A THEORY OF HOUSEHOLD CONSUMPTION BEHAVIOR*

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I.

In case of planning over the life span, it is far most important for us to know the amount of assets which a consumer is to have both for his own consumption after retirement and for inheritance to his children.

From this point of view, our object is to derive this sum of consumption after retirement and inheritance.

We must in the first place make clear the frameworks for deriving the sum.

At the same time we have to present relations between specifications of utility function and portfolio selections for many sorts of assets, and prepare a model to combine micro-variables to macro-variables.

II.

First we see the simple case of maximizing consumer's utility under some constraints. We can usually define the following model based on a utility function consisting of the rate of consumption at every moment of time.

Consider a consumer unit and let T be its horizen [14].

$$U=f(c_0, c_1, \cdots, c_T) \tag{1}$$

The quantity U, a real number, is the utility of the consumption plan c.

Next, let c_t be consumption plan at time t, y_t income at time t, x_t saving holdings (net worth) at the end of time t and r_t interest rate. And for each time, we would have the following relation.

$$c_t+x_t=(1+r_{t-1})x_{t-1}+y_t \tag{2}$$

Then we extend (2) into the model including portfolio selection in every moment of time. If we assume that asset, x_t, held at time t is distributed

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among many types of assets $x_{1t}, x_{2t}, \cdots, x_{mt}$, and each profit from types $r_{1t}, r_{2t}, \cdots, r_{mt}$, (2) is changed in the following relation

$$c_t + x_t = \sum_{i=1}^{m} (1 + r_{i \cdot t-1}) x_{i \cdot t-1} + y_t \qquad t = 0 \cdots T \tag{3}$$

From the above assumption

$$x_t = x_{1t} + x_{2t} + \cdots x_{mt} \tag{4}$$

and if U of (1) is maximized moving x_{it}'s when the special values of the x_0's or x_T are given under constraints (3), (4), we could obtain the solutions of x_{it}'s ($t=1, \cdots, T-1$).

But we cannot solve this by the Lagrange multiplier method. We, therefore, maximize mathematical expectation of utility [11, 16, 17], i. e.

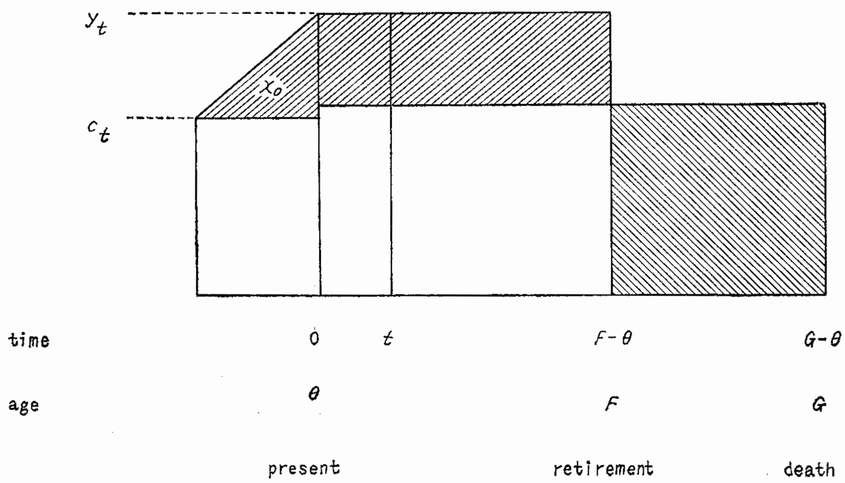
$$EU = Ef(c_0, c_1, \cdots, c_T) \tag{5}$$

III.

It is important to know what T is and how x_T is to be determined. Then we assume the following assumptions;

Consumer unit refers to consumption plan over his life span. From this point of view, T and x_T are the notions including viewpoint of consumer's life cycle. First we see t as the time of consumer's life cycle stages; $t=0$, present time (he is now at the age θ), the time of retirement equals $F-\theta$ (he retires at the age F), the time of death equals $G-\theta$ (dies at the age G) [1, 2, 4, 5, 15].

Then we make a simple figure for these relations.



According to the budget constraint over the life span, it must be assumed that $x_{F-\theta}$, net worth at the time of retiring, equals the sum of consumption for his enjoying $G-F$ years after retirement and inheritance to his children [6, 7], so called the hump-saving [9].

Accordingly, it is very important for us to have $x_{F-\theta}$.

From this point, we examine some of the way of deriving $x_{F-\theta}$.

