MONOPOLISTIC COMPETITION AND GENERAL EQUILIBRIUM THEORY:

A Particular Equilibrium Version of The Monopolistic Firm with its Subjective Demand Curve

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1. INTRODUCTION

In spite of the same title as that of Triffin's famous critical work,[1] this paper is intended to "pour the new wine into the old goatskins": i.e., we shall discuss theoretical problems of monopolistic competition within the compass of particular equilibrium methodology. The author believes in the pedagogical usefulness of Marshallian methodology even now when we have a fullfledged theory of Walrasian general equilibrium. He also believes that the particular equilibrium discussion should be founded firmly upon the general equilibrium ground.

From this point of view, the well-known Chamberlinian construction (Figure 1) is not completely satisfactory. In *Section 2*, we shall examine Chamberlin's theory of equilibrium within a large group,[2] and show that the theory cannot explain how the number of sellers in the general market is determined because of its simplifying assumption. In order to overcome this difficulty and others, we need a theory of general equilibrium into which monopolistic firms are explicitly incorporated. And it has been with us now for ten years.

In the early 1960's, twenty years after Triffin's criticism, the first pioneering work was done by Takashi Negishi.[3] In *Section 3*, we shall illustrate his theory in terms of particular equilibrium methodology, and show that the monopolists' modes of expectations have vital influence upon the dynamic properties of the market equilibrium. This point is not explicit in Negishi's statement. We shall assert that we should take this point into account when we explain the number of monopolistic firms in the general market.

In addition to a set of ordinary assumptions of the purely competitive general equilibrium theory, Negishi introduced two special assumptions into his model. The first asserts that every monopolistic firm is separated from every other such firm. In other words, there can be at most only one monopolistic firm in every market. According to his remark, the aim of this assumption is to exclude from his model such situations as the bilateral monopoly, oligopoly, etc., and confine himself to the study of monopolistic competition.[4]

In view of importance of its role, this assumption deserves to have a

1) Robert Triffin, *Monopolistic Competition and General Equilibrium Theory*, Harvard University Press, 1940.

2) E. H. Chamberlin, *The Theory of Monopolistic Competition*, Harvard University Press, the 1st ed. 1933, and the 8th ed. 1962: esp. pp. 81–100.

3) "Monopolistic Competition and General Equilibrium", *Review of Economic Studies*, XXVIII, (1961), pp. 196–201.

4) Ibid., p. 197.

particular name. The reader should imagine a frog spawn. Metaphorically, it could be regarded as the market system. Each egg coated with a piece of jelly represents each market. The jelly is a group of purely competitive units (buyers and sellers) of each market; and the egg, a black kernel, is the monopolistic firm which "dominates"[5] the market. What the first assumption asserts is that there may be pieces of jelly which have no kernels in them, but there cannot exist any that contains two or more kernels in a piece. Thus we shall name it the *monokernel* assumption.

The second is a set of assumptions which state that each monopolistic firm has its own *subjective* demand curves for its products.[6] They are subjective because they show the monopolist's expectations about the market demand schedule. His expectations are formed on the basis of the market data: i.e., the current price and the quantity currently demanded. By virtue of the monokernel assumption, the excess demand curve of the purely competitive group (a piece of jelly) serves, as it is, as the *objective* demand curve for the monopolist's product.[7] The market data given to the monopolist are nothing but co-ordinates of a revealed point on the objective demand

5) The usage of "to dominate" is Negishi's (See ibid. p. 196). It does not imply anything active or aggresive. It implies merely "to exist in" or "to coexist in".

6) Although Arrow and Hahn remarked that "Negishi assumed that each monopolist produced only one commodity" (*General Competitive Analysis*, San Francisco, 1971, p. 167), Negishi's assumption of productive technoloty permits joint production; and his statement about subjective demand curves implies, in accordance with the monokernel assumption, that one monopolistic firm can have more-than-one subjective demand curves for its more-than-one products. According to our metaphor, one monopolistic firm can be represented by more-than-one kernels in the frog spawn. Of course, we are virtually assuming that each monopolist produces only one product, since we are encumbered with the fetters of the particular equilibrium methodology.

7) Recently H. Nikaido wrote: ("Negishi's) perceived demand functions embody only firms' subjective perception of the economic situation and conjectures as to rival firms' behaviors rather than a direct recognition of the interdependence of firms in the objective sense. A monopolist controls prices or outputs through the interdepent relations among economic agents in the objective sense even if his decision making is based on a profit estimate in terms of his perceived demand function. So we are still not completely satisfied with our knowledge about monopolistic competition in the general equilibrium context. The purpose of this work is to attempt to shed some light on the interdependence of agents in the general equilibrium context in the objective sense by constructing objective demand functions." ("Monopolistic Competition, Objective Demand Functions and the Marxian Labor Value in the Leontief System", *Discussion Paper*, No. 15, May 1972, University of Minnesota, p. 3).

Unfortunately, this statement seems to suggest that there can be no explicit objective demand curves in Negishi's model. This is because, in the general equilibrium context, he has in his mind the interrelationship between the market demand schedules and the consumers' incomes out of the monopolistic firms' profits. Within the framework of the particular equilibrium analysis, however, we neglect this interrelationship; and a set of ordinary assumptions of general equilibrium theory guarantees the existence of the negatively sloping excess demand schedules of the purely competitive groups. These are, by virtue of the monokernel assumption, the objective demand curves for the monopolistic sellers.

curve.

If one point on the objective demand curve is revealed in the market, the monopolistic seller forms his subjective demand curve on the basis of this information. It is not identical, in general, with the objective demand curve for his product: i.e., he cannot estimate the market demand correctly. With some reservations of uncertainty, he will plan his supply so as to maximize his expected profit. In other words, he will be able to calculate his output and selling price under which subjective marginal revenue is equal to marginal cost. Thus we shall establish a relationship between the revealed market-price-and-demand and the seller's planned selling-price-and-supply. This implies that we can deal with the monopolistic seller as if he were a price-taker. In *Section 3*, we shall define such *a la* monopolists' supply curves as the maximal curves.

In *Section 4*, we shall relax the restriction of the monokernel assumption. Within the compass of diagrammatic treatment, we confine our remarks to the duopolistic case in which two monopolistic sellers dominate the market. We shall assert that the monopolistic sellers' modes of expectations are vitally important in determining the number of sellers in the market.

