Advertising Costs, Monopolistic Competition, and International Trade*

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We examine the effects of trade and integration on the level of utility of consumers in two trading countries with a different size of population by using an extended love-of-variety approach with endogenously determined fixed costs. We expose that i) trade is not always good to all countries, and ii) there is some conflict between the large country and the small country on "trade or integration." Thus we state that the result of Krugman (1979) is not generally correct. And we introduce a new idea, the brand jam effect, which brings a similar result to Kikuchi (1996), i.e., trade pushes some inefficient firms out of the market.

Keywords: Monopolistic competition; International trade; Advertising costs; Integrated economy.

1. Introduction

Since Krugman (1979) showed up in a neat way, the love-of-variety approach has been used so many times to describe monopolistic competition. Countries trade their brands one another and everyone is happy with trade because there is no duplication of brands and firms do not have to pay any additional cost to trade so that available brands increase.

This approach deals with product differentiation but it is related to the price of goods only. The common price firms set are of course derived from the utility function which presumes that a representative consumer loves variety.¹ A consumer is willing to purchase a brand since he thinks that it is well differentiated. Firms incur an exogenous fixed cost to differentiate their products from the others.

However, as Spence (1976) and Kikuchi (1996) pointed out, such fixed costs play a crucial role: the number of firms actually producing products is limited by the amount of fixed costs. If the fixed costs are large and the size of the la-

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¹ More on the utility function see Dixit and Stiglitz (1977).
bor force is constant, then the number of firms will be small. Kikuchi (1996) describes the model with increasing fixed costs, where a firm that enters the market later has to pay more fixed costs to differentiate from all the existing products. But the fixed cost is exogenously given in Krugman (1979, 1980, 1981), and Kikuchi (1996) takes the fixed cost for the first firm as given.

We think the level of fixed costs a firm incurs will be affected if the utility function in the earlier models is a little modified, just as the markup rate is affected by a change in the price elasticity parameter.

We regard these fixed costs as advertising costs. When a new brand is introduced in the market, a consumer may not notice it. If he notices, he does not buy it when he does not recognize it, that is, when he does not realize that it is really differentiated from the ones he usually purchases. Firms must advertise their brands for consumers to recognize it. If they succeed, they will earn positive (and more) profits. Thus, we will construct a model in which a firm's fixed cost is endogenously determined by its optimal behavior. And we will use an extended love-of-variety approach, which has the rate of recognition, i.e., how many people recognize a brand (i.e., how many people accept a good as a new brand). This rate depends positively on the amount of advertising costs of one firm, and negatively on advertising costs of the other firms and the number of brands per capita.

Section 2 describes the model and examines an equilibrium in a closed economy while section 3 examines the effect of trade. Section 4 analyzes the effect of integration and section 5 wraps up the paper.

2. Closed Economy

We use an extended love-of-variety approach to describe monopolistic competition. There is only one industry, producing differentiated products indexed by \( i \). The number of varieties of these goods produced is \( n \). However, a consumer may not know some of them. When he knows one brand, he may not feel it's well differentiated from the others that he feels so. Here we suppose that a consumer knows and purchases only a fraction of the whole brands.\(^2\)

There are \( L \) people and consumer \( k \) has the utility function as below:

\[
U_k = \left( \sum_{i=1}^{n} c_i^\theta \right)^{1/\theta}, \quad 0 < \theta < 1, \quad 0 < u_k < 1, \tag{1}
\]

where \( c_i \geq 0 \) is the consumption of brand \( i \) (positive as long as he recognizes it). (1) implies this consumer buys a fraction \( u_k \) of brands. Maximizing with respect to brand \( i \) leads to

\(^2\) If a consumer thinks that brand \( j \) is not well differentiated from brand \( i \), then he will purchase either brand \( i \) or \( j \).
where $p_i$ is the price of brand $i$ and $w$ is the wage rate.

Firm $i$ producing brand $i$ uses labor, which is the only factor of production, by the amount of

$$l_i = a_i + bx_i, \quad a_i, b > 0,$$

where $a_i$ is the fixed cost and $x_i$ is output. Firms have to pay the fixed cost for consumers to know that their products are well differentiated. Therefore, it is reasonable to take $a_i$ as advertising costs.\(^3\) With the advertising cost the profit of firm $i$ is expressed as

$$\pi_i = p_i x_i - (a_i + bx_i)w.$$

The profit maximization with respect to the price for a given level of $a_i$, using (2), amounts to

$$\frac{p_i}{w} = \frac{\varepsilon}{\varepsilon - 1} b, \quad \forall i,$$

where $\varepsilon = 1 / (1 - \theta)$ is the price elasticity of demand for each brand.\(^4\) Free entry drives profits to zero. Then, in the long run, the output of brand $i$ turns out to depend on the advertising cost of firm $i$:

$$x_i = \frac{a_i}{b} (\varepsilon - 1).$$

Thus, when a firm advertises harder, its brand becomes known to more people to consume it.