It is assumed that consumer recieves utility only from present and prospective consumption, c_t ($t=0, \cdots, G-\theta$) and from inheritance to his children, $h_{G-\theta}$. If we assume further that the price level of consumer goods is not expected to change appreciably over the balance of the life span, so that the volume of consumption is uniquely related to its value, than for a consumer at the age θ, the utility function is given by

$$U=f(c_0, c_1, \cdots, c_{G-\theta}, h_{G-\theta}) \tag{6}$$

And the hump-saving equals to (7).

$$x_{F-\theta}=\sum_{t=F-\theta+1}^{G-\theta} c_t+h_{G-\theta} \tag{7}$$

On the other hand, from (3), we have

$$c_t+x_t=\sum_{i=1}^{m}(1+r_{i\cdot t-1})x_{i\cdot t-1}+y_t \qquad t=0, \cdots, F-\theta \tag{8}$$

Next we write the following form as a new utility function

$$U=f(c_0, \cdots, c_{F-\theta}, x_{F-\theta}) \tag{9}$$

This function is a kind of modification of (6).

Now, in the present case it is reasonable to assume that (9) is of the form

$$U=\sum_{t=0}^{F-\theta}\alpha_t g[c_t]+\varphi[x_{F-\theta}(G)] \tag{9$'$}$$

Where α may be interpreted as a subjective discount function being a non-negative real valued function on the interval $[0, F-\theta]$ and having a continuous first derivative. g is the utility associated with the rate of consumption at every moment of time and should be a concave real valued function and has two continuous derivatives the second of which is everywhere negative and the first everywhere non-negative. φ is linear function defined on the entire real line and then, which implies that a positive $x_{F-\theta}$ (G) adds to utility while a negative $x_{F-\theta}(G)$ subtracts from utility [13, 19].

Here, we take account of the fact that he does not know how long he will live. Then T is not a fixed number but a random variable with a known, probability distribution.

If (9)' is maximized under constraints (4), (7), (8), we can obtain the values of the $x_{it}'s$. and also, as we noted above ((5)), (9) must be written as

$$EU = Ef(c_0, \cdots, c_{F-\theta}, x_{F-\theta})$$

Then we can rewrite (9)′ as

$$EU = E\Big[\sum_{t=0}^{F-\theta} \alpha_t g[c_t] + \varphi[x_{F-\theta}(G)]\Big] \tag{9}''$$

IV.

Let us assume that we find $x_{F-\theta}$ only in a model of certain macro-relationships, and this is equal to start the discussion along the lines of Meade's model [13].

He consideres first the following notations ;

the rate of growth of the working population, 1
the rate of Harrod-neutral technical progress (i. e. of labor-expanding progress), 1'
the rate of growth of total output, of total consumption, of total savings, $1+1'$
the rate of growth of output per head, of consumption per head, of the wage rate, $1'$
the age of consumer unit, θ
time, t
net worth which consumer holds at the age θ, X_θ
the amount of property which consumer at the age θ will inherit to his children, I_θ
consumption of person at the age θ, C_θ
the rate of wage of person at the age θ, W_θ
the proportion of total national income saved, S
the proportion of the national income going to profits, R
the rate of interest, i

and if I_θ, C_θ, W_θ, X_θ are written as (here, as a modification of Meade's discussion)

$$I_\theta = Ie^{lH+(l'-i)(H-\theta)}$$

$$C_\theta = Ce^{(\sigma i-l')\theta} \sum_{t=0}^{G-\theta} e^{(\sigma-1)it}$$

$$W_\theta = W\sum_{t=0}^{F-\theta} e^{(l'-i)t}$$

$$X_\theta = I_\theta + C_\theta - W_\theta$$

(a budget constraint over the life span),

After all, he develops his model at the macroeconomic level in the following relations

(i) $$K_0 = NIe^{(l+l'-i)H}\sum_{\theta=0}^{H} e^{(i-l-l')\theta} + NC\sum_{\theta=0}^{G}\left(\sum_{t=0}^{G-\theta} e^{(\sigma-1)it}\right)e^{(\sigma i-l'-l)\theta} - NW\sum_{\theta=0}^{F}\left(\sum_{t=0}^{F-\theta} e^{(l'-i)t}\right)e^{-l\theta}$$

(ii) $$\frac{iKo}{WN\sum_{\theta=0}^{F} e^{-l\theta}} = \frac{R}{1-R}$$

(iii) $$\frac{C\sum_{\theta=0}^{G} e^{(\sigma i-l-l')t}}{W\sum_{\theta=0}^{F} e^{-l\theta}} = \frac{1-S}{1-R}$$

(iv) $$W = W(i)$$

(v) $$R = R(i)$$

(vi) $$S = \frac{l+l'}{i}R$$

(vii) Specification of I

Where the variables of his model are I, C, W, S, R, i, K_0 and the parameters, N, 1, 1′, σ, F, G, H.

We can, therefore, find the equilibrium levels of all variables as the solution to a system of simultaneous equations. Finally, we must obtain the following results.