2. A CRITICAL SURVEY OF THE CHAMBERLINIAN CONSTRUCTION

As is pointed out by Triffin,[8] the Chamberlinian diagram (Figure 1) is constructed on the basis of two assumptions. The first ensures the invariability of the cost curve (PP') throughout the argument: i.e., the cost curve presents the firm's costs as a function of its own level of output, independently of the output of the group. In reality, expansion or contraction of production by the group may have certain effects upon the supply-and-demand position of other factor markets, so that the changes in factor prices bring about the changes in the expenditure of individual firms. The assumption excludes from the argument not only an account of the genuine external effects but also that of the pecuniary external effects due to the market interdependence. It

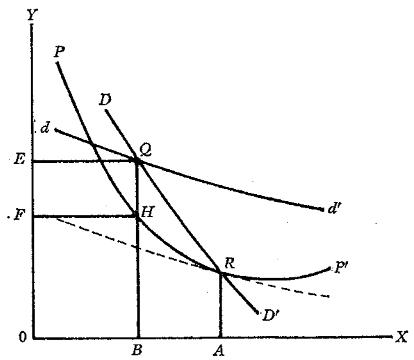

Figure 1.[9]

[8] op. cit., pp. 23–5.

[9] This is a reproduction of Figure 14 of Chamberlin's, op. cit., p. 91. It is also reproduced as Figure 1 in Triffin, op. cit., p. 26.

is inevitable for the particular equilibrium methodology, and we shall assume it througout the following discussion.

The second, so-called "symmetry" assumption, asserts that both demand and cost curves for all the products are uniform throughout the group. As regards demand curves in particular, this implies that all markets are of equal size. DD' is therefore a demand curve for the product of any one seller, which is constructed on the basis of the *ceteris paribus* clause asserting that all prices of rival's products are always identical with his own. It is a fractional part of the demand curve for the general class of product. Thus, "the position of DD' depends upon the number of sellers in the field. It lies further to the left as there are more of them, since the share of each in the total is then smaller; and further to the right as there are fewer of them, since the share of each in the total is then larger."[10)]

Tautologically, the theory constructed on the basis of the symmetry assumption cannot explain the differences among the market shares of sellers, while they may be a matter of little relevance in the case of the large group. The average share, or the number, of the sellers in the group (i. e., the position of DD') is, however, what the theory must explain, even in the case of the large group. Can the symmetry assumption be justified for this purpose? Suppose a situation in which the number of sellers is so great that their losses are larger than ever. Then, equilibrium must be achieved only by the elimination of firms. But, how is it done? By virtue of the symmetry assumption, all firms must be marginal firms. Does this imply that all firms must be eliminated? Perhaps, not. Then, must all firms contract their outputs by the equal amounts? This does not change the number of sellers. Under the symmetry assumption, therefore, the position of DD' cannot be adjusted so as to attain the tangency solution at R.

dd' is the demand curve for the product of the same seller, and is constructed on the basis of the *ceteris paribus* clause asserting that all rivals' prices are fixed at the level BQ. Comparing both *ceteris paribus* clauses of DD' and dd', we can easily see the reason why DD' is steeper than dd'. dd' shifts with every change in the general level of rivals' prices, taken as parameters. For example, it slides downwards along DD' to the position of the broken curve which passes through at R, as the general level of rivals' prices falls from BQ to AR.

It is assumed that the firm will maximize profits on the basis of dd'. According to Chamberlin's argument, "the sales of each (producer) are OB, and the profits of each are $FHQE$. …Evidently, profits may be increased for any individual seller by moving to the right along dd'.[11)] In other

10) Chamberlin, op. cit., p. 92.
11) Chamberlin, op. cit., pp. 90–91.

words, he asserts that the marginal revenue curve derived from *dd'* crosses the marginal cost curve derived from *PP'* at a certain point in the right-hand region of *BQ*. It is evident in Figure 1, because the tangent of *PP'* at *H* is steeper than that of *dd'* at *Q*. This conclusion, however, does not follow solely from the *ceteris paribus* conditions of *DD'* and *dd'*. In fact, Chamberlin assumes implicitly that *dd'* slides downwards along *DD'* in such parallel ways that the tangent of *dd'* at *Q* has the same slope as that of the broken curve at *R*.[12]

The proof of the necessity of such an implicit assumption (or similar restrictions) is trivial. If we are free from such a restriction, we can draw curves so that the tangent of *dd'* at *Q* is so much steeper than that of *PP*, at *H* that profits may be increased for any individual seller by moving to the left along *dd'*, in no conflict with the *ceteris paribus* clauses of *DD'* and *dd'*.

The final equilibrium holds at *R* where the well-known tangency condition is satisfied: i.e., tangency between the average cost curve and the sales curve[13] of the seller is attained. Is the existence of the unique equilibrium point guaranteed? As a matter of fact, Chamberlin assumes implicity that the shift of *DD'* does not bring about a change in the slope of the sales curve. Suppose that we have a little steeper sales curve in Figure 1 than that actually drawn. Then, *R* is no longer the equilibrium point. Under the above implicit assumption, however, we will be able to find the new equilibrium point by shifting *DD'* to the left along *PP'* (although the shift cannot be justified under the symmetry assumption). Similarly, if we have a little flatter sales curve than that actually drawn in Figure 1, we will be able to reach the new equilibrium point by shifting *DD'* to the right along *PP'*.

The assumption that the sales curve has a constant slope, therefore, is needed to ensure the existence of the unique equilibrium. Without such an assumption, it is possible to have no equilibrium positions. Suppose, for example, that we have a little steeper sales curve in Figure 1 than that actually drawn, and that a shift to the right of *DD'* brings about no changes in the slope of the sales curve, while a shift to the left of *DD'* makes the slope much steeper. Then, we can find no equilibrium points in the diagram, whether we shift *DD'* to the left or to the right.

In the next section, we shall draw two demand curves (objective and

12) This assumption (or its generalized versions) seems to be necessary for ensuring the existence of the normal equilibrium. We shall also adopt this assumption in the next section. It is a wonder that Negishi's subjective demand curve is free from such a restriction. See Negishi, op. cit., p. 197.

13) Taking after Triffin (op. cit., p. 5), we designate *dd'* as the sales curve. Thus, the broken curve in Figure 1 is the sales curve which passes through at *R*.

subjective) in one diagram, the positions of which are just similar to those of DD' and dd' in Figure 1. It is noted that Chamberlin's sales curve is not of subjective nature but of objective one. He explicitly remarks that the elasticity of the sales curve could be regarded as a rough index of buyers' preferences for the product of one seller over that of another.[14] In other words, he assumes that the seller has a perfect knowledge about the shape of his sales curve. This seems to be implausible. Surely, "when products are differentiated, buyers are given a basis for preference, and will therefore be paired with sellers, not in random fashion, but according to these preferences."[15] It is one thing, however, to assert the presence of a particular relationship between sellers and buyers, and it is another to maintain that the seller perceives the relationship correctly. The possibility of the product differentiation brings about a new type of uncertainty, and the seller cannot help his uncertain expectation about the sales conditions. Under such circumstances, the sales curve may be rather of subjective nature than of objective nature.