On the other hand, by using the full employment condition, $L = \sum_{i=1}^{n} l_i$, we find out that the number of brands in the long run is

$$n = \frac{L}{a \varepsilon'}$$

if firms are symmetric, i.e. $a_i = a$, $\forall i$.

To close the model, we must have something to determine each firm’s advertising cost level. As mentioned above, a firm has to pay the advertising cost

\(^3\) Advertising that firms give has to be differentiated as well. But we do not explore it here.

\(^4\) In deriving (5), we set $x_i L_i$, where $L_i$ is constant. Later we have (9) instead, though, we don’t use it here to show we’re following the same procedure (1)-(7) as Krugman (1979).
to differentiate its product to consumers’ eyes. We assume when firm \( i \) pays \( a_i \), a fraction \( \gamma \) of consumers recognizes brand \( i \), and the rate of recognition for brand \( i \) is defined as

\[
\gamma = \left[ \frac{a_i}{L} \right]^{\alpha} - \left[ \frac{\sum_{j \neq i} a_j}{n} \right]^{\beta} \left[ \frac{n}{L} \right]^{\chi}, \quad 0 < \beta < \alpha < 1, \quad 0 < \chi < 1. \tag{8}
\]

The rate of recognition increases with the advertising cost that firm \( i \) pays and decreases with those costs of all firms \( j \neq i \) and the number of brands per capita. An increase in the number of brands reduces the rate of recognition through increases in both the total amount of advertising of the other firms and the number of per-capita brands. Therefore, this specification is general. The assumption \( \beta < \alpha \) implies the own advertising effect overwhelms the cross advertising ones.

Now the output of firm \( i \) is represented by

\[
x_i = \gamma L c_i, \tag{9}
\]

where \( c_i \) is given by (2). Advertising raises sales since \( \partial \gamma / \partial a_i > 0 \). As we take firms to be symmetric, the level of advertising becomes identical among firms, \( a_i = a \). Each firm maximizes profits with respect to the fixed cost under the identical price in (5). The result of the maximization is as follows:

\[
\left[ \frac{P_i}{w} - b \right] \frac{\partial x_i}{\partial a_i} = 1. \tag{10}
\]

Therefore, any firm chooses the advertising cost which realizes the marginal revenue equal to the marginal cost in wage units. Using (9), this equation is transformed into

\[
\frac{\partial \gamma}{\partial a_i} = \frac{\gamma}{a_i}. \tag{11}
\]

The marginal rate must meet the average rate of recognition. This is depicted in Fig. 1. The GG curve represents the function of \( \gamma_i \). The optimal advertising cost is determined at point \( E \). When the number of firms rises, this curve shifts downward and the recognition rate falls. Then, a firm will advertise harder to raise the rate, ending up with a higher advertising cost.

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5 There is no reason for one firm to compete on another firm’s brand. Then, the number of firms is equal to the number of brands.

6 Note that \( a_i \) is positive when \( \gamma_i \) is zero. A plausible interpretation is that the product differentiation incurs another fixed cost as well.
Differentiate $\gamma$ in (8) and substitute (7) for $n$ in (8) to obtain
\begin{equation}
\alpha \gamma = \frac{L^n}{(1 - \alpha)^{\alpha+\gamma}}. \tag{12}
\end{equation}

Therefore, when the population size is larger, each firm will pay more advertising costs. To explain that, let us take a close look at (8). By symmetry, the recognition rate turns out to be
\begin{equation}
\gamma = \gamma_i = \left( \frac{\alpha}{L} \right)^a - \left( \frac{1}{\varepsilon} \right)^b + \left[ \frac{1}{a \varepsilon} \right]^x, \quad \forall i. \tag{8'}
\end{equation}

The effect of the cross advertising does not depend on $L$, and the effect of per-capita brands does not directly either. This is because the long-run number of firms is proportional to the work force as in (7).\(^8\) Fig. 1 illustrates the $GG$ curve rotating down to $GG'$ when $L$ increases and the optimal advertising cost is determined at point $E'$. After all, the advertising cost in equilibrium increases with the size of population.

