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Adjustment Costs and an Overlapping Generations Model

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In researches into the standard overlapping generations model, the firm's intertemporal planning such as equip investment has not been a matter of main concerns since the seminal work by Diamond [4]. This paper examines the dynamic equilibrium and stability of an overlapping generations model in which a firm's flow investment is explicitly considered. In order to formulate an extended model, we introduce adjustment costs, equity financing by firms, and household saving by stock holding. These modifications complicate dynamics of an overlapping generations economy and have a negative effect on the steady-state capital per capita compared to results in the standard model.

1. Introduction

In this paper, we examine the problem of how equilibrium conditions and steady state results are formulated if we allow for the existence of adjustment costs of investment in an overlapping generations model. In researches into the standard overlapping generations model with productive capital (for example, see chapter 3 of Blanchard and Fisher [3] and Galor and Ryder [5]), decision of a flow investment demand by firms has not been properly formulated. In most cases, the firm's intertemporal planning such as equip investment has not been a matter of main concerns since the seminal work of Diamond [4]: the firm's objective is assumed to maximize the one-period profit under the perfect mobility of labor and capital. As a result, the amount of capital employed in production is determined completely by the static considerations in each period and it is well known that finiteness of planning horizons in both parts of individuals and firms leads to possibility of dynamic inefficiency of capital accumulation in an overlapping generations framework.

In a model proposed in this paper, the firm's investment demand is explicitly derived on the basis of Tobin's q theory. We assume that costs to buy a physical good for investment are financed by new issuance of stocks. Then, as Bailey and Scarth [2] pointed out, rents are generated in the stock market because the investing firm can raise funds more than actual costs. If the firm has no retained earnings, the rents are added to stockholders' disposable income. As for an individual's behavior, we assume that saving is carried out in the form of purchasing and holding stocks that are financial claims to dividends. In what follows, we show that introduction of adjustment costs, equity financing by firms, and household saving by stock holding complicates dynamics of an overlapping generations economy and has a negative effect on the steady-state capital per capita.

The rest of paper is organized as follows. In Section 2 , we briefly describe the standard overlapping generations model in order to make reference and comparison easy and clear. In Section 3 , we propose a modified model and formulate agents' maximization problems. Section 4 discusses the equilibrium and steady state of the model and Section 5 concludes the paper.

2. The Standard Overlapping Generations Model

In order to make our discussions intelligible and clear, we begin with reviewing structures and dynamics of the standard one-good overlapping generations model in which no explicit considerations are taken into adjustment cost of investment.

The two-period lived population in the economy grows at a positive constant rate $n>0$: $L_{t+1}=(1+n)L_t$, $t\geq 0$. The number of the initial old people L_0 and the initial level of capital K_1 are given. The representative individual of generation t supplies one unit of labor inelastically and receives the wage income w_t in period t when he is young. He allocates the labor income between consumption c_t^t and savings s_t. The individual carries out saving by purchasing s_t amount of productive capital when he is young and then renting it to the firm for one period to earn the rental income $r_{t+1}s_t$ where r_{t+1} is one period rate of return for renting capital in period $(t+1)$. Then, his total income in the retiring period $(t+1)$ is the sum of rental $r_{t+1}s_t$ and sales revenue of capital s_t. He spends this total amount of income on consumption c_{t+1}^t without any altruistic or bequest motives. We assume a simple log-linear type of utility function. Then the lifetime utility maximization problem of the representative individual of generation t is written to

$$\max U(c_t^t, c_{t+1}^t)=\ln c_t^t+\beta \ln c_{t+1}^t,\ 0<\beta<1, \tag{1}$$

$$\text{s. t. } c_t^t+s_t\leq w_t \tag{2}$$

$$c_{t+1}^t\leq(1+r_{t+1})s_t, \tag{3}$$

where β is a discount factor.

Solving this lifetime utility maximization problem, we have the saving function

$$s_t=\{\beta/(1+\beta)\}w_t \tag{4}$$

A firm is assumed to produce under the Cobb-Douglas function $F(K, L) = K^{\alpha}L^{1-\alpha}$, $0<\alpha<1$ and the profit maximization conditions in period t are spelled out in per capita forms as follows:

$$f'(k_t) = \alpha k_t^{\alpha-1} = r_t \tag{5}$$
$$f(k_t) - k_t f'(k_t) = (1-\alpha)k_t^{\alpha} = w_t \tag{6}$$

where $f(k_t) = F(k_t, 1)$ and $k_t = K_t/L_t$.

