THE BALANCED GROWTH POLICY BY FISCAL POLICY

—A Critique of Musgrave's Theory—

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I. PREFACE

The main purpose of this paper is a thorough criticism of the balanced growth policy of R. A. Musgrave, especially of the required growth rate (capacity growth rate or supply growth rate).

It is Musgrave's theory that is the most popular as balanced growth policy by fiscal policy. As you know, his theory has been quoted in a great many books and articles not only in Japan but also in many other countries. The thoery that Musgrave presented in Fiscal Dynamics and Growth, Part 4 Chapter 20 of "The Theory of Public Finance", is a policy to balance required growth rate (growth rate of supply) R^r and spending growth rate (growth rate of demand) R^e by the means of an increase and decrease policy on taxation or fiscal spending. Moreover, it intends to raise the level of balance by giving priority to the policy that affects sensitively on the lower growth rate. As inflation means the excess of demand over supply, we should adopt a policy that efficiently affects the lower growth rate of R^r. On the contrary, deflation means the excess of supply over demand, so we should adopt a policy that efficiently affects the lower growth rate of R^e. His conclusion, however, is a mistake. He concluds that the supply growth rate rises with increased taxation and falls with decreased taxation. Actulally we know that a corporation tax reductive policy has been put into practice effectively in order to raise the growth rate. Well, then why does such a contradictory phenomenon occur? The answer is this: his supply growth rate deals with only the tax that is paid by reduction of consumption and the consuming fiscal expenditure, so he is obliged to make such a mistake as above.

As he showed in his model, taxes should be paid by reduction of private consumption and savings, and at the same time we know that fiscal spending consists of consuming expenditures and investment expenditures.

Without these two parts, we will be unable to come to an accurate conclusion of the analysis.

II. SUPPLY GROWTH RATE

In order to obtain the supply growth rate, that is, Capacity Growth Rate

(R^r of Musgrave's), we multiply the rate of savings (=investment) by productivity.

First of all we must confirm that the national income consists of private consumption, savings and the revenue of government. And taxes are paid by a reduction of what is to be alloted for private consumption and savings.

$$Y = C + S + T \tag{II–1}$$

$$C = c(1-t)\,Y \tag{II–2}$$

$$S = \alpha(1-t)\,Y \tag{II–3}$$

$$T = t\,Y = ct\,Y + \alpha t\,Y \tag{II–4}$$

Next, let's assume the balanced budget. It's done only to symplify and you will soon recognize that the following analysis is more useful in the case of unbalanced budget.

We can divide fiscal expenditures into two parts, namely consuming expenditure dgY and investment expenditure γgY. Then the saving (=investment) fund of the economic society is as follows: if we assume

$$T = G$$

$$G = dgY + \gamma gY$$

the formula (II–1) is thus,

$$Y = c(1-t)\,Y + \alpha(1-t)\,Y + dgY + \gamma gY$$

$$Y - c(1-t)\,Y - dgY = \alpha(1-t)\,Y + pgY \tag{II–5}$$

Now suppose. $\varDelta Y^c$ is the increment of capacity income under full employment and s is productivity the ratio of the capital stock increment to the capacity income increment, and we shall obtain,

$$\varDelta Y^c = s\Big[Y - c(1-t)\,Y - dgY\Big] = s\Big[\alpha(1-t)\,Y + \gamma gY\Big]$$

Hence, the supply growth rate (capacity growth rate) is thus:

$$R^r = \frac{\varDelta Y^c}{Y} = s\Big[1 - c(1-t) - dg\Big] = s\Big[\alpha(1-t) + \gamma g\Big]$$

$$= s[\alpha + ct - dg] = s[\alpha - \alpha t + \gamma g] \tag{II–6}$$

The left side of this formula (II–6) is R^r_c and the right side is R^r_α, that is,

$$R^r_c = s[\alpha + ct - dg] \tag{II–7}$$

$$R^r_\alpha = s[\alpha - \alpha t + \gamma g] \tag{II–8}$$

R^r_c expresses that the saving rate ex ante plus the discrepancy between the rate of tax yields paid by reduction of consumption and the consuming fiscal expenditure rate is the saving (=investment) rate. R^r_α expresses that the saving rate ex ante plus the discrepancy between the rate of tax yields paid by reduction of savings and the investment fiscal expenditure rate is the saving (=investment) rate.