The rate of recognition is determined by substituting (12) for $\alpha$ in (8) as
\begin{equation}
\gamma = \alpha \left( \frac{\alpha}{L} \right)^a = \alpha \frac{\alpha}{\left[ (1 - \alpha)^{\alpha+\gamma} \right]}^{\alpha(a+\alpha)} \tag{13}
\end{equation}

The recognition rate falls with the population size since it depends on the amount of advertising per capita and the two negative effects are independent

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7 We approximate $n^{-1} = n$.

8 In the term of the per-capita brand effect it seems the advertisement by the rest of the firms raises the recognition rate of firm $i$. But this precisely implies an increase in the level of advertising per firm leads to a decrease in the number of brands per capita, which in turn raises the recognition rate of firm $i$.\(^7\)
of the size.

The equilibrium number of brands and output are obtained by (7) and (6) together with (12). By (7) and (12) we have

\[ n = \frac{1}{\varepsilon} \left[ \frac{1 - a}{\varepsilon^{\delta + x} L^x} \right]^{1/(\delta + x)}. \tag{14} \]

When \( L \) is larger, firms have to pay more advertising costs so that some of them cannot take it and exit out of the market. This reduces the number of firms. On the other hand, a larger \( L \) implies a larger market/labor force so that this allows for more firms to exist. From (14) we see that a larger size of the market ends up with a greater number of firms. But the output per firm results in being larger. This is because a larger population brings a larger potential demand. This effect is not captured in the conventional models because they do not take account of the linkage between the fixed cost and the market size.

Next we consider welfare implications. It is pretty useful to get along with symmetry among consumers. Suppose \( v = v_k = \gamma, \forall k \). Then, using (2) and (5), the utility function (1) is rewritten as

\[ U = U_k = (\gamma n)^{(1 - a)/\alpha} \frac{W}{p}, \forall k. \tag{15} \]

Thus, the utility level depends on the real wage and how many brands a consumer purchases. From (5) \( W/p \) is always constant while \( \gamma n \) is expressed according to (13) and (14) as

\[ \gamma n = \frac{\alpha}{\varepsilon} \left[ \frac{(1 - a)}{\varepsilon^{\delta + x} L^x} \right]^{(1 - a)/(\delta + x)}. \tag{16} \]

A larger size results in a larger number of brands for a consumer to buy because an increase in the number of brands overwhelms a decrease in the rate of recognition due to a larger amount of the advertising cost.

We figured out that the factors reducing the rate of recognition are independent of \( L \) in the closed economy. What if trade is allowed to open among nations? We check it out in the next section.

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9 This is the case when there are \( n \) groups of consumers and firms are symmetric. Because this seems so crucial that the results we will get below might alter, we must be careful for this. The recognition rate need not equal \( v \). However, there must be some relation between them. When a firm is successful in raising \( \gamma \), there will be somebody who starts to buy its variety. This implies \( \partial v / \partial \gamma > 0 \). Besides, we don’t have to specify who will when which firm is successful. Thus we make it symmetric for simplicity and this lets us assume \( dv/d\gamma = 1 \), i.e. \( v = \gamma \).
3. Effects of Trade

Now trade begins between the two countries described above, Home and Foreign. Home is endowed with \(L\) while Foreign with \(L^* = \delta L\), where \(0 < \delta < 1\). There are \(N\) brands in the world due to trade. At the beginning of trade, Foreign products flow into the Home market and Foreign firms start to advertise their brands so that the \(GG\) curve in Fig. 1 sifts down to \(G"G"\) and the advertising cost increases as described at point \(E"\). Increases of both advertising by Foreign firms and brands per capita reduce the rate of recognition, which we call the brand jam effect.

Now each firm has to advertise in both markets. The total advertising costs are defined by \(A = a_i + a_i^*\), where the star on variables stands for "in the Foreign market," while the star on the brand index for a Foreign firm's and its brand's name.\(^\text{10}\) A firm chooses the output level to maximize its profits earned in each market. The total output is denoted by \(X = x_i + x_i^*\), where \(x_i = \gamma_i L c_i\) and \(x_i^* = \gamma_i^* L' c_i\). The profit maximizing price is the same as before trade and the total output and the number of brands in equilibrium are obtained by replacing \(a\) by \(A\) for all \(i\) and \(i^*\).

