -q(1 - \delta) < 0 \).

It should be noted that the price of used durable goods tends to push up both current and future durable goods prices. Furthermore, when goods are not durable (\( \delta = 1 \)), the secondhand market does not exist.

3. The Monopoly Case

Suppose that durable goods are supplied by a foreign monopolist whose domestic production is assumed to be zero. For analytical simplicity, we assume that the monopolist’s cost function has the following characteristic:

A3. Constant (zero) marginal cost and no fixed cost.

As for the choice variable, we assume that

A4. Quantity is the choice variable.

Consider a typical monopolist’s behaviour within the traditional two-period framework. The monopolist solves the two-period problem backward. In period 2 the monopolist determines that the amount to be produced, which maximizes his
profits during the same period, will be solved as a function of the amount produced in period 1. Thus the monopolist maximizes overall profits and selects the optimal amount of output for period 1.

Now we state the profit function in each period.

\[ \pi^i = D^i(x_1, x_2)x_i. \quad [i=1, 2] \]

Consider the optimization problem in period 2. The necessary condition will be expressed by equation (4).

\[ \frac{\partial \pi^2}{\partial x_2} = p^2 + D^2_2 x_2 = 0. \quad (4) \]

Substituting equation (2) into equation (4) yields the optimal choice of \( x_2 \).

\[ x_2^F = \frac{1 - (1 - \delta)x_1}{2} = F(x_1). \quad (5) \]

Superscript \( F \) in \( x_2^F \) stands for "free trade." Define \( F_1 = \frac{\partial F(x_1)}{\partial x_1} = \frac{1 - \delta}{2} < 0 \).

The price of the used durable goods is easily determined as follows:

\[ p_u^{F} = \frac{(1 - \delta)^2(1 - \frac{1 - \delta}{2}q)}{2[2 + (1 - \delta^2)q]} . \]

When the commodities concerned are not durable, the equilibrium quantity in each period will be derived by substituting \( \delta = 1 \) and \( p_u = 0 \). We summarize the above results as Lemma 1.

**Lemma 1.** (Free trade equilibrium in the case of durable goods monopoly)

Compared to the non-durable goods case, the durable goods monopolist produces less output during each period and creates price increases through the existence of the secondhand market.

The overall profit function will be shown as follows:

\[ \pi^2 = D^1(x_1, F(x_1))x_1 + qD^2(x_1, F(x_1))F(x_1). \]

We can derive the following first order condition for maximizing overall profits:

\[ \frac{d \pi}{dx_1} = D^1 + (D^1_1 + D^1_2 F_1)x_1 + qD^2_2 F(x_1) = 0. \]

Using equation (2), (3) and (5), we obtain the optimal value of \( x_1 \).

\[ x_1^F = \frac{1 + \frac{1}{2}(1 - \delta)q}{2 + (1 - \delta^2)q} . \quad (6) \]

Substituting this into equation (5), we obtain
\[ x_2^F = \frac{1 + \delta^2}{2 + (1 - \delta^2)q} \cdot \frac{1}{1 + \frac{1}{2}(1 - \delta)q} \cdot \frac{1}{2[2 + (1 - \delta^2)q]} \]. 

(7)

It should be noted that when the commodities concerned are not durable (\( \delta = 1 \)), then the monopolist produces the same amount of output without taking into account the price changes of used commodities.

Next, suppose that the durable goods monopolist cannot choose in period 2 the number of new durable goods to be supplied in period 2, but rather he is forced to declare it in period 1. This kind of voluntary choice seems credible because the importing country expects the monopolist to keep to his commitment. Then it is reasonable to assume that the monopolist will maximize his overall profits and simultaneously determines \( x_1 \) and \( x_2 \) in period 1.