Judging from the formula (II–6), you can easily see that R_c^r and R_α^r show the same growth rate. Nevertheless, the functional efficiency of growth is quite opposite according to the variety from the sources of taxation and the composition of fiscal expenditure. So let us examine the efficiency of growth of increase and decrease of taxation by the difierential, using t just as Musgrave did.

$$\frac{dR_c^r}{dt}=s\left[c-d\frac{dg}{dt}\right] \qquad \text{(II–9)}$$

$$\frac{dR_\alpha^r}{dt}=s\left[-\alpha+\gamma\frac{dg}{dt}\right] \qquad \text{(II–10)}$$

In the case of balanced budget, $\frac{dg}{dt}=1.$

And when the fiscal expenditure is given, $\frac{dg}{dt}=0.$ So,

$$\frac{dR_c^r}{dt}=sc>0 \qquad \text{(II–11)}$$

$$\frac{dR_\alpha^r}{dt}=-s\alpha<0 \qquad \text{(II–12)}$$

As the result in the case of consumption tax R_c^r shows that the increase or decrease of taxation adds to or reduces the growth rate, but in an opposite case the saving tax reduces or adds to the growth rate.

Therefore, when we examine the efficiency of the increase and decrease of taxes in the general system of taxation, we find it necessary to compare the offset between the parts which is paid by dis-consuming and dis-savings.

$$|sc|\gtreqless|s\alpha|$$

$$c\gtreqless\alpha$$

$$1-\alpha\gtreqless\alpha$$

$$\frac{1}{2}\gtreqless\alpha$$

When c is larger than α, that is, the saving rate is less than 0.5, the increase or decrease of taxes adds to or reduces the growth rate and in the opposite condition they reduce or add to the growth rate.

We can say the same thing about the fiscal expenditure. For if we assume that the tax yields are given, $\frac{dt}{dg}=0.$

$$\frac{dR_\alpha^r}{dg}=-sd<0 \qquad \text{(II–13)}$$

$$\frac{dR_\alpha^r}{dg} = sr > 0 \tag{II-14}$$

So that, the increase of the rate of fiscal consuming expenditures reduces the growth rate and the increase of the rate of fiscal investment expenditure adds to the growth rate on the contrary. On the other hand, The ratio of consuming expenditure and investment expenditure, that is, the scale of d and γ, shows the effects of the increase and decrease of fiscal expenditure upon the growth rate. It is necessary that the rate of investment expenditure is at least mote than 0.5, so that the increased expenditure might raise the growth rate.

Let us revert to the above-mentioned subject and reexamine R_c^r and R_α^r. These two rates are equal, namely the same degree of growth. In other words, in the case of only the balanced budget both R_c^r and R_α^r come to show the same growth rate. Now let us explain it according to such a table.

	private sector	public sector	
consuming expenditure	ct	$(-)\,dg$	$ct-dg$
saving and investment	$(-)\,\alpha t$	γg	$-\alpha t+\gamma g$

That is to say, taxes from private sectors are paid by the reduction of the consuming expenditure and saving (=investment) funds, and the government spends the tax yields on consuming expenditure or investment expenditure. For example, now the tax yields paid by economy of consumption are 10, and 8 of them are spent on consuming fiscal expenditures. Then the balance 2 are appropriated for investment expenditure γg. In this case, $ct-dg=10-8=2$ and on the other hand $-\alpha t+\gamma g=0+2=2$ and the saving (=investment) funds are 2, either way.

Now suppose that if there are tax yields of 8 by the reduction of saving and 5 of them are appropriated for the investment fiscal expenditures, the balance of 3 is comsumed by fiscal expenditures. You can see by $-\alpha t+\gamma g=8+5=-3$ and $ct-dg=0-3$, that, saving (=investment) funds are reduced by 3, either way.

As the above facts, the growth rate itself is just the same R_r and R_c, but effects of the increased and decreased taxes and the increased and decreased fiscal expenditure on R_α^r and R_c^r is completely opposite.