The recognition rate under trade is written as

\[
\eta = \left( \frac{a_i}{L} \right)^\alpha - \left( \sum_{i=1}^{N} \frac{a_i + a_i^*}{L} \right)^\beta \left( \frac{N}{L} \right)^\gamma,
\]

where \(N = n + n^* = (1 + \delta)L/A\varepsilon\). By symmetry this equation reduces to

\[
\eta = \left( \frac{a_i}{L} \right)^\alpha - \left( N \frac{a_i + a_i^*}{L} \right)^\beta \left( \frac{N}{L} \right)^\gamma = \left( \frac{a_i}{L} \right)^\alpha - \left( \frac{1 + \delta}{\varepsilon} \frac{a_i + a_i^*}{A} \right)^\beta \left( \frac{1 + \delta}{A\varepsilon} \right)^\gamma.
\]

In contrast to \(\text{c8}\) in the case of the closed economy above, the two negative effects depend on the ratio of the number of consumers in the two trading countries. Holding \(L\) fixed and ceteris paribus, a large \(\delta\) means a large labor force in Foreign so that a large number of firms produce actively. Thus, a large amount of the advertising by the rest of firms and a large number of per-capita brands reduce the recognition rate of brand \(i\).

The conditions to maximize profits in each market have the same form as \(\text{11}\). These two equations result in the ratio of the advertising costs for both markets:

\[
\frac{a_i^*}{a} = \delta^\gamma,
\]

where \(\rho = 1 - \chi/(\alpha - \beta) < 1\). If the per-capita brand effect is large, that is

\(^{10}\text{Since the real wage is the same in each country, the nationality of an employee for advertising doesn't matter.}\)
$\chi > \alpha - \beta$, the advertising cost for the Foreign market is greater than for the Home market. Moreover, the sign of $\rho$ is important in another sense: we can write the total number of brands as $N = (L + L^*)/(1 + \delta^\rho)$. If $\rho > 0$, a rise in $\delta$ implies a fall in $N$ because the total amount of the advertising cost a firm pays gets larger. Together with the implication above, the consequence of the rise in $\delta$ becomes ambiguous. If $\rho < 0$, on the other hand, it unambiguously raises $N$ so that the recognition rate declines.$^{11}$

Using (14) together with (17) and (18), the advertising cost in the Home market is given by

$$a^{a+x} = D^{\beta+x} \bar{a}^{a+x} < \bar{a}^{*a+x}, \quad (19)$$

where $D = (1 + \delta)/(1 + \delta^\rho) < 1$ and the upper bar represents a value before trade. We can see that the advertising cost for the Home market decreases after trade. On the other hand, the advertising cost for the Foreign market increases after trade:

$$a^{*a+x} = D^{*\beta+x} \bar{a}^{*a+x} < \bar{a}^{*a+x}, \quad (20)$$

where $D^* = [(1+\delta)/\delta]/[(1+\delta^\rho)/\delta^\rho] < 1$. It turns out, the total advertising costs satisfy

$$A^{a+x} = (1 + \delta)^{\beta+x}(1 + \delta^\rho)^{\alpha-\beta} \bar{a}^{a+x} > \bar{a}^{a+x} > \bar{a}^{*a+x}. \quad (21)$$

Thus, the total advertising costs increase after trade. And in turn the total number of firms is given by

$$N = \frac{L + L^*}{A^\epsilon} = D^{(\alpha-\beta)/(\alpha+x)}\bar{a} = D^{*\epsilon} \bar{a}^{*}. \quad (22)$$

Therefore, the total number of firms falls in the Home market and rises in the Foreign market after trade since $D < 1$ and $D^* > 1$.

What's this all about? At the beginning of trade new brands come from abroad into the domestic markets. Then, the firms in both countries try to increase their advertising costs because the introduction of those new brands reduces the recognition rate. Some marginal firms cannot afford and exit from the market. As we have seen in (12), a Foreign firm’s advertising cost before trade is smaller than a Home firm’s so that the number of firms is greater in Foreign. Since the required amount of the advertising costs after trade is the

$^{11}$ We'll give a numerical example later.
same for any firms in both countries, an increment of the costs is greater for a Foreign firm, and hence, a % decrease in the number of national firms is larger in Foreign. This result is similar to Kikuchi (1996), that is, trade crowds out some inefficient firms, though he concludes in Proposition 2 as:

*By the opening of trade, the number of firms . . . . increases in the small country, while it decreases in the large country.*

The difference stems from his assumption of asymmetric fixed costs among earlier entering firms and later entering ones. In our model trade boils down to \( n^* < N < \bar{N} \) so that \( a < \bar{a} \) and \( a^* > \bar{a}^* \).