3. THEORY OF MONOPOLISTIC COMPETITION IN THE PRESENCE OF THE MONOKERNEL ASSUMPTION: A PARTICULAR EQUILIBRIUM VERSION OF NEGISHI'S MODEL

3–1. *Some Fundamental Concepts*

We have already mentioned that Negishi's model incorporates two characteristic assumptions: the monokernel assumption and the assumption that specifies the form of the subjective demand curve. The latter states that each monopolistic firm has its own "subjective inverse demand function" for its products:

(1) $$p_i = a_i y_i + b_i$$

where p_i and y_i are the i-th monopolistic firm's expected price and expected sales of its product, respectively; and both parameters a_i (<0) and b_i (>0) depend upon the market price and the excess demand which are revealed in a current state of the purely competitive group. For simplicity, we shall assume that a_i is constant, although Negishi's original work is free from this specification.

The monopolistic seller, as Negishi assumed implicitly, has no certain knowledge about the whole shape of the excess demand curve of the purely competitive group. In such an uncertain situation, he cannot help making his expectation on the basis of his subjective valuation of the effects of his

14) Chamberlin, op. cit., p. 93.
15) Ibid., p. 69.

product differntiation with reference to the currently revealed market data. The slope of the subjective demand curve a_i, therefore, differs from the actual slope of the objective demand curve. Though it is not necessarily linear in Negishi's original version, we shall assume that the objective demand curve is given by the linear equation:

$$p = \alpha y + p_0 \tag{2}$$

where p and y are the market price and the market demand for the i-th firm's products; and p_0 is the purely competitive equilibrium price which would hold if the monopolistic seller were excluded from the market.

Following in the wake of what was originally suggested by Bushaw and Clower,[16] Negishi assumes that the expected price is realized in the market if the expected output is identical with the excess demand of the purely competitive market. In other words, the subjective demand curve has a point at which the expected relationship between p_i and y_i is consistent with the given market data. This implies that the subjective demand curve given by (1) intersects the objective demand curve given by (2) at a point in the first quadrant. This point will be called the *consistency* point of the subjective demand curve.

In view of (2), the consistency point Q has the coordinates:

$$Q = (y, y + p_0)$$

Thus, the equation of the subjective demand curve passing through at Q is given by

$$p_i = a_i y_i + (\alpha - a_i) y + p_0 . \tag{3}$$

Multiplying the expected price p_i by the expected sales y_i, we obtain the expected total revenue TR_i on the basis of the revealed point Q:

$$TR_i = a_i y_i^2 + (\alpha - a_i) y y_i + p_0 y_i \tag{4}$$

Differentiating TR_i with respect to y_i, we obtain the expected marginal revenue MR_i:

$$MR_i = 2a_i y_i + \gamma(\alpha - a_i) y_i + (\alpha - a_i) y + p_0 \tag{5}$$

where $\gamma = \dfrac{dy}{dy_i}$. We shall call γ the coefficient of data-adjustment. It is noted that the relationship between (3) and (5) is reduced to the ordinary average-marginal relationship (i. e., the marginal curve is twice as steep as the average curve), when the seller's expectation is perfectly correct ($\alpha = a_i$), or when the coefficient of data-adjustment is zero. In the latter case, we have

16) D. W. Bushaw and R. W. Clower, *Introduction to Mathematical Economics*, 1957.

$$MR_i = 2a_i y_i + (\alpha - a_i) y + p_0 . \tag{5'}$$

Equating marginal revenue to marginal cost ($MR_i = MC_i$), we obtain the level of output at which the expected profit is maximized:

$$y_i = \frac{MC_i - p_0 - (\alpha - a_i) y}{2a_i + \gamma(\alpha - a_i)} . \tag{6}$$

Substituting (6) into (3), we have the expected price corresponding to the level of output given by (6):

$$p_i = \frac{a_i \{MC_i - p_0 - (\alpha - a_i) y\}}{2a_i + \gamma(\alpha - a_i)} + (\alpha - a_i) y + p_0 . \tag{7}$$

Eliminating y from (6) and (7), we obtain the relationship between y_i and p_i at which the seller's expected profit is maximized:

$$p_i = -\left\{a_i + \gamma(\alpha - a_i)\right\} y_i + MC_i \tag{8}$$

which is reduced to:

$$p_i = -a_i y_i + MC_i \tag{8'}$$

when the coefficient of data-adjustment is zero.

3–2. *Equilibrium in the Case of Pessimistic Expectation*

We shall call the graph of the equation (8) or (8′) the maximal curve. For simplicity, assume that the coefficient of data-adjustment is zero, and that MC_i is constant. Figure 2 shows the case in which the subjective demand curve has a steeper slope than the objective one, i. e., $a_i < \alpha$. Along the horizontal axis, the market demand (as to the objective demand curve DD'), the expected sales (as to the subjective demand curves d_0d_0', $d_1d'_1$, etc.), and the level of output (as to the marginal cost curve CC') are measured. Along the vertical axis, the market price (as to DD'), the expected sales price (as to dd'), and the marginal cost (as to CC') are measured.

By definition (see the equation (2)), the vertical axis' intercept Q_0 of DD' is identical with the level of competitive equilibrium price p_0. The subjective demand curve d_0d_0' has Q_0 as its consistency point. This implies that the seller expects his sales conditions given by $d_0'd_0'$, when purely competitive equilibrium holds in the market.

If $a_i < \alpha$ as is in Figure 2, the subjective demand curve d_0d_0' lies below the objective demand curve DD' within the relevant region (i. e., the region of positive output). This implies that his expectation is *pessimistic*, both in a Marshallian sense that his expected prices are lower than the market prices which will hold if the market demand is balanced against the same

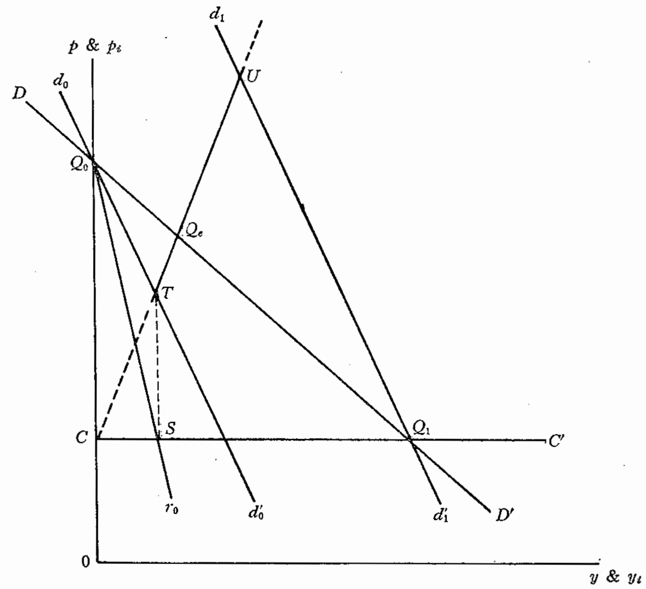

Figure 2. Equilibrium in the Case of Pessimistic Expectation

amount of expected output, and in a Walrasian sense that his expected sales are short of the amounts which he will be able to sell at any given price level. We shall call his sales expectation, therefore, the *pessimistic* one.