We solve the following maximization problem with respect to \( x_1 \) and \( x_2 \):

\[
\max \pi = \pi^1 + q \pi^2 = D^1(x_1, x_2)x_1 + qD^2(x_1, x_2)x_2.
\]

We have the following first order conditions:

\[
\frac{\partial \pi}{\partial x_1} = P_1 + D_1^1x_1 + qD_1^2x_2 = 0.
\]

\[
\frac{\partial \pi}{\partial x_2} = D_2^1x_1 + q(P_2 + D_2^2x_2) = 0.
\]

Simple calculation can derive the equilibrium values of \( x_1 \) and \( x_2 \).

\[
x_1^V = \frac{1}{2[1 + \delta(1 - \delta)q]}.
\]

(8)

\[
x_2^V = \frac{\delta[1 + (1 - \delta)q]}{2[1 + \delta(1 - \delta)q]}.
\]

(9)

Comparing equations (6) and (7) with (8) and (9), we can derive Lemma 2.

**Lemma 2**

For \( 0 < \delta < 1 \), \( x_1^V < x_1^F \) and \( x_2^V < x_2^F \).

Lemma 1 says that when a durable goods monopolist must commit to \( x_2 \) in period 1, he will choose the amount of \( x_2 \) when it is less than that in a free trade case in period 1, so as to maximize overall profits. This possibility occurs only when dealing with durable goods. Letting \( P_i^F = D^i(x_1^F, x_2^F) \) and \( P_i^V = D^i(x_1^V, x_2^V)(i = 1, 2, U) \), we have Lemma 2.

**Lemma 3**

For \( 0 < \delta < 1 \), \( P_i^V > P_i^F \), \( P_2^V > P_2^F \), and \( P_U^V > P_U^F \).
When the monopoly exporter encounters a political situation where he cannot choose the level of future production at a future date and must voluntarily restrict his future production, he will rationally commit to announcing at current date the future production level which maximizes overall profits. Lemmas 2 and 3 show that the monopolist successfully reduces future production in keeping with his commitment and increases his overtime profits. By switching future production to the present, the future price of durable goods goes up. Since the supply of secondhand durable goods in the future decreases, this raises the price of secondhand goods and also pushes up their current price. We summarize this section by Proposition 1.

Proposition 1
Suppose that the foreign firm monopolizes the domestic durable goods market. When the foreign firm is forced to involuntarily restrict his exports, the monopolist is committed to announcing future production levels. Then the monopolist may rationally maximize his overall profits, resulting in a decrease in future output which raises both current and future prices. Since every commodity price rises, consumer surplus decreases, while the foreign monopolist may increase his profits compared to that in the free trade case.

4. The Duopoly Case
Two firms produce a homogeneous durable good. Firm $i$ is a domestic firm and Firm $j$ a foreign firm. For simplicity, we continue to assume A3 and A4. A3 implies that there is no cost advantage to either firm. Furthermore, we add that

A5. Each firm behaves in a Cournot fashion.

First, we consider the traditional two-period case where the second period production level is decided in period 2 and the first period production level is chosen so as to maximize overall profits. Then, we investigate the case where Firm $j$ must commit the level of period 2 production in period 1.

A. Free Trade Case
Consider the optimal behaviour of Firm $i$ in period 2. Firm $i$ chooses $x_1$ to maximize second period profits.

$$x^*_i = D^2(x_1, x_2) x_{2i}$$

where $x_{2i}$ denotes the amount of output produced by Firm $i$ in period 2. By substituting equation (2) into the profit function and maximizing profits in period 2, we can derive equation (10).

$$x_{2i} = \frac{1 - (1 - \delta) x_1 - x_{2i}}{2}.$$  \hspace{1cm} (10)
\( x_{2j} \) also is similarly derived. Then the Cournot-Nash subgame perfect equilibrium in period 2 will be determined as equation (11).

\[
x_{2k}^F = \frac{1-(1-\delta)x_1}{3} = F^k(x_1) \quad [k=i, j]
\]

Now we consider the maximization problem in period 1. Using equation (11), we can express the overall profit function as a function of both \( x_{1i} \) and \( x_{1j} \).

\[
\pi_i = D^1(x_1, F^i(x_1))x_{1i} + qD^2(x_1, F^i(x_1))F^i(x_1).
\]

The first-order condition is given by equation (12).