Here, if we difine $\alpha=0.3$, $c=0.7$, $d=0.4$, $r=0.6$, $t=g=0.2$,

$$R_c^r = s[0.3+0.7\times 0.2-0.4\times 0.2] = s[0.36]$$
$$R_\alpha^r = s[0.3-0.3\times 0.2+0.6\times 0.2] = s[0.36]$$

So, we can obtain the same growth rate from both R_c^r and R_α^r.

Well then how about the increased tax?

If $t=0.21$,

$$R_c^r = s[0.3+0.7\times0.21-0.4\times0.2] = s[0.367]\cdots[+0.007]$$
$$R_\alpha^r = s[0.3-0.3\times0.21+0.6\times0.2] = s[0.357]\cdots[-0.003]$$

Here R_c^r shows the risng of growth rate and R_α^r shows the quite opposite. And then, how about the increased fiscal expemditure?

If $g=0.21$,

$$R_c^r = s[0.3+0.7\times0.2-0.4\times0.21] = s[0.356]\cdots[-0.004]$$
$$R_\alpha^r = s[0.3-0.3\times0.2+0.6\times0.21] = s[0.366]\cdots[+0.006]$$

Therefore if a taxation system depends on the taxes paid by the reduction of concumption and saving, it is necessary to find out the real net effect on growth by the offset between these two$\cdots$the model of Musgrave also shows that consuming function is this $C=(1-\alpha)(1-t)Y$, so a part of the tax ctY is paid by the reduction of consumption and the rest $(Y-ctY=\alpha tY)$ is paid by the reduction of savings. But actually R^r of Musgrave expresses only R_c^r.

Musgrave's R^r is,

$$\begin{aligned}R^r &= s[t+\alpha(1-t)-g(1-r)]\\ &= s[ct+\alpha t+\alpha-\alpha t-dg]\\ &= s[\alpha+ct-dg] = R_c^r\end{aligned}$$

That is to say, the formula (20–47) of Musgrave deals with only the comparison between consumption tax and consuming fiscal expenditure. So, according to his balanced growth policy, the growth rate is risen by an increase of taxation and fallen by increased fiscal expenditure.

To be accurate, the effects on growth rate of an increase and decrease of taxation and an increased and decreased fiscal expenditure need the offset and net effect of each other, and at least in order to lead the conclusion of Musgrave it is necessary to consider the condition that $c>\alpha$, $d>\gamma$.

If we want to count the effect of a general increase and decrease of taxation and a general increased and decreased financial spending by using numbers above, it is necessary for us to make a synthesis which to offset the portion of increase and decrease between R_c^r and R_α^r.

$$R^r = 1/2(R_c^r+R_\alpha^r) = s\Big[\alpha+1/2\big\{(c-\alpha)t+(\gamma-d)g\big\}\Big] \qquad \text{(II–15)}$$

$t=g=0.21$

$$R^r = s\Big[0.3+1/2\big\{(0.7-0.3)\times0.21+(0.06-0.04)\times0.21\big\}\Big]$$

$$= s\left[0.3 + 1/2(0.084 + 0.042)\right] = s\left[0.363\right]$$

$$R_c^r = s[0.3 + 0.147 - 0.084] = s[0.363]$$

$$R_\alpha^r = s[0.3 - 0.063 + 0.126] = s[0.363]$$

As you see above, R^r, R_c^r and R^r show the same rate of growth but we can measure the change of growth rate as below:

$$\varDelta R^r = s\left[1/2\left\{(c-d)\varDelta t + (\gamma - d)\varDelta g\right\}\right] \tag{II–16}$$

$\varDelta t = \varDelta g = 0.01$

$$\varDelta R^r = s\left[1/2\left\{(0.7-0.3)0.01 + (0.6-0.4)0.01\right\}\right]$$

$$= s\left[1/2(0.004 + 0.002)\right] = s\left[0.003\right]$$

As you saw before, the growth rate is $s[0.36]$ when $t = g = 0.2$ and it is $s[0.363]$ when $t = g = 0.21$, so this $s[0.003]$ is just the same at the difference between these two. Therefore, we can obtain the growth efficiency of an increasing tax and fiscal expenditure by adapting the same figures to the equation (II–16).

Well, I have developed my argument regarding a balanced budget as precondition. How about the effects of this analysis with an unbalanced budget? It is easy for us to explain the effect.