The line segment Q_0r_0 whose equation is given by (5′) with $y=0$ is the marginal revenue curve attached to the subjective demand curve d_0d_0'. Q_0r_0 crosses the marginal cost curve CC' at S. T which lies on the subjective demand curve right above S, therefore, gives us the expected price and output at which his expected profits are maximized on the basis of the given Q_0. As the consistency point slides down along DD' from Q_0 to Q_1, the maximum-profit point on the subjective demand curve moves up along the line segment TU. This is the maximal curve defined by the equation (8′) with the appropriate constraints of its range.

All the points on the maximal curve are not necessarily the equilibrium points, because most of them are inconsistent with the market demand. The only point Q_e at which the maximal curve crosses the objective demand curve is the equilibrium point. Solving the system of the two equations (2) and (8) together with conditions $y_i=y$ and $p_i=p$, we obtain the coordinates of the equilibrium point:

$$(9)\qquad Q_e=(y_e,p_e)=\left(\frac{MC_i-p_0}{\alpha+a_i+\gamma(\alpha-a_i)},\ \frac{\alpha MC_i+\{a_i+\gamma(\alpha-a_i)\}p_0}{\alpha+a_i+\gamma(\alpha-a_i)}\right)$$

which is reduced to:

$$(9') \qquad Q_e = \left(\frac{MC_i - p_0}{\alpha + a_i},\ \frac{\alpha MC_i + a_i p_0}{\alpha + a_i} \right)$$

when $\gamma = 0$ as in Figure 2.

To sum up, Q_e shows the equilibrium position in the sense that both the following conditions are satisfied simultaneously: (a) Subjective equilibrium: the monopolistic firm maximizes its expected profit at Q_e because it lies on the maximal curve; and (b) Market equilibrium: the market demand and supply are balanced at Q_e because it lies on the objective demand curve.

3–3. *Eqilibrium in the Case of Optimistic Expectation*

Figure 3 shows the maximal curve TU which is derived from the *optimistic* subjective demand curves together with the constant marginal cost assumption. By *optimistic,* we mean that the slope of the objective demand curve is steeper than that of the subjective one ($a_i > \alpha$): therefore, the seller's expected prices are higher than the market prices which will reveal themselves if the market demand is balanced aginst the same amount of expected output (on the right side of the consistency point).

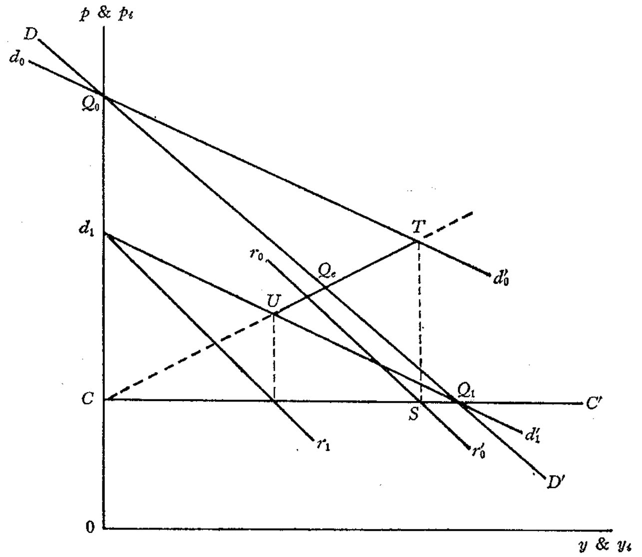

Figure 3. Equilibrium in the Case of Optimistic Expectation

The subjective demand curve d_0d_0' has its consistency point Q_0 at the equiliblium point of the purely competitive market. Its marginal revenue curve is r_0r_0' which crosses the marginal cost curve CC' at S: therefore, the co-ordinates of point T give the planned output and selling price which are consistent with the expected-profit maximization and the given market data Q_0.

As the consistency point slides down along DD' from Q_0 to Q_1, the maximum-profit point on the subjective demand curve moves down along the line segment TU. Of course, d_1r_1 is the marginal revenue curve of the subjective demand curue d_1d_1' whose consistency point is Q_1. As is shown by the equation (8′), the maximal curve TU is a segment of the straight line whose slope is the same as that of the subjective demand curve in absolute terms $(-a_i)$ and whose vertical axis' intercept is equal to the level of constant marginal cost C.

Clearly, there exists an equilibrium point Q_e at which the maximal curve crosses the objective demand curve: i.e., both conditions of subjective and market equilibria are satisfied simultaneously at Q_e.

3–4. *Effects of Changes in Expectation*

We defined the seller's expectation as pessimistic when $a_i<\alpha$ holds, and as optimistic when $a_i>\alpha$. We shall say that the seller's expectation tends to be optimistic when a_i increases (i. e., the subjective demand curve comes to be flatter), and that his expectation tends to be pessimistic when a_i decreases (i. e., the subjective demand curve comes to be steeper).

The partial derivatives of the equilibrium values of y_e and p_e given by (9) with respect to a_i are as follows:

$$\frac{\partial y_e}{\partial a_i}=\frac{(1-\gamma)(p_0-MC_i)}{\{\alpha+a_i+\gamma(\alpha-a_i)\}^2}, \quad \text{and} \tag{10}$$

$$\frac{\partial p_e}{\partial a_i}=\alpha\frac{\partial y_e}{\partial a_i}. \tag{11}$$

Both derivatives of (10) and (11) are positive and negative, respectively, for $\gamma<1$. This implies that the equilibrium output of the monopolistic firm increases and the equilibrium price decreases as its expectation tends to be optimistic, and *vice versa*.