\[
\frac{\partial \pi_i}{\partial x_{1i}} = D^1 + D^1 x_{1i} + qD^2 F^i(x_1) = 0,
\]

which yields the following relation:

\[
a_1 - 2b_1 x_{1i} - c_1 x_{1j} = 0,
\]

where \( a_1 = 1 + \frac{(1-\delta)(1+8\delta)}{9} q, \) \( b_1 = 1 + \frac{(1-\delta)(2+7\delta)}{9} q, \) and \( c_1 = 1 + \frac{(1-\delta)(1+8\delta)}{9} q. \)

Similarly,

\[
a_1 - 2b_1 x_{1j} - c_1 x_{1i} = 0.
\]

It should be noted that \( b_1 > c_1 > a_1 \) for \( 0 < \delta < 1 \). If we do not assume any durability \( (\delta = 1) \), then \( b_1 = c_1 = a_1 \). Because of symmetry,

\[
x_{1i}^F = x_{1j}^F = \frac{a_1}{2b_1 + c_1} = \frac{1 + \frac{(1-\delta)}{9} q}{3 + \frac{(1-\delta)(5+22\delta)}{9} q}.
\]

Substituting this into equation (11) yields

\[
x_{2i}^F = x_{2j}^F = \frac{1 + 2\delta + \frac{(1-\delta)(1+8\delta)}{9} q}{3[3 + \frac{(1-\delta)(5+22\delta)}{9} q]}.
\]

Note that if the commodities concerned are not durable, both domestic and foreign firm production is one third in each period (the non-durable Cournot case). We can easily prove Lemma 4.

**Lemma 4**

\[
x_{1i}^F = x_{1j}^F < \frac{1}{3} \quad \text{and} \quad x_{2i}^F = x_{2j}^F < \frac{1}{3}.
\]

When the commodities concerned are durable, the monopolist takes into account the
effect of price changes caused by used goods at the current price. By reducing the
supply of newly produced goods in period 2, a decrease in supply of used goods in
period 2 raises the prices of used goods, which tends to increase prices in both
period 1 and period 2.

B. The VER Case

Consider the case where foreign Firm \( j \) must commit in period 1 his supply of
output in period 2. In the previous section where durable goods are supplied solely
by the foreign monopolist, then the VER level of production is always less than the
free trade level of production in period 2. In the duopoly case, the necessary condi-
tion for profit maximization in period 2 is the same as in the free trade case for Firm
\( i \), which is given by equation (10).

\[
x_{2i} = \frac{1 - (1 - \delta)x_{1i} - x_{2j}}{2} = G(x_1, x_2).
\]

Define \( G_1 = \frac{\partial G}{\partial x_1} = -\frac{1 - \delta}{2} < 0 \) and \( G_2 = \frac{\partial G}{\partial x_{2j}} = -\frac{1}{2} < 0 \).

Firm \( j \) now commits by announcing at the beginning of period 1 his supply of
durable goods in period 2. It should be noted that by abandoning his choice of output in
period 2, Firm \( j \) can now behave as a leader in a Stackelberg duopoly model during
period 2.

We now consider the optimal decisions for period 1. Firm \( i \)'s overall profit
function is given by equation (13).

\[
\pi_i = D^1(x_1, G^1(x_1, x_{2j}))(x_{1i} + qD^2(x_1, G^1(x_1, x_{2j}))x_{1i} + qD_1^2 = 0.
\]

Differentiating the profit function with respect to \( x_{1i} \) yields the following first-order
condition:

\[
\frac{\partial \pi_i}{\partial x_{1i}} = D^1 + \{D^1_i + D^1_jG_{ij}\}x_{1i} + qD^2 + D^2_jG_{ij}x_{1i} = 0.
\]

Rearranging yields equation (14).

\[
1 - 2\alpha x_{1i} = \beta x_{1j} = 0,
\]

where \( \alpha = 1 + \frac{(1 - \delta)(1 + 3\delta)}{4} q \) and \( \beta = 1 + \delta(1 - \delta)q \).