Let us define the surplus or deficit of a budget as these $B = btY$ or $B = bgY$. Under the condition of an overbalanced budget, the surplus can be divided into two parts: one is a consumption tax and the other is a savings tax.

$$btY = bctY + b\alpha tY$$

The deficit in an unbalanced budget can also be divided into two parts: fiscal consuming expenditures and investment expenditures.

$$bgY = bdgY + b\gamma gY$$

Hence, under an overbalanced budget,

$$s\left[\alpha + ct(1+b) - dg\right] > s\left[\alpha - \alpha t(1+b) + \gamma g\right] \tag{II–17}$$

On the contrary under an unbalanced budget,

$$s\left[\alpha + ct - \alpha g(1+b)\right] < s\left[\alpha - \alpha t - \alpha y(1+b)\right] \tag{II–18}$$

In the above mentioned analysis about the effect on growth of an increase or decrease of taxation and an increase or decrease expenditure can be adapted to theses cases. Which is more accurate growth rate, R_c^r or R_α^r? This is a very difficult question, for it comes to complex according to the treatment of the balance of budget. When each half of the surplus under an overbalanced budget consists of reduction of consumption and reduction of savings, you can recognize briefly by the above example that neither R_c^r nor R_α^r shows the accurate growth rate when $t=0.21$ and $g=0.2$, and yet they are different. And so does it under an unbalanced budget. In other words, R^r of Musgrave has an effect on counting of growth rate only under a balanced budget. That is he who is stood under this assumption of an balanced budget. Then, using the former equation (II–15), we get

$$R^r = s\Big[\alpha + 1/2\Big\{(c-\alpha)\,t(1+b)+(\gamma-d)\,g(1+b)\Big\}\Big] \qquad \text{(II–19)}$$

and so we can count the growth rate under an unbalanced budget. It is natural that the great difference comes out, according to the difference of the part for which the surplus under an overbalanced budget should be paid or the method of financing the deficit of an unbalanced budget. But it is hardly possible to seek after and measure this, as the case may be. For example, when surplus is hoarded up, goods and services which correspond to a value of the surplus come to be released for private investment funds, so now it can be added to the fiscal investment rate γ. Onthe other hand, when surplus is applied to redemption of a national debt, we should reconsider the treatment according to circumstances whether the redemption is applied for consuming expenditure, savings or investment. On the deficit of an unbalanced budget, we must consider whether it is paid by private free savings or by credit creation. In the latter case, for instance, we must consider degree of rising prices (inflation) in order to count the saving (=investment) funds, regarding full employment as a precondition. Either way, it is very difficult for me to grasp in detail how to use the public funds in private sectors. So I will stop here only to show the equation (II–19) concerning the grasp of growth rate. R_c^r represents Musgrave's Capacity Growth Rate, but you can recognize easily that it shows quite imperfect growth rate connecting with R_α^r under an unbalanced budget. It is only under an balanced budget that both R_c^r and R_α^r show the same growth rate.

There is one more problem in respect to show an actual and accurate growth rate. For, as a matter of fact, a tax ctY is not always paid the sum multiplied by the average rate of taxation from consumption, nor a tax αtY is paid the sum multiplied by the average rate of the taxation from savings. Hence it is necessary for us to count an accurate growth rate using a figure that expresses a ratio between average rate and effective rate of taxation.

III. NECESSITY OF TAX SOUCE MATRIX

The payments of taxes are done by reducing of ex ante consumption or by reducing of ex ante savings and occasionally by the reducing of both of these.

Now suppose a tax paid by the reduction of consumption to be a first category tax, while a tax paid by the reduction of savins to he a second category tax and a tax paid by the reduction of both these to be a third category tax. Here let's examine a general example. We can find a consumption tax belongs to first category, and the corporation tax that is supposed to be paid by reduction of a retained surplus, belongs to second category, and individual income tax which is a tax as you earn with reduction both of these belonging the third category.