Together with the budget constraints (2)-(3) and the firm's profit maximization conditions (5)—(6), the good market clearing condition in period $(t+1)$:

$$L_t c_{t+1}^{t} + L_{t+1}(c_{t+1}^{t+1} + s_{t+1}) = F(K_{t+1}, L_{t+1}) + K_{t+1} = L_{t+1}\{f(k_{t+1}) + k_{t+1}\} \tag{7}$$

implies $s_t = (1+n)k_{t+1}$. From this equation and the saving function (4), the fundamental equation of the standard overlapping generations model is derived as follows:

$$k_{t+1} = \frac{s_t}{1+n} = \frac{(1-\alpha)\beta}{(1+n)(1+\beta)} k_t^{\alpha}. \tag{8}$$

The non-trivial steady state value of per capita capital is

$$k^* = \left\{ \frac{(1-\alpha)\beta}{(1+n)(1+\beta)} \right\}^{\frac{1}{1-\alpha}}. \tag{9}$$

Linearizing the fundamental equation around the steady state, we have

$$k_{t+1} - k^* = \alpha(k_t - k^*). \tag{10}$$

Because $0<\alpha<1$, the steady state is locally stable.[1]

3. A Modified Model

In this section, we modify the standard overlapping generations model described in the previous section by introducing adjustment costs of investment. The behavior of the representative individual of generation t is the same as in the standard model except for the way of saving. When the individual is young, he allocates labor income between consumption c_t^t and purchase of s_t units of stocks. That is, saving is carried out by buying stocks at the current market price q_t and by holding them for one period. When he is old in period $(t+1)$,

1 Actually, the steady state is globally stable under the present specification of the standard model. For the existence and stability conditions, see Galor and Ryder [5].

he retires from work and his income is the sum of sales revenue of stocks $q_{t+1}s_t$ and dividends $d_{t+1}s_t$. He spends this income on consumption of c^t_{t+1} during the retiring period.

The lifetime utility maximization of the representative agent of generation t is written as follows :

$$\max U(c^t_t, c^t_{t+1}) = \ln c^t_t + \beta \ln c^t_{t+1}, \quad 0 < \beta < 1$$

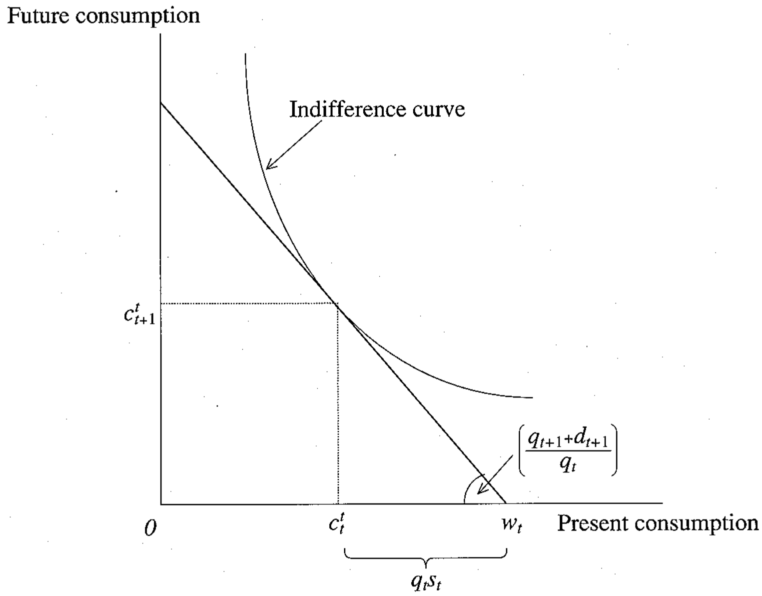
$$\text{s. t. } c^t_t + q_t s_t \leq w_t, \tag{11}$$

$$c^t_{t+1} \leq (q_{t+1} + d_{t+1}) s_t, \tag{12}$$

where β is the discount factor. Refer to Figure 1 for the graphical representation of this decision problem. Solving the above maximization problem, we have the saving function (or the stock demand function)

$$s_t = \beta (1+\beta)^{-1} (w_t / q_t). \tag{13}$$

Figure 1 . Consumption and Savings of the Representative Individual of Generation t