Firm \( j \)'s overall profit function is given by equation (15).

\[
\pi_j = D_1(x_1, G^1(x_1, x_{2j}))(x_{1j} + qD^2(x_1, G^1(x_1, x_{2j}))x_{1j} + x_{2j}x_{2j} = 0.
\]

Differentiating with respect to \( x_{1j} \) and \( x_{2j} \) yields

\[
\frac{\partial \pi_j}{\partial x_{1j}} = D^1_i + \{D^1_j + D^1_jG_{ij}\}x_{1j} + q(D^2_i + D^2_jG_{ij})x_{1j} = 0.
\]

\[
\frac{\partial \pi_j}{\partial x_{2j}} = D^2_i[G^2_i + 1]x_{1j} + q(D^2_i + D^2_j[G^2_i + 1])x_{2j} = 0.
\]
Using the \( G^i \) function and equation (15), we obtain the following:
\[
x_{2i} = \frac{1}{2} - \frac{1 - \delta}{2} x_{1i} - (1 - \delta) x_{1j}.
\]
Substituting this into equation (14) yields equation (18).
\[
1 - 2 \beta_1 x_{1j} - \beta_1 x_{1i} = 0.
\]
Using equations (12) and (16), we can simultaneously solve \( x_{1i} \) and \( x_{1j} \):
\[
x_{1i} = \frac{1}{3 + (1 - \delta)(1 + 2 \delta)q}, \quad x_{1j} = \frac{1}{1 + \frac{1 - \delta}{2} q}
\]
\[
[1 + \delta (1 - \delta) q][3 + (1 - \delta)(1 + 2 \delta)q]
\]
Substituting the above into \( x_{2i} \) and \( x_{2j} \), we obtain the following equilibrium solutions:
\[
x_{2i}^* = \frac{2 + \delta + (1 - \delta)(1 + 2 \delta)q}{4[3 + (1 - \delta)(1 + 2 \delta)q]}, \quad x_{2j}^* = \frac{\delta [1 + (1 - \delta)q]}{2[1 + \delta (1 - \delta)q]}
\]
Comparing \( x_{2i} \) and \( x_{2j} \) with \( x_{2i}^* = x_{2j}^* \), we may derive Proposition 2.

Figure 1. Equilibrium in Period 2
Proposition 2

Suppose that the commodities concerned are sufficiently durable. Then the foreign exporter voluntarily restricts his supply of output in period 2. That is, for \( \delta < \delta^* \), the following relation holds:

\[
x^F_{2j} < x^F_{2i} = x^V_{2j} < x^V_{2i},
\]

Where \( \delta^* = \min(\delta_1, \delta_2) \), \( \delta_1 = \{ \delta (q) : x^F_{2i} = x^V_{2i} \} \), and \( \delta_2 = \{ \delta (q) : x^F_{ij} = x^V_{ij} \} \).

(For proof see Appendix)

In order to understand Proposition 2, we use Figure 1. Point A shows the Cournot equilibrium in the non-durable goods case. Points B and C respectively show the equilibrium quantity of foreign Firm \( j \) as a leader and the equilibrium quantity of domestic Firm \( i \) as a follower in the non-durable goods case. As durability increases (\( \delta \to 0 \)), foreign Firm \( j \) seriously considers the interaction of price changes.

Proposition 2 suggests that the commitment of a VER by a foreign firm can be reasonably explained by introducing the durability property of commodities concerned. Proposition 1 is related to Lemma 1 in the monopoly case. In the latter case, rational behaviour by the foreign monopolist reasonably explains the VER activity, where the volume of exports is less in the VER case than in the case of free trade. In a duopoly case, we find another important factor, which influences firm behaviour. When the foreign firm abandons the choice of output in period 2 and is forced to commit to announcing its output level in period 2 at the beginning of period 1, it comes to behave like a leader in the Stackelberg case. Supposing that the commodities are not so durable, so that \( \delta = 1 \), then

\[
\delta_2 = \{ \delta (q) : x^F_{ij} = x^V_{ij} \} > x^F_{2i} = x^V_{2i} = \frac{1}{3} > x^F_{2i} = \frac{1}{4}.
\]

As \( \delta \) approaches 0, the validity of Proposition 2 increases. From Proposition 2, we have Corollary.