Now, assuming that a_{ij} is the rate of effective tax burden against average tax rates, each tax burden is as follows:

$$\begin{array}{l} \\ \text{first category tax} \\ \text{second category tax} \\ \text{third category tax} \end{array} \begin{array}{c} \\ \\ = \\ \\ \end{array} \begin{array}{c} \text{expenditure} \quad \text{saving} \\ \begin{bmatrix} a_{11}, & 0 \\ 0, & a_{22} \\ a_{31}, & a_{32} \end{bmatrix} \end{array} t \begin{bmatrix} cY \\ \\ \alpha Y \end{bmatrix} = \begin{bmatrix} a_{11}tcY + & 0 \\ 0 & +a_{22}t\alpha Y \\ a_{31}tcY + a_{32}t\alpha Y \end{bmatrix}$$

$$T = tY = a_{i1}tcY + a_{i2}t\alpha Y = a_{11}tcY + a_{22}t\alpha Y + a_{31}tcY + a_{32}t\alpha Y$$

$$\therefore \quad a_{i1}c + a_{i2}\alpha = a_{11}c + a_{22}\alpha + a_{31}c + a_{32}\alpha = 1$$

$\begin{bmatrix} a_{11}, & 0 \\ 0, & a_{22} \\ a_{31}, & a_{32} \end{bmatrix}$ are called tax-souce Matrix.

$\begin{bmatrix} a_{11}t, & 0 \\ 0, & a_{22}t \\ a_{31}t + a_{32}t \end{bmatrix}$ are average tax rates of each taxation.

If such a table of tax-souce can be formed as above, formula (II–15) is followed like this:

$$s\left[\alpha + 1/2\left\{(a_{i1}c - a_{i2}\alpha)\,t + (\gamma - d)\,g\right\}\right] \qquad \text{(III–1)}$$

Formula (II–16) is

$$\varDelta R^r = s\left[1/2\left\{(a_{i1}c - a_{i2}\alpha)\,\varDelta t + (\gamma - d)\,\varDelta g\right\}\right] \qquad \text{(III–2)}$$

so we can find here it possible to count an effect of growth of increase and decrease in each tax. R^r_c and R^r_α come to thus;

$$R^r_c = s[\alpha + a_{i1}ct - dg] \qquad \text{(III–3)}$$

$$R^r_a = s[\alpha - a_{i2}\alpha t + \gamma g] \qquad \text{(III–4)}$$

I will show you a real example to measure by applying almost real numbers to the equation above. According to the data of the Taxation System Investigation Committee in Japan, it's possible to define as follows; the total of indirect taxes conforms to first category tax, the corporation tax to second category and the indivudual income tax to third category, therefore each category tax holds 45%, 30% and 25%. The rate of tax burden of national income is about 20 percent, i.e. $t=0.2$, while the propensity of consuming is about 70 percent, i.e. $c=0.7$ ($\therefore\ \alpha=0.3$).

$$\begin{array}{l} \text{first category tax} \\ \text{second category tax} \\ \text{third category tax} \end{array} \begin{bmatrix} a_{11}tcY + & 0 \\ 0 & + a_{22}t\alpha Y \\ a_{31}tcY + a_{32}t\alpha Y \end{bmatrix} = \begin{bmatrix} 9/14, & 0 \\ 0\ , & 1 \\ 1/4\ , & 1/4 \end{bmatrix} [0.2] \begin{bmatrix} 0.7Y \\ \\ 0.3Y \end{bmatrix} = \begin{bmatrix} 0.09Y \\ 0.06Y \\ 0.05Y \end{bmatrix}$$

The numbers in place of a_{11}, a_{22}, a_{31} and a_{32} were obtained in this way: now indirect taxes as first category tax occupy 45 percent of a tax yields, i.e. 20 percent of Y, so I counted the ratio of 0.45 to the propensity to consume 0.7. Actually not always paying tax from consumption in proportion to the propensity to consume, we must adjust with insertion of numerical value a_{11}. As the result, the indirect tax which holds 45% of 20% tax burden is equivalent to 0.09 of national income Y.

Next, let us move to the ratio of investment expenditure among fiscal expenditures. The total of industrial economic expenses, conservation of national land and development expenses in the general account of public expenditure comes to about 30 percent, i.e. $\gamma=0.3$ ($\therefore\ d=0.7$).