Figure 4 shows the above results. It is the same diagram as Figure 2 and 3 except that no subjective demand curves are drawn here. The straight line CA has the same slope in terms of the absolute value $(-\alpha)$ as that of the objective demrnd curve DD'. When the seller's expectation is pessimistic $a_i<\alpha$, the slope $(-a_i)$ of the maximal curve is steeper than that $(-\alpha)$ of the line CA: e.g., segment T_1U_1 of CA_1 is one of the maximal curves in this case. When his expectation is optimistic $(a_i>\alpha)$, the maximal curve T_2U_2 which is a segment of CA_2 lies below CA. As the expectation tends to be optimistic, the half line CA_1 rotates clockwise around the central

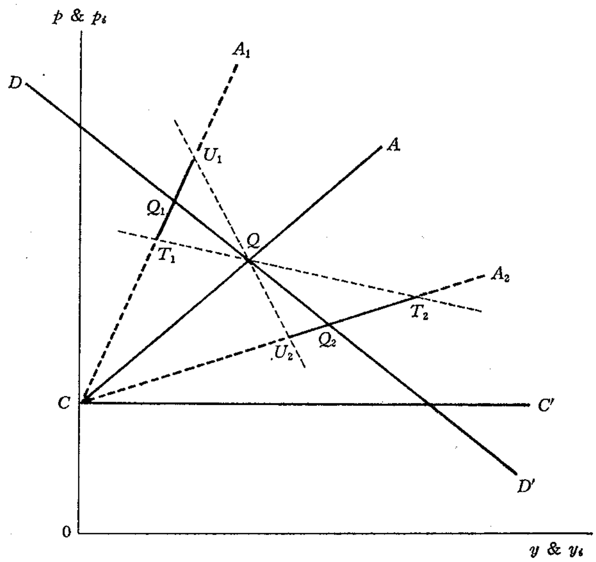

Figure 4. Effects of Changes in Expectation upon the Equilibrium Position

point C. Thus, the equilibrium point slides down from Q_1 to Q_2 along the objective demand curve DD'.

3–5. *Stability of the Equilibrium: A Cobweb Approach*

Let us examine the stability properties of the equilibrium position (9) or (9'). Figures 5 is a modified reproduction of the relevant part of Figure 2: i. e., DD' and TU are the same objective demand curve and the maximal curve, respectively, as those in Figure 2. The arrows, Q_0T, $Q_0'T'$, $\cdots$, $Q_1'U'$, and Q_1U are the segments of subjective demand curves; and they connect consistency points Q_0, Q_0', $\cdots$, Q_1', and Q_1 with the corresponding maximal points T, T', $\cdots$, U', and U.

Consider the initial situation in which the purely competitive equilibrium happens to hold in the market in the absence of the monopolistic firm. And let the monopolistic firm enter the market. Then, Q_0 is the initially given market data to the firm. On the basis of Q_0, the firm will plan its supply conditions shown by point T: i. e., the planned output and the planned selling price are the abscissa and the ordinate of T, respectively. Obviously, this plan is inconsistent with the market demand. At this selling price, we have a considerable excess demand. If the firm will adhere to its plan, the market price will diverge from its selling price: i. e., seeking extra profits, middlemen will enter the market. The total of the extra profits which they obtain will reach the amount equivalent to the area of the quadrilateral

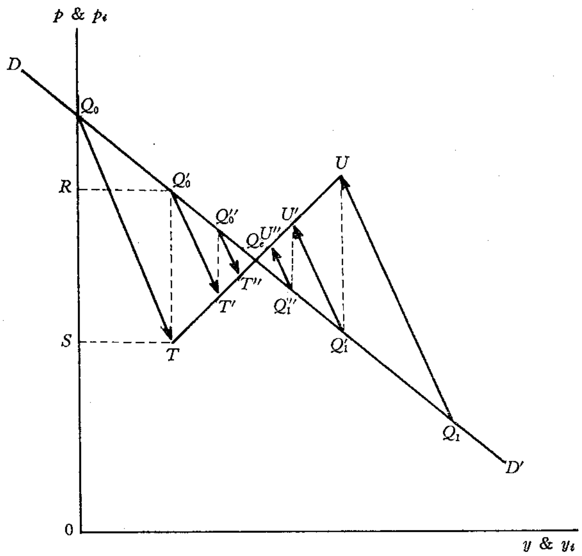

Figure 5. Stable Process to the Equilibrium in the Case of Pessimistic Expectation

$RSTQ_0'$, as long as the firm insists that the plan shall be followed.

Of course, there are a number of ways in which the firm adjusts its initial plan to the market demand. For simplicity, let us assume that the monopolistic seller revises his selling price so promptly that no middlemen can enter the market. As a result, the altered plan will be to sell the output $ST(=RQ_0')$ at the market price OR: i.e., the monopolistic firm will abosorb all the extra profits. This position is given by point Q_0'.

Although the altered plan Q_0' is consistent with the market demand, it does not satisfy the condition of profit maximization. Thus, we are on the second stage of adjustment. On the basis of Q_0', the firm will plan its supply condition shown by point T' on the maximal curve. This plan, however, is inconsistent with the market demand and must be altered. By the same logic as the above, we will reach the position Q_0''. Similarly, we shall be on the third, the fourth, the fifth stages of adjustment and so on, until we reach the final equilibrium position Q_e.

Next, let us consider the opposite initial situation Q_1 in which the firm supplies so great an amount of output that the market price falls below the equilibrium level. Given the initial position Q_1, the firm will plan its supply conditions shown by point U on the maximal curve. Since the plan U is inconsistent with the market demand, it must be altered. We shall assume that the adjustment will be made only in terms of selling price. The planned output is cleared at point Q_1'. This point serves as the market data for the second stage of adjustment. Applying the same logic in succession, we will be on the third, the fourth, the fifth stages of adjustment, and so on, until the final equilibrium will be attained at Q_e.

Figure 6 shows that, when the seller's expectation is optimistic, the equilibrium at Q_e is also stable, but oscillatory. Here, DD' and TU are the objective demand curve and the maximal curve, respectively. The arrows, Q_0T, Q_1U_1, Q_2T_1, are the segments of subjective demand curves which connect their consistency points Q_0, Q_1, and Q_2 with the corresponding maximal points T, U_1, and T_1.

Consider the same initial situation Q_0 as that in the pessimistic case. Given the competitive equilibrium price and zero excess demand at Q_0, the monopolistic seller will plan his supply conditions at T: the co-ordinates of T show his planned output and selling price. Of course, this plan is not consistent with the market demand shown by the objective demand curve DD'. The monopolistic seller, therefore, cannot help adjusting his plan to the market demand. If he insists on his planned output, the market price will fall to the level shown by Q_1, and this point will serve as the market data at his second stage of supply planning. U_1 shows his second supply conditions, and his adjustment of the second plan to the market demand will lead to Q_2. Then, his third stage of supply planning will be started. This process of successive stages of planning will continue until the final equilibrium will be attained at Q_e.