First, total assets at time $F-\theta$ is given by

$$K_{F-\theta} = K_0 + NW\sum_{\theta=0}^{F}\left(\sum_{t=0}^{F-\theta} e^{(l'-i)t}\right)e^{-l\theta} - NC\sum_{\theta=0}^{F}\left(\sum_{t=0}^{F-\theta} e^{(\sigma-1)it}\right)e^{(\sigma i-l-l')\theta}$$

And the population at time $F-\theta$ is *defined*

$$\sum_{\theta=0}^{F} e^{l(F-\theta)}Ne^{-l\theta} = \sum_{\theta=0}^{F} Ne^{lF-l\theta-l\theta} = N\sum_{\theta=0}^{F} e^{l(F-2\theta)}$$

We can thus establish that total assets at time $F-\theta$ per head is

$$x_{F-\theta} = K_{F-\theta}\Big/N\sum_{\theta=0}^{F} e^{l(F-2\theta)}$$

V.

Let us consider a specific example for our problems. We assume to specify (9)′ as

$$U=\sum_{t=0}^{F-\theta}\alpha_t\left(p-\frac{q}{c_t}\right)+\beta(aG+b) \tag{13}$$

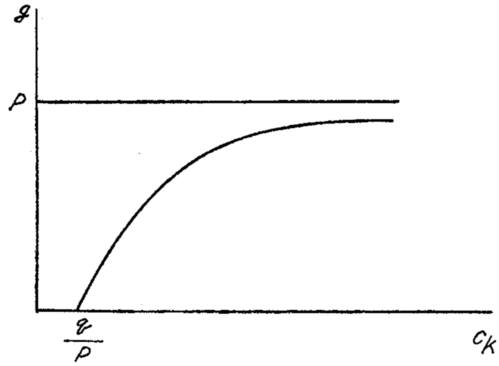
This utility function satisfies the general assumptions of the economic theories, i. e.

$$\frac{\partial U}{\partial c_t}=\frac{\alpha_t q}{c_t{}^2}\geqq 0$$

$$\frac{\partial^2 U}{\partial c_t{}^2}=\frac{2\alpha_t q}{c_t{}^3}\leqq 0$$

$$\frac{\partial U}{\partial G}=a\beta>0$$

$$\frac{\partial^2 U}{\partial G^2}=0$$



And from (9)″

$$EU=\sum\alpha_t(p-qEc_t^{-1})+\beta(aEG+b) \tag{13$'$}$$

Here, we will use Taylor expansion on c_t^{-1} about Ec_t.

$$\begin{aligned}C^{-1}&=(Ec_t)^{-1}-(c_t-Ec_t)(Ec_t)^{-2}-(c_t-Ec_t)^2(Ec_t)^{-3}\\&\fallingdotseq(Ec_t)^{-1}-(c_t-Ec_t)(Ec_t)^{-2}-(c_t-Ec_t)^2(Ec_t)^{-3}\end{aligned}$$

As a result, we can obtain

$$E(c_t^{-1})=(Ec_t)^{-1}-E(c_t-Ec_t)^2(Ec_t)^{-3}$$

On the other hand, consumption c_t at each time is from (8)

$$c_t=y_t+\sum_{i=1}^{m}(1+r_{i\cdot t-1})x_{i\cdot t-1}-x_t \tag{14}$$

Then, if we assume the following [8]

$$\begin{aligned}&Ey_t=\mu y_t \qquad Er_{it}=\mu_{it}\\&E(r_{it}-Er_{it})(r_{jt}-Er_{jt})=\sigma_{ijt}\\&E(y_t-\mu_{yt})^2=\sigma_{yt}\\&E(y_t-\mu_{yt})(r_{it}-\mu_{it})=\sigma y_{it}\end{aligned}$$

we have

$$Ec_t=\sum_{i=1}^{m}(1-\mu_{it})x_{i\cdot t-1}+\mu_{yt}-x_t \tag{15}$$

$$E(c_t-Ec_t)^2=\sum_{j=1}^{m}\sum_{i=1}^{m}\sigma_{ijt}x_{i\cdot t-1}x_{j\cdot t-1}+2\sum_{i=1}^{m}\sigma_{yit}x_{i\cdot t-1}+\sigma_{yt} \tag{16}$$

And if $EG=\mu_G$, we have

$$E\varphi=\beta a\mu_G+b\beta \tag{17}$$

By substituting (15), (16), (17) into (13)′ we can see

$$\begin{aligned}EU=\sum_{t=0}^{F-\theta}\alpha_t\Bigg[p-q\Bigg\{&\left(\sum_{i=1}^{m}(1-\mu_{it})\,x_{i\cdot t-1}+\mu_{yt}-x_t\right)^{-1}\\&-\left(\sum_{j=1}^{m}\sum_{i=1}^{m}\sigma_{ijt}x_{i\cdot t-1}x_{j\cdot t-1}+2\sum_{i=1}^{m}\sigma_{yit}x_{i\cdot t-1}+\sigma_{yt}\right)\\&\times\left(\sum_{i=1}^{m}(1-\mu_{it})\,x_{i\cdot t-1}+\mu_{yt}-x_t\right)^{-3}\Bigg\}\Bigg]\\&+a\beta\mu_G+b\beta\end{aligned} \tag{18}$$