Now the marginal capital coefficient of Japan is 3 with the United Nations Statistic Figure, from which we can derive that productivity s is one third as a reciprocal number. Consequently the growth rate is

$$R^r_c = 1/3[0.3 + 25/28 \times 0.2 \times 0.7 - 0.7 \times 0.2] = 0.095$$
$$R^r_a = 1/3[0.3 - 13/4 \times 0.2 \times 0.3 + 0.3 \times 0.2] = 0.095$$
$$R^r = 1/3\Big[0.3 + 1/2\big\{25/28 \times 0.2 \times 0.7 - 13/4 \times 0.2 \times 0.3 + (0.3 - 0.7) \times 0.2\big\}\Big] = 0.095$$

In any case, the growth rate comes to 0.095, and it shows that the growth rate is less than $1/3[0.3]=0.1$ by 0.005 without fiscal expenditure.

Here I want to compute the effect of growth of an increase or derease of taxation. If first category tax (the total of indirect taxes) is increased by 30 percent, and the sccond one (corporation tax) is decreased by 10 percent and the third one (individual income tax) is decreased by 20 percent, fiscal expenditure is also increased or decreased according to these. In other words, it is to maintain the balanced budget.

$$\begin{bmatrix} 11.70/70, & 0 \\ 0 \quad , & 5.4/30 \\ 0.4/10 \ , & 0.4/10 \end{bmatrix} \begin{bmatrix} 0.7Y \\ \\ 0.3Y \end{bmatrix} = \begin{bmatrix} 0.117Y \\ 0.054Y \\ 0.040Y \end{bmatrix}$$

You can see that 11.7/70, 5.4/30, 0.4/10 are the average tax rates of each first, second, third category tax after increase or decrease of taxation. The rate of tax burden rises to 0.211. And the growth rate is

$$R_c^r = 1/3\Big[0.3+0.7(11.7/70+0.4/10)-0.7\times 0.211\Big] = 0.0991$$

$$R_\alpha^r = 1/3\Big[0.3-0.3(5.4/30+0.4/10)+0.3\times 0.211\Big] = 0.0991$$

or

$$R^r = 1/3\Big[0.3+1/2\Big\{(0.7\times 14.5/70-0.3\times 6.6/30)+(0.7-0.3)\,0.211\Big\}\Big] = 0.0991$$

Hence, the growth rate rises up by $0.0991-0.0095=0.041$. If we count a rising part only,

$$\begin{aligned} \varDelta R^r = 1/3\Big[1/2\Big\{&(2.7/70-0.1/10)\,0.7 \\ &-(0.6/30+0.1/10)\,0.3+(0.3-0.7)\,0.011\Big\}\Big] = 0.0041 \end{aligned}$$

that is, it is 0.0041.

Thus, you can easily agree with my opinion that it is necessary for growth policy of tax to form a figure of taxation souce matrix or a table of taxation souce.

IV. DEMAND INCREASING

Musgrave refered to required growth rate (spending growth rate) R^e in chapter 20 of The Theory of Public Einance. R^e differs from above-mentioned required growth rate (supply growth rate or capacity growth rate) R^r on investment function only in model. In R^r

$$\varDelta Y_n^e \frac{1}{\alpha(1-t)+t-g}\,\varDelta I_n \tag{20-44}$$

$$\varDelta Y_n^e = \varDelta Y_n^c = \varDelta Y_n^r \tag{20-45}$$

$$\frac{\varDelta I_n}{I_{n-1}+\gamma g Y_{n-1}^r} = s\Big[\alpha(1-t)+t-g\Big] \tag{20-46}$$

From thses formulas, one just like an equation of E. Domar

$$R^r = s\Big[t+\alpha(1-t)-g(1-\gamma)\Big] \tag{20-47}$$

can be led, but in R^e an investment function is corresponds to a fixed percentage of disposal income. That is,

$$I_n = b(Y^e_{n-1} - T_{n-1}) = b(1-t)\,Y_{n-1} \tag{20–58}$$

And

$$Y^e_n = C_n + I_n + G_n \tag{20–54}$$

$$Y^e_n = (1-\alpha)(1-t)\,Y^e_n + b(1-t)\,Y^e_{n-1} + gY^e_n \tag{20–59}$$

$$R^e = \frac{Y^e_n - Y^e_{n-1}}{Y^e_{n-1}} = \frac{(b-\alpha)(1-t)+g-t}{\alpha(1-t)+t-g} \tag{20–60}$$

Thus R^e is gotten. Moreover, R^e shows such a relation as follows,

$$S_n = \alpha(1-t)\,Y^e_n + T - G$$
$$I_n = b(1-t)\,Y^e_{n-1}$$
$$S_n = I_n$$

Well, in above formula a tax rate on S_n is t (average tax rate) in n *term* and a tax rate on I_n is t in $n-1$ *term*. You can see it briefly in the formula (20–58) of Musgrave also. Therefore, the tax policy to increase or decrease t effects on the tax rate in not only n term but also $n-1$ term. but the fact as this is irrational and impossible actually.