Corollary

Let us suppose that the commodity concerned does not have strong durability. If the foreign firm undertakes VER, then it involuntarily chooses a volume of exports less than the amount in a free trade case. Next, consider equilibrium quantities in period 1 and equilibrium prices.

Lemma 5. (Equilibrium quantities in period 1)

We have the following relations:

(a) \( x^V_{ij} > x^V_{1i} \), and (b) \( x^F_{ij} > x^F_{1i} \).

Furthermore, for reasonably strong durability (\( \delta < \frac{3}{4} \)), (c) \( x^V_{ij} > x^F_{ij} \).
Lemma 5 says that irrespective of durability, foreign Firm $j$ as a leader in period 2 produces more than domestic Firm $i$ as a follower. Since property (b) in Lemma 5 always holds, the foreign firm can enjoy price increases. Lemma 5 also implies $x_{i2}^F > x_{i1}^F$, always.

By limiting our interest on the range of $\delta$ less than $\delta^*$, we obtain Lemma 6.

**Lemma 6 (Price comparison)**

\[(a) \: p_1^Y > p_1^F, \: (b) \: p_2^Y > p_2^F, \text{ and } (c) \: p_{12}^Y > p_{12}^F.\]

**(Proof)**

We know that $x_1^Y < x_1^F$. Then the definitions of commodity prices imply Lemma 6.

5. **Concluding Remarks**

In this paper, we introduced a new factor, durability, into the trade model. We have shown the possibility that a foreign firm will voluntarily undertake VER. For the strong durability case, if the foreign firm restricts his supply in period 2 to less than free trade, it can enjoy greater profits through price increases, including price rises in used durable goods. Contrary to Harris (1992), the domestic firm increases its share in period 2 and definitely enjoys an increase in profits due to the VER. However, this paper also allows for cases in which the foreign firm involuntarily restricts exports. If the property of durability is not strong, the foreign firm may stick to the free trade level of production, in which case the Harris results hold.

*Professor of Economics, Hokkaido Univ.*

**References**


**Appendix**

1. We show that $x_{ij}^V > x_{ij}^F = x_{ij}^V > x_{ij}^F$. Note that $x_{ij}^V = x_{ij}^F = \frac{A_1}{B_1}$, $x_{ij}^V = \frac{A_2}{B_2}$, and $x_{ij}^V = \frac{A_3}{B_3}$, where $A_1 = 1 + \frac{1 - \delta}{9} q$, $B_1 = 3 + \frac{(1 - \delta)(5 + 22 \delta)}{9} q$, $A_2 = 1$, $B_2 = 3 + (1 - \delta)(1 + 2 \delta) q$, $A_3 = 1 + \frac{1 - \delta^2}{2} q$, and $B_3 = [1 + \delta(1 - \delta)q]B_2$.

(a) $x_{i1}^V < x_{i1}^F$
\[
x_{i1}^V - x_{i1}^F = \frac{1}{B_2B_3} \frac{(1 - \delta)2}{2} q > 0.
\]

(b) $x_{ij}^V$ and $x_{ij}^F$
\[
x_{ij}^V - x_{ij}^F = \frac{1}{B_1B_3} \left\{ \frac{(1 - \delta)(13 - 25 \delta)}{18} q + \frac{(3 - 7 \delta)(2 \delta + 1)}{18} q^2 \right\} + \frac{1 - \delta}{9} (3 - \delta(1 - \delta)q(1 + 2 \delta)q^2).
\]

If $\delta < \frac{3}{7}$, then $x_{i1}^V > x_{i1}^F$.