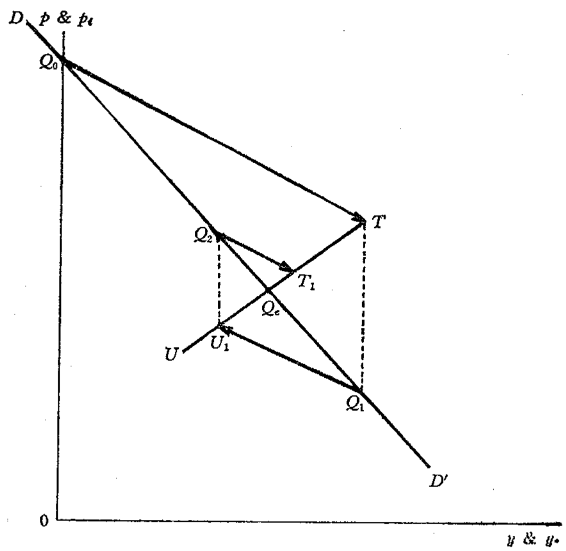

Figure 6. Stable Process to the Equilibrium in the Case of Optimistic Expectation

Although Figure 6 shows a stable equilibrium, there remain other possibilities in the case of the optimistic expectation. This is illustrated by Figure 7. Here, d_1d_1' and d_2d_2' are the subjective demand curves. And let MM' be the maximal curve whose position is independent of the parameters of the objective demand curve as is shown by the equation (8′). Note that the slope of MM' is equal, in absolute value, to those of subjective demand curves. Let MQ_1 and Q_2M' be the vertical lines which start from M and M', respectively. Then, we obtain the parallelogram Q_1MQ_2M'. Now, consider a position of the objective demand curve DD' in which it is identical with the diagonal Q_1Q_2 of the parallelogram. If the objective demand curve is in this position, the equilibrium point Q_e at which the maximal and objective demand curves intersect each other is the middle point between M and M'.

The position DD' is a critical position of the objective demand curve in which the dynamic process is neither converging nor explosive. This is easily seen as follows. Let Q_1, for example, be the initial point. On the basis of this point, the seller will plan the supply conditions shown by point M' on the maximal curve. If he insists on his planned output being cleared in the market, the market price will fall to the level shown by point Q_2. Then, on the basis of the market data Q_2, he will plan his second supply conditions shown by point M on the maximal curve. As a result, the market position will return to the initial one Q_1, and the same cycle will continue endlessly. Thus, we have a neutral equilibrium when the

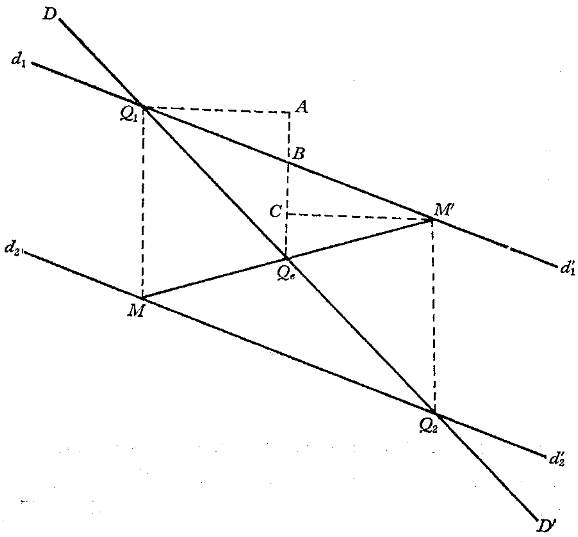

Figure 7. A Critical Position of the Objective Demand Curve in the Case of Optimistic Expectation

objective demand curve is in the position DD'.

The slope of the critically-positioned objective demand curve DD' is easily expressed in terms of the slope of the subjective demand curve. By definition, the triangle Q_1MM' is isosceles (i.e., $Q_1M'=MM'$), and $MQ_e=Q_eM'$; therefore, the perpendicular AQ_e bisects the line segment Q_1M' at B. Thus, it follows that $Q_eM'=BM'=Q_1B$. Let CM' and Q_1A be horizontal. Then, it follows that $AB=BC=CQ_e$. Thus, we conclude that $AQ_e=3AB$.

The slope of the subjective demand curve d_1d_1' is a_i, and

$$-a_i=\frac{AB}{Q_1A}.$$

On the other hand, the slope of the critically-positioned objective demand curve is α^*, and

$$-\alpha^*=\frac{AQ_e}{Q_1A}=\frac{3AB}{Q_1A}.$$

Thus, we have the condition of the neutral equilibrium:

$$-\alpha=-3a_i \tag{12}$$

If the slope of the objective demand curve is steeper than that of the critically-positioned one (i.e., $\alpha<\alpha^*$), it will be proved that the dynamic process is explosive: i.e., the equilibrium is unstable, if

$$-\alpha>-3a_i, \quad \text{or} \quad a_i-\alpha>-2a_i. \tag{13}$$

In other words, the equilibrium is unstable, when the monopolistic seller is so optimistic that his overestimation about the slope of the demand curve $(a_i-\alpha)$ goes beyond the value $-2a_i$.

As an opposite possibility, we obtain the stability condition in the case of the optimistic expectation:

$$-\alpha<-3a_i, \quad \text{or} \quad a_i-\alpha<-2a_i. \tag{14}$$

3–6. *A Marshallian Approach to Stability: An Example of the Wrong Formulation*

We have discussed the stability properties of the market equilibrium in terms of *a la* cobwed approach, and concluded that the stability depends upon the seller's mode of expectation in essential manners. It is well-known, however, that the stability depends upon how the adjustment process is formulated, and that there may be various ways of formulation. Apparently, there may be a Marshallian approach in which we have a coefficient of the speed of adjustment m. The process in which the seller adjusts his supply to the excess demand price is formulated in terms of the difference equation of the first order:

$$y_i(t)-y_i(t-1)=m\left\{p(t-1)-p_i(t-1)\right\} \tag{15}$$

where

$$p_i(t)=-a_i y_i(t)+MC_i \tag{16}$$

$$p(t)=\alpha y(t)+p_0, \quad \text{and} \tag{17}$$

$$y(t)=y_i(t). \tag{18}$$

Note that the maximal curve (16) serves as a Marshallian supply curve in the above formulation. It was derived from the marginal cost, the subjective demand, and the objective demand curves. Thus, it must be a "mongrel curve" so to speak. On the surface, however, it is completely independent of the objective demand curve when the data-adjustment coefficient γ is zero.

From the above equations, we obtain the first order linear difference equation with respect to the quantity supplied:

$$y_i(t)-\left\{(1+m(\alpha+a_i)\right\}y_i(t-1)=m(p_0-MC_i). \tag{19}$$

Put $y_i(t)=\bar{y}$ for all t. Then, we can easily see that the static equilibrium (9′) is consistent with this formulation. Given the initial condition y_0 at $t=0$ ($y_0=0$ if the initial situation is the competitive equilibrium point), the soluation of (19) is obtained by iteration:

$$y_i(t)=\bar{y}+(y_0-\bar{y})\left\{1+m(\alpha+a_i)\right\}^t \tag{20}$$

In view of the restrictions upon parameters ($m>0$, $\alpha<0$, and $a_i<0$), the value of the factor $\{1+m(\alpha+a_i)\}$ must be less than unity. There are five possibilities:

Case 1: $m<\dfrac{-1}{\alpha+a_i}$: the slowest region in speed of adjustment. Here, $\{1+m(\alpha+a_i)\}^t$ decreases steadily to zero as t increases indefinetely. Hence, $y_i(t)$ converges steadily to the equilibrium level $\bar{y}$.