While, the saving increases or decreases with the increasing or decreasing tax policy. You can recognise these facts with differentiating by t, that is,

$$\frac{d(S_n/Y_n)}{dt} = \frac{d\{\alpha(1-t)+t-g\}}{dt} = 1-\alpha \qquad >0$$

As clearly shown in the above differentiation saving rate shows the tendency to increase despite of the effect of increased taxation, because the total sum of tax yields is regarded as the same amount of savings. But this regard is unacceptable, but the part by reduction of saving may be involved into the total sum of tax yields. It can be accepted that ex post private saving plus not the total sum of tax yields but the balance of budget equals savings. How to treat when balanced budget $T=G$ will show us the reason.

Now my supply growth rate (capacity growth rate) R^r shows a long range balanced growth as G_w of Harod. Then, in addition to the policy, is the long range growth policy of demand required? Rather than that, is the fiscal policy, which holds the demand increasing rate temporaly in order to maintain the capacity growth rate under full employment, more practical?

When capacity growth rate is effective under full employment, supply growth rate is rather a long range one and impossible to fluctuate temporaly. Until the enlargement of producitve facilities begin to operate actually, it needs a time to construct, so to speak gestation term, and at the same time it has much technical difficulty to get (skilled) labour force. The adoption of reduction of operation for supply decrease is objected to by enterprises, so labour policy will be much more difficult. Therefore, we cannot help but choose a long-range policy.

On the other hand, as a demand policy can be expected immediate effect without any objection, firstly a tight money policy under over heating condition and a pump-priming policy under stagnated condition may be adopted as a counter-cycle policy.

Then I want to assume a temporaly demand increasing rate of expenditure. Spending income is as follows:

$$Y_n^e = C(1-a_{i1}t)+I(1-bt)+G \qquad \text{(IV–1)}$$

bt represent here a degree of checking against willingness to invest money by taxation. Increasing rate of spending income is

$$R^e = c'(1-a_{i1}t)+i'(1-bt)+g' \qquad \text{(IV–2)}$$

each c', i' and g' represents increasing rates for a year.

V. CONCLUSION

Here I want to sum up what I have said before.

1. An Anti-inflation Policy

In this case the given condition is $R^e > R^r$, that is, rate of demand increasing is over capacity growth rate. In such a case of inflation, Musgrave insists, the balanced growth rate, that is, the growth rate when R^e equals with R^r, should be risen as high as possible through making the lower growth rate rise. On the other hand, I regard R^e as a temporary rate of demand increasing and R^r as a long-range capacity growth rate. So I need not take the step as Musgrave did. I believe that we should rather aim at stable growth which maintains the process of growth by capacity growth rate, by lowering the rate of demand increasing that is over capacity growth rate. I will explain this theory by using arranged table.

$$\text{anti-inflation policy} \quad \begin{matrix} R^e \downarrow \\ \vee \\ R^r \uparrow \end{matrix} \quad \text{by} \begin{cases} t \uparrow & \text{about } R^e \text{ only when } \gamma < d \\ g \downarrow & \text{about } R^r \text{ only when } a_{i1}c > a_{i2}\alpha \end{cases}$$

As you see the table above, it is necessary to take a tax increse policy and a spending reductive policy for to fall the rate of demand increasing and to rise the capacity growth rate. But on capacity growth rate R^r, we must notice three conditions: under the condition of $a_{i1}c > a_{i2}\alpha$, a tax increase policy has effets on Gapacity Growth Rate R^r, under the condition of $a_{i1}c < a_{i2}\alpha$, a tax reduction policy is effective, and under the condition of $a_{i1}c=a_{i2}\alpha$, a general taxation policy has no power yet. A fiscal spending reductive policy under the condition of $\gamma<d$ or an increased spending policy under the condition of $\gamma>d$ has each efficiency, and under the condition of $\gamma=d$, a general spending policy has no power yet. You can easily understand these phenomena through the formula (III–2). Now we move to the question which is more effective, taxation policy or spending policy. First, let's examine R^r.

$$a_{i1}c\sim a_{i2}\alpha>\gamma\sim d \qquad \text{taxation policy}$$
$$a_{i1}c\sim a_{i2}\alpha<\gamma\sim d \qquad \text{spending policy}$$

in other words, more effective policy is one that bring a larger variation of the rate of saving (=investment) funds by an increased or decreased taxation and by a decreased or increased spending.