The rate of recognition in each market is expressed as the same as (13):

\[
\gamma = \alpha \left[ \frac{a}{L} \right]^a \\
\gamma^* = \alpha \left[ \frac{a^*}{L^*} \right]^a
\]

Since \( a < \bar{a} \) and \( a^* > \bar{a}^* \), the recognition rate decreases in the Home market and increases in the Foreign market after trade. In the Home market a fall in the total number of firms results in less advertising, which implies a lower rate of recognition. On the contrary, a rise in the total number of firms in the Foreign market makes competition tighter and results in a higher rate of recognition.

Finally, we consider a change in the utility. Since \( \gamma < \gamma^* \), \( \gamma^* > \bar{\gamma}^* \) and \( \bar{n}^* < N < \bar{n} \), we have

\[
\bar{\gamma} \bar{n} > \gamma N \\
\bar{\gamma}^* \bar{n}^* < \gamma^* N
\]

The number of brands a consumer in Home purchases thus decreases while it increases in Foreign after trade. Trade reduces the recognition rate in the first place. This is a direct brand jam effect. Because of the difference in the population size the advertising costs increase in the Foreign market and decrease in the Home market. As a consequence, the rate of recognition rises in the For-
eign market and falls in the Home market, and therefore Foreign gains and Home loses. This is an indirect brand jam effect, which would not work when $\delta = 1$. Since a consumer loves variety as in (15), the welfare deteriorates in Home and improves in Foreign. Now we get the following proposition:

PROPOSITION 1: when a firm determines its fixed costs of advertising according to the rate of recognition in each market, it incurs more advertising costs in a small country than in a large country after trade. Because of the brand jam effect, trade harms the large country and benefits the small country.

In Krugman (1979, 1980, 1981) the recognition rate is constant and one all the time, and trade gives consumers all the brands from abroad. This sounds like there is no competition. But as Spence (1976) and Kikuchi (1996) pointed out the fixed costs play a crucial role in differentiating products. We took those costs as advertising costs and firms really competed. As a result, not all the firms survived and a consumer had less brands in Home and more brands in Foreign after trade. 13

4. Effects of Integration and Difference in Size

Imagine the two countries are integrated into one. Then it is evident by replacing $L$ by $L_w = L + L^*$ in section 2 that the welfare is always improved in proportionate to the population size, which implies integration is superior to isolation for both Home and Foreign. Comparing to trade, integration is sure to be superior to trade for Home. Though for Foreign it depends on parameter values, trade is very likely to be superior to integration. 14

We consider what if the difference in size is larger and restrict ourselves to the case of $x > a - \beta$, that is, the per-capita brand effect is large. In this case a firm pays more advertising costs for the Foreign market than for the Home market, $a < a^*$. Since $\partial D/\partial \delta > 0$ and $\partial D^*/\partial \delta < 0$, the deterioration in Home and the improvement in Foreign get larger when the difference in size becomes larger (i.e., $\delta$ becomes smaller) [see (19), (21), (22) and (23)]. This results from the per-capita brand effect and we have the proposition as follows:

PROPOSITION 2: a large country prefers integration to trade and a small country prefers trade to integration, and this tendency is growing when the per-

13 Also notice a Foreign consumer had less brands than Krugman (1979)'s result.
14 We examine for a particular value of parameters: for I) $x = 0.7, \beta = 0.3, \chi = 0.4, \delta = 0.9, 0.5, 0.1 \Rightarrow g = 0.59, 0.47, 0.19$; II) $x = 0.7, \delta = 0.9, 0.5, 0.1 \Rightarrow g = 1.05, 0.72, 0.24$, where $g = \chi_{\text{offw}}/\chi_{\text{offw}}$ and the subscript $w$ stands for variables under integration. Thus even if $\chi$ is large, a sufficiently small $\delta$ ensures the result.
capita brand effect is sufficiently large and the countries are more dissimilar in their size.

Krugman (1979) showed trade and integration are perfect substitutes. But in our model this is not true and there is some conflict between countries on "trade or integration."

5. Conclusions

We have presented an extended love-of-variety model with an endogenous fixed cost. We have shown that the intraindustry trade does not always benefit all the countries. If the market expands, a firm obtains another place to compete. Some of them cannot survive so that the large country loses from trade because of the love of variety.

Integration is superior for the large country and there must be an argument on labor mobility, although we did not examine it. And we treated firms as symmetric. If we allow firms to be asymmetric, only a large firm will go abroad and earn additional profits. These issues are left for a future work.

References


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