We assume that the firm undertakes investment as well as produces a good using labor and capital under the same Cobb-Douglas production function $F(K, L) = K^{\alpha} L^{1-\alpha}$, $0 < \alpha < 1$ as in Section 2. The firm finances investment expenditures by issuing stocks that sells at the current market price q_t in period t. Besides, retained earnings are assumed not to be held within the firm so that all profit and rent are distributed to the stockholders. The firm's objective is to maximize the dividend D in each period that is composed of profit and rent.

$$\max_{\{L_t,\ I_t\}} D_t = \underbrace{\{F(K_t, L_t) - w_t L_t\}}_{\text{profit}} + \underbrace{[q_t I_t - \{I_t + C(K_t, I_t)\}]}_{\text{rent}} \tag{14}$$

Note that the dividend is the sum of products net of wage payments and rents generated in the stock market.

In the present study, we use following form of adjustment cost function :

$$C(K, I) = aI^2/K, a > 0 \quad \text{for} \quad I \geq 0, \tag{15}$$

where negative investment is assumed to be impossible. Then, the necessary conditions for dividend maximization are

$$(1-\alpha) k_t^a = w_t \ , \tag{16}$$

$$1 + (2aI_t/K_t) = q_t \ , \tag{17}$$

where $k_t = K_t/L_t$.

The left-hand side of (17) stands for the marginal cost of investment that is abbreviated to MCI in Figure 2 . From this condition and assumed irreversibility of investment, we have the following Tobin's q investment function.

$$I_t = \begin{cases} (2a)^{-1}(q_t - 1)K_t > 0, & \text{if } q_t > 1 \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

From this result, we have the capital accumulation equation

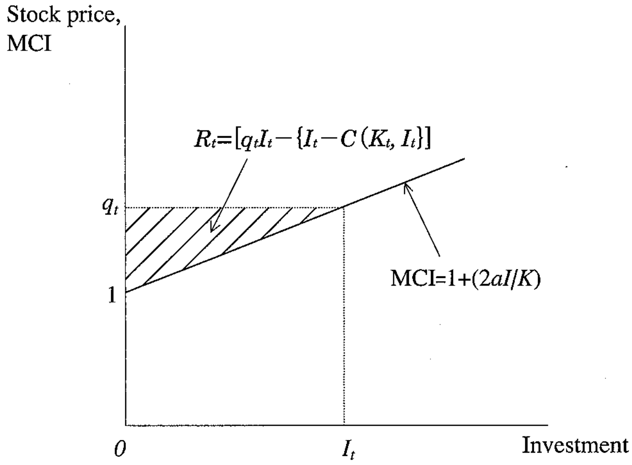
$$K_{t+1} = \{1 + (2a)^{-1}(q_t - 1)\} K_t \ . \tag{19}$$

The amount of rent generated in the stock market is calculated as follows :

$$R_t = (4a)^{-1}(q_t - 1)^2 K_t \ . \tag{20}$$

Figure 2 shows determination of the firm's investment and the amount of rent generated in the stock market.

Figure 2. Determination of the Firm's Investment and Rent Generated in the Stock Market



4. Steady State and Stability

In our modified model, the good market clearing condition in period t is

$$L_{t-1}c_t^{t-1}+L_tc_t^t+I_t=F(K_t, L_t)-C(K_t, I_t). \tag{21}$$

This condition means that products net of adjustment costs are available for consumption and investment. On the other hand, the stock market equilibrium condition is