Next, let us see about R^e.

$$(a_{i1}c'+b')>1 \qquad \text{taxation policy}$$
$$(a_{i1}c'+b')<1 \qquad \text{spending policy}$$

that is, comparing the variation of rate of saving and investment to a taxation and a fiscal spending increased or decreased rate, a policy that has larger differcence is more effective.

2. An Anti-deflation Policy

In this case the given condition is $R^e<R^r$, i.e. increasing rate of demand is lower than Capacity Growth Rate. Productive facilities have fallen into under-employment.

From the stand point of my stable growth I come to agree with Musgrave's theory that we should hold full employment by rising temporary rate of demand increasing and make productive capacity work efficiently. This is shown on the table below:

$$\text{anti-deflation policy} \quad \begin{matrix} R^r\downarrow \\ \vee \\ R^e\uparrow \end{matrix} \quad \text{by} \begin{cases} g\uparrow & (R^r \text{ under } a_{i1}c<a_{i2}\alpha) \\ t\downarrow & (R^e \text{ under } \gamma<d) \end{cases}$$

As you see above, it's necessary as anti-deflation policy to lower capacity growth rate and to rise the rate of demand increaseing, as a consequence a fiscal increased spending policy or a tax reduction policy is required. But on capacity growth rate R^r, both a tax reduction policy when $a_{i1}c>a_{i2}\alpha$ and a tax increase policy when $a_{i1}c<a_{i2}\alpha$ are effective, therefore under the condition of $a_{i1}c=a_{i2}\alpha$, a general taxation policy is effectless. On the other hand on a fiscal expenditure policy an increased spending policy under the condition of $\gamma<d$ and a spending deductive policy under the condition of $\gamma>d$ are effective, so under the condition of $\gamma=d$, a general spending policy has no power itself. Which is more effective, taxation policy or spending policy? There is no difference as I have said before about anti-inflation policy on both R^r and R^e.

The invalidity of general taxation policy or general spending policy means to increase or decrease the total of tax yields by a unity average rate without any particular variation of tax souce or same to do with the total of fiscal spending (i. e. to deal with the investment and consuming fiscal spending without any variation of the ratio of investment and consuming expenditure). An increase or decrease of a certain speific tax and an increase or decrease of spending which variate the ratio of investment and consuming expenditure is quite different from the former one and it effects on the change of growth rate dfferently. And realistic phenomenon can be seen in the latter one. The equation (III–2) makes it possible to count even such a case.

The condition of efficiency of taxation policy and spending policy can be derived as the result of a partial derivation of the equation (III–1) by t and g. The equation (III–1) is

$$R^r = s\left[\alpha+\frac{1}{2}\left\{(a_{i1}c-a_{i2}\alpha)\,t+(\gamma-d)\,g\right\}\right]$$

Here, let's derivate patially this equation by t and g.

$$\frac{\partial R^r}{\partial t}=\frac{1}{2}(a_{i1}c-a_{i2}\alpha)\begin{cases}>0 & \text{when} \quad a_{i1}c>a_{i2}\alpha\\ =0 & \qquad\quad\;\; a_{i1}c=a_{i2}\alpha\\ <0 & \qquad\quad\;\; a_{i1}c<a_{i2}\alpha\end{cases}$$

$$\frac{\partial R^r}{\partial g}=\frac{1}{2}(\gamma-d)\begin{cases}<0 & \text{when} \quad \gamma<d\\ =0 & \qquad\quad\;\; \gamma=d\\ >0 & \qquad\quad\;\; \gamma>d\end{cases}$$

This is the explanations of these conditions as an anti-inflation policy and an anti-deflation policy.