Case 2: $m=\dfrac{-1}{\alpha+a_i}$: the lower critical level of speed of adjustment. Here, $\{1+m(\alpha+a_i)\}^t$ is zero for all t. Hence, $y_i(t)$ is always identical with $\bar{y}$. The adjustment is done correctly and instanteneously.

Case 3: $\dfrac{-1}{\alpha+a_i}<m<\dfrac{-2}{\alpha+a_i}$: the intermediate region. Here, $\{1+m(\alpha+a_i)\}^t$ decreases in absolute value, and alternates its sign, as t increases. There is a damped oscillation towards the equilibrium level.

Case 4: $m=\dfrac{-2}{\alpha+a_i}$: the upper criticatl level. Here, $\{1+m(\alpha+a_i)\}^t$ is always unity in absolute value, and alternates its sign, as t increases. There is a regular oscillation.

Case 5: $\dfrac{-2}{\alpha+a_i}<m$: the highest region in speed of adjustment. Here, $\{1+m(\alpha+a_i)\}^t$ increases indefinitely in absolute value, and alternates its sign. Hence, there is an explosive oscillation.

To sum up: In the case of a Marshallian approach, the stability of equilibium depends upon values of the coefficient of speed of adjustment in relation to the sum of both slopes of the objective and subjective demand curves. It is not essential to the stability property whether the seller's expectation is pessimistic or optimistic.

This conclusion has some similarities to that of Negishi's. According to this formulation, the stability of the dynamic process can be proven without any restrictions on the seller's mode of expectation. Unfortunately, however, our Marshallian formulation makes a slip in its beginning. The maximal curve cannot serve as the Marshallian supply curve without any restrictions. It is meaningful only when the relationship between each point on it and each consistency point on the objective demand curve is explicitly stated. Thus, the dynamic process must be formulated in terms of the above cobweb approach.

Anyhow, it is a wonder that Negishi could prove the stability of the dynamic process of his monopolistically competitive equilibrium without more specifications about the parameters of his subjective demand functions.

3–7. *The Maximal Curve Derived From the U-Shaped Marginal Cost Curve*

It is high time to relax our assumption of constant marginal cost. Let MC_i be an ordinary U-shaped function of output y_i. For simplicity, we shall consider a case in which $\gamma=0$. Then, we have the equation of the maximal curve, instead of (8′), as follows:

$$p_i=-a_iy_i+MC_i(y_i)\,. \tag{21}$$

Differentiating (21) with respect to y_i, we obtain:

$$\frac{dp_i}{dy_i}=\frac{dMC_i}{dy_i}-a_i\,. \tag{22}$$

In view of the sufficient condition for maximizing profits that the derivative of (MR_i-MC_i) must be negative, the relevant range of the gradient of the marginal cost curve is given by:

$$2a_i < \frac{dMC_i}{dy_i} \tag{23}$$

which implies that the slope of the marginal revenue curve must be steeper than that of the marginal cost curve. Within this range, there are three possibilities. From (22), we deduce:

Range 1: When the negatively inclined segment of the marginal cost curve has a gentler slope than that of the marginal revenue curve, and has a steeper slope than that of the subjective demand curve $\left(2a_i < \frac{dMC_i}{dy_i} < a_i\right)$; the maximal curve is inclined negatively.

Range 2: At the point where the negatively inclined segment of the marginal cost curve has the same slope as that of the subjective demand curve $\left(\frac{dMC_i}{dy_i} = a_i\right)$, the gradient of the maximal curve is flat.

Range 3: When the negatively inclined segment of the marginal cost curve has a gentler slope than that of the subjective demand curve, or when the marginal cost curve is increasing $\left(a_i < \frac{dMC_i}{dy_i}\right)$; the maximal curve is increasing.

To sum up: A U-shaped maximal curve follows from a U-shaped marginal cost curve. There are some exceptions, of course, because not all U-shaped marginal cost curves have their segments which fall within *Range 1*.

In order to construct the maximal curve on the basis of a given U-shaped marginal cost curve in the diagram, it will be helpful to note that the slope of the maximal curve is equal, in absolute value, to the subjective demand curve $\left(\frac{dp_i}{dy_i} = -a_i\right)$ at the level of output at the bottom of the marginal cost curve.

In Figure 8, CC' is the U-shaped marginal cost curve on which we shall construct the maximal curve MM'. For convenience' sake, we shall ignore the position of the objective demand curve so that we shall be able to shift the subjective demand curve at will.

The southwestmost position of the subjective demand curve whose marginal revenue curve can cross, or touch at least, the marginal cost curve is shown by dd'. In this position, its marginal revenue curve dr touches CC' at point C. Point M on dd' at the same level of output as that of C is the left-side extreme point of the maximal curve.

Now, let us make the subjective demand curve shift upwards in parallel from dd' to d_1d_1'. Here, the marginal revenue curve d_1r_1 crosses CC' at

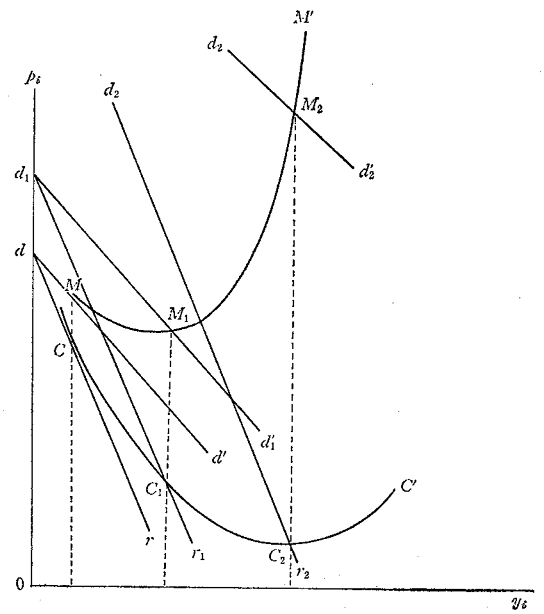

Figure 8. The Skewedly U-Shaped Maximal Curve Derived from the U-Shaped Marginal Cost Curve

point C_1 where the tangency gradient of CC' is equal to the slope of d_1d_1'. In view of the case of *Range 2* above, the slope of the maximal curve at M_1 is perfectly flat. And the above result of *Range 1* tells us that the segment MM_1 of the maximal curve is negatively inclined.

Slide the subjective demand curve still more upwards from the position d_1d_1'. It will be seen that the maximal curve is increasing, and the above result of *Range 3* will be confirmed. When the subjective demand curve reaches the position d_2d_2', its marginal revenue curve d_2r_2 crosses CC' at the bottom point C_2. Point M_2 on the subjective demand curve d_2d_2' at the same level of output as that of point C_2 is the maximal point at which the slope of the maximal curve is identical, in absolute value, with that of the subjective demand curve. Thus, we obtain the skewedly U-shaped maximal curve MM' in the diagram.

3–8. *Equilibrium on the Skewedly U-Shaped Maximal Curve*

Now, we are in a position to combine the skewedly U-shaped maximal curve with the objective demand curve. Obviously, the discussion about the existence and stability of the equilibrium in this case is essentially the same as what we have already discussed under constant marginal cost, provided that the purely competitive equilibrium price is so high that the objective demand curve crosses the maximal curve at a point on the latter's rising segment. We shall discuss, therefore, only a special case in which the competitive equilibrium price is so low that the objective demand curve crosses the falling segment of the maximal curve.

In Figure 9, the seller's expectation is pessimistic. Here, dd' is the southwestmost subjective demand curve whose marginal revenue curve can touch or cross the U-shaped marginal cost curve (Of course, it is omitted in this diagram). DD' is the objective demand curve which crosses the maximal curve MM' twice at E and E'. Strictly speaking, the latter E' need not lie on the falling segment of MM', while the former E must.

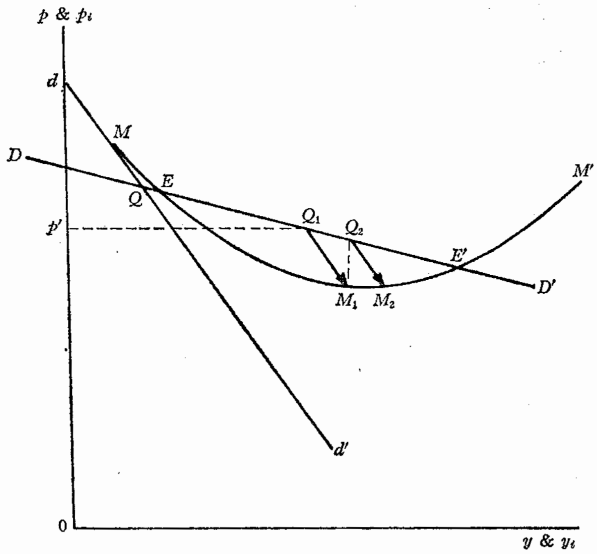

Figure 9. Two Equilibrium Points on the U-Shaped Maximal Curve and their Stability

It is easily shown that the equilibrium point E' is stable in its neighborhood. Suppose that the market price rises to the level of p' since the initial equilibrium position E' has been disturbed by certain causes. Given the market position Q_1, the seller will plan his supply conditions at M_1. The arrow Q_1M_1 is the segment of the subjective demand curve whose consistency point is Q_1. Since M_1 is not on the objective demand curve, his supply plan is inconsistent with the market demand. Thus, he alters his planned selling price, and the market position shifts to Q_2. Given Q_2, the second stage of planning and adjustment starts, and the succeeding stages will follow until the final equilibrium is attained at E'.

Needless to say, a similar stable process will occur when the market price falls below the equilibrium level of E'.

In order to examine the stability of the equilibrium point E at which the tangent gradient of the falling maximal curve is steeper than the slope of the objective demand curve, let us turn to Figure 10 where the relevant part of Figure 9 is enlarged. Here, all the same as in Figure 9, the south-westmost subjective demand curve dd′ crosses the objective demand curve DD' at Q. The ordinate of Q is a critical level of price in the sense that the seller cannot plan his supply conditions and gives up his domination of the market if the market price is higher than that. Though this is a remarkable phenomenon, we shall examine the stability of the equilibrium point E first.

Suppose that the market price deviates upwards from the equilibrium

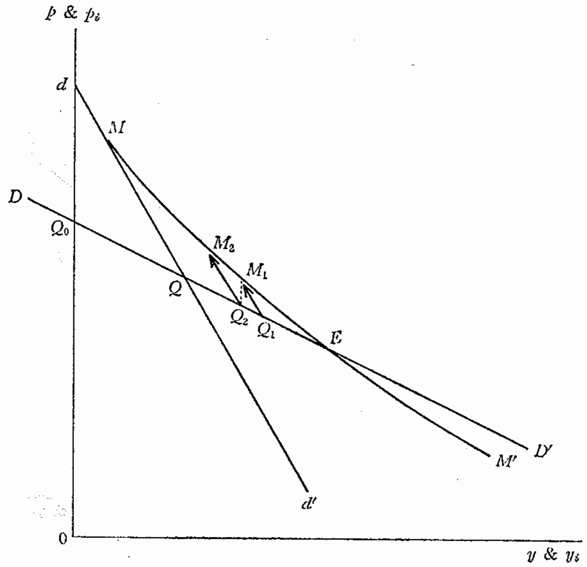

Figure 10. Unstable Equilibrium on the U-Shaped Maximal Curve

level at E. Let the initial position, for example, be point Q_1 on the objective demand curve. Given Q_1, the seller will plan his supply conditions at M_1, because the arrow Q_1M_1 is the segment of his subjective demand curve which connects its consistency point Q_1 with its corresponding maximal point M_1. Obviously, his planned supply position M_1 is inconsistent with the market demand, and he alters his planned selling price. Thus, the second market position Q_2 will be realized. Given Q_2, he will start the second stage of his planning and adjustment. It is clear that the equilibrium position E is unstable leftwards. Its rightward unstability follows directly from the stability of the other equilibrium point E'. If this leftward unstable process continues, the market price will rise sooner or later beyond the critical level at Q. This implies that the monopolistic seller cannot stay in the market if the initial market price is higher than the equilibrium level at E.

Suppose that the purely cometitive equilibrium holds in the market before the monopolistic firm enters it. This situation implies, in Figure 10, that the initially given market position is point Q_0 which is the vertical intercept of the objective demand curve. Since the market price at Q_0 (=the purely competitive equilibrium price) is higher than the critical level at Q, the monopolistic firm cannot enter the market. The actual competitive market price may fluctuate. The monopolistic seller cannot be motivated to dominate the market, as long as the downward deviation of the market price from the competitive equilibrium level is so small that the market price does not fall below the monopolistic equilibrium level at E.

Note that the monopolistic firm with constant marginal cost can straightforwardly enter the market in which the competitive equilibrium has already established, as long as there exists the monopolistically competitive equilibrium position (Q_e in Figures 2–6). In this section, we show that the monopolistic firm with