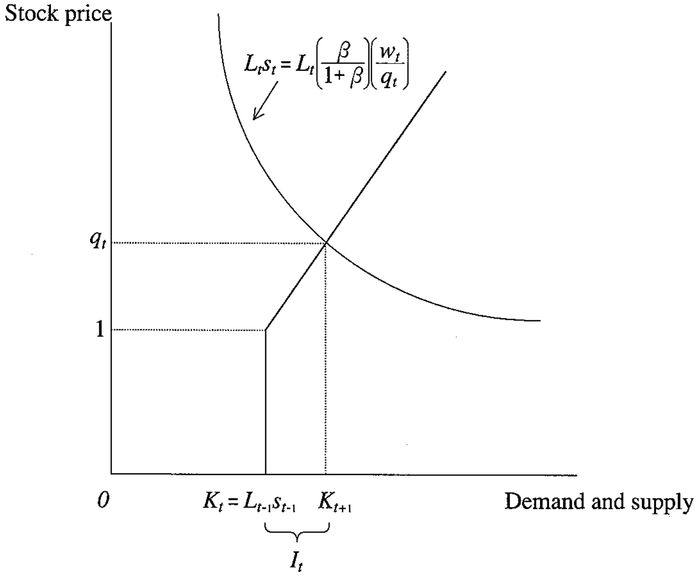
$$L_{t-1}s_{t-1}+I_t=L_ts_t\ , \tag{22}$$

which is illustrated in Figure 3. The left-hand side of this condition represents that the total stock supply is the sum of sales by the old generation and new issuance by the investing firm. An equilibrium requires the total supply to be equal to demands by the young who plan to save. It is readily seen that this condition is equivalent to the capital accumulation equation $(K_t+I_t=K_{t+1})$ because all units of claims to productive capital are owned by the current old in each period, i.e., $L_{t-1}s_{t-1}=K_t$, $\forall\, t\geq 1$. By Walras law, if budget constraints of individual and firm are satisfied, these two equilibrium conditions (21) and (22) are not independent from each other. Therefore, the equilibrium path is characterized by the sequence $\{k_t, q_t\}_{t=1}^{\infty}$ that satisfies the following set of difference equations:

$$k_{t+1}=(1+n)^{-1}\{(2a)^{-1}(q_t-1)+1\}k_t \ , \tag{23}$$

$$k_{t+1}=(1+n)^{-1}(1+\beta)^{-1}\beta(1-\alpha)k_t^{\alpha}q_t^{-1} \ . \tag{24}$$

Figure 3 . The Stock Market Equilibrium in Period t



Eliminating q_t from both of these equations, we have the dynamic equation of per capita capital

$$k_{t+1}^2=\frac{\beta(1-\alpha)}{2a(1+n)^2(1+\beta)}k_t^{1+\alpha}+\frac{(2a-1)}{2a(1+n)}k_t k_{t+1} \ . \tag{25}$$

The steady state values of k and q become respectively as follows:

$$k_s=\left\{\frac{\beta(1-\alpha)}{(1+2an)(1+n)(1+\beta)}\right\}^{\frac{1}{1-\alpha}} \ . \tag{26}$$

$$q_s=1+2an \ . \tag{27}$$

Linearizing the dynamic equation around the steady state, we have

$$k_{t+1}-k_s=\frac{2a(1+n)+\alpha(1+2an)}{1+2a(1+2n)}(k_t-k_s). \tag{28}$$

$$q_t-q_s=-\frac{2a(1-\alpha)(1+n)+(1+2an)}{\{1+2a(1+2n)\}k_s}(k_t-k_s). \tag{29}$$

The coefficient in the right-hand side lies in the open interval $(0, 1)$.[2] Therefore, the steady state is locally stable. Besides, if the converging sequence of k_t is increasing, that of q_t is decreasing and vice versa. From comparison between the steady state capital per capita (9) and (26), it is obvious that $k_s<k^*$. That is, in the presence of adjustment costs, products available for investment are smaller than in the standard model.

5. Conclusion

So far, we have examined the way to formulate an overlapping generations model in which a firm's flow investment is explicitly considered. In the present attempt, we assumed equity financing by a firm and household saving by stock holding. Such a model building makes the dynamics rather complicated than the standard model and makes global analysis difficult even under the simplest utility and production functions used in this paper. Furthermore, in Section 3, we derived an investment function from the dividend maximization problem in a static manner. Exploring for an alternative extension consistent with the firm's dynamic optimization and global dynamics of the equilibrium path are the author's future task.

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2 In order to evaluate a value of the coefficient, we take a difference between numerator and denominator to obtain

$2a(1+n)+\alpha(1+2an)-\{1+2a(1+2n)\}=-(1-\alpha)(1+2an)<0.$

Both of numerator and denominator are positive and numerator is less than denominator. Therefore, the coefficient is in $(0, 1)$.

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