(c) $x_1^V$ and $x_1^F$
\[
x_1^V - x_1^F = \frac{2}{B_1B_3} \left\{ \frac{11 \delta(1 - \delta)}{108} q + \frac{(1 - \delta)q}{108} [1 - (1 - \delta)q] \right\} + \frac{\delta(1 - \delta)^2(19 + 6 \delta)}{108} q^2 + \frac{\delta(1 - \delta)^3(1 + 2 \delta)}{27} q^3 < 0.
\]

2. Proof of Proposition 2

First, we show that $x_{2j}^F$, $x_{2j}^V$, and $x_{2j}^Y$ are increasing functions of $\delta$. Note that
\[
x_{2j}^F = x_{2j}^V = \frac{1 + 2 \delta + \frac{(1 - \delta)(1 + 8 \delta)}{9} q}{3[3 + \frac{(1 - \delta)(5 + 22 \delta)}{9} q]}.
\]
\[ x_{2i}^V = \frac{2 + \delta + (1 - \delta)(1 + 2 \delta)q}{4[3 + (1 - \delta)(1 + 2 \delta)q]}, \text{ and} \]
\[ x_{2j}^V = \frac{\delta[1 + (1 - \delta)q]}{2[1 + \delta(1 - \delta)q]}. \]

\[ \frac{\partial x_{2j}^V}{\partial \delta} = \frac{R_1}{Q_1}, \text{ where } R_1 = 6 + \frac{(1 - \delta)(56 - 44 \delta)q}{9} + \frac{(1 - \delta)^2q^2}{27} > 0, \text{ and} \]

\[ Q_1 = 3[3 + \frac{(1 - \delta)(5 + 22 \delta)q}{9}] > 0. \]

\[ \frac{\partial x_{2i}^V}{\partial \delta} = \frac{R_2}{Q_2}, \text{ where } R_2 = 1 + (1 - \delta)^2q > 0, \text{ and } Q_2 = 2[1 + \delta(1 - \delta)q]^2. \]

\[ \frac{\partial x_{2i}^V}{\partial \delta} = \frac{R_3}{Q_3}, \text{ where } R_3 = 4[3 + (1 - \delta)(1 + 2 \delta)q]^2, \text{ and } Q_3 = 3 + 2(1 - \delta)^2 > 0. \]

Therefore, \( x_{2i}^V, x_{2j}^V, \) and \( x_{2k}^V \) will be depicted as in Figure 1.

Next, we shall derive sufficient conditions for \( x_{2i}^V > x_{2j}^V \), that is, in such a case where the level of the VER is rationally chosen. We can derive the following:

\[ x_{2j}^V - x_{2i}^V = \frac{R_4}{Q_4}, \]

where \( R_4 = \frac{5 \delta - 2}{3} + \frac{(1 - \delta)(10 \delta^2 + 10 \delta - 2)}{9} + \frac{(1 - \delta)^2(1 + 2 \delta)q^2}{3} \) and \( Q_4 = 6[1 + \delta(1 - \delta)q][1 + \frac{(1 - \delta)(5 + 22 \delta)q}{27}] \).

We can easily check that if \( d < -\frac{-5 + 3\sqrt{5}}{10} = 0.169 \), then the second term in \( R_4 \) is negative, and the first and third terms together are also negative.

Finally, we derive a sufficient condition for \( x_{2i}^V > x_{2k}^V \). Simple calculation yields

\[ x_{2i}^V - x_{2k}^V = \frac{R_5}{Q_5}, \]

where \( R_5 = \frac{2 - 5 \delta}{4} + \frac{(1 - \delta)q}{3}[13 - 41 \delta - 26 \delta^2] + \frac{(1 - \delta)^2(1 + 2 \delta)(1 - 10 \delta)q^2}{9} \) and \( Q_5 = [3 + (1 - \delta)(1 + 2 \delta)q][3 + \frac{(1 - \delta)(5 + 22 \delta)q}{9}] \).

\( R_5 \) can be arranged as follows:

\[ R_5 > \frac{6 - 25 \delta}{20} + \frac{(1 - \delta)q}{36}[13 - 41 \delta - 26 \delta^2] + \frac{(1 - \delta)^2(1 + 2 \delta)(1.6 - 10 \delta)q^2}{9}. \]

The above was derived by nothing that

\[ (1 - \delta)^2(1 + 2 \delta)q^2 < \frac{1}{3}. \]

Therefore, for \( \delta < 0.16 \), \( x_{2i}^V > x_{2k}^V \).