Giving the expectations and variances-covariances of income y_t and rate of profit r_{it};

$$\mu_t=\begin{pmatrix}\mu_{yt}\\ \mu_{1t}\\ \vdots\\ \mu_{mt}\end{pmatrix}\qquad V_t=\begin{pmatrix}\sigma_{yt} & \sigma_{y1t} & \cdots & \sigma_{ymt}\\ \sigma_{10t} & \sigma_{11t} & \cdots & \sigma_{1mt}\\ \vdots & \vdots & & \vdots\\ \sigma_{m1t} & \sigma_{m2t} & \cdots & \sigma_{mmt}\end{pmatrix}$$

And the expectation of G; μG, x_0 or $x_{F-\theta}$ under constraint

$$x_t=\sum_{i=1}^{m}x_{it}$$

$$x_{it}\geqq 0\qquad i=1\cdots m\qquad t=0\cdots(F-\theta-1)$$

we can determine the x_{it} $(i=1\cdots m,\ t=0\cdots(F-\theta-1))$ so that EU is maximized.

VI.

A whole class of the maximization problem, e.g. (18) has, in general as we pointed out before, severe difficulties for obtaining the solutions. We note, here, that dynamic programming approach (D.P.) has no procedual difficulties, i.e. D.P. may, computationally, be easier [3, 18]. And also the value of D.P. lies, in the insight it supplies concerning optimum strategies and how, in principle, they could be computed.

Now we assume that (18) can be reduced to the form

$$EU=\sum_{t=1}^{F-\theta}R_t(x_{1\cdot t-1},\cdots,x_{m\cdot t-1},x_t) \tag{19}$$

The manner in which (19) is obtained is as follows [10].

$$EU = R_1(x_{1\cdot 0}, \cdots, x_{m\cdot 0}, x_1) + R_2(x_{1\cdot 1}, \cdots, x_{m\cdot 1}, x_2) + \cdots + R_{F-\theta}(x_{1\cdot F-\theta-1}, \cdots, x_{m\cdot F-\theta-1}, x_{F-\theta}) \tag{20}$$

Where we assume that $x_{F-\theta}$, hump-saving, has already been given by Meade's macrorelationships. From (20), the sum of R_t over time (s to $F-\theta$) is given by

$$Z_s = \sum_{t=s}^{F-\theta} R_t(x_{1\cdot t-1}, \cdots, x_{m\cdot t-1}, x_t) \tag{21}$$

In this case, we can maximize Z_s on x_{it} ($i=1\cdots m, t=s\cdots F-\theta$) when the special value is given x_t, especially, in case that R_t is a specific form.

Therefore, taking $P_{s-1}(x_s)$ as the value of Z_s maximized, we have

$$P_{s-1}(x_s) = \text{Max}\, Z_s \tag{22}$$

And when $s=F-\theta$, (22) is

$$P_{F-\theta-1}(x_{F-\theta}) = \text{Max}\, Z_{F-\theta} = \text{Max}\, R_{F-\theta}(x_{1\cdot F-\theta-1}, \cdots, x_{m\cdot F-\theta-1}, x_{F-\theta}) \tag{23}$$

Accordingly, we can easily obtain each value of $x_{1\cdot F-\theta-1}, \cdots, x_{m\cdot F-\theta-1}$ so that $R_{F-\theta}$ is maximized when $x_{F-\theta}$ was given.

On the other hand, (22) is expanded in the following form.

$$\begin{aligned} P_{s-1}(x_s) &= \underset{x_s=\sum^m x_{is}}{\text{Max}} \; \underset{x_{s+1}=\sum^m x_{i\cdot s+1}}{\text{Max}} \cdots \underset{x_{F-\theta-1}=\sum^m x_{i\cdot F-\theta-1}}{\text{Max}} \sum_{t=s}^{F-\theta} R_t(x_{1\cdot t-1}, \cdots, x_{m\cdot t-1}, x_t) \\ &= P_s(x_{s+1}) + \text{Max}\,\{R_s(x_{1\cdot s-1}, \cdots, x_{m\cdot s-1}, x_s)\} \end{aligned} \tag{24}$$

This function equals to "recurrence relation" in D.P..

Finally, we know the values of $x_{i\cdot 0}$'s so that R_1 is maximized when x_1 was given.

$$P_0(x_1) = P_1(x_2) + \text{Max}\,\{R_1(x_{1\cdot 0}, \cdots, x_{m\cdot 0}, x_1)\} \tag{25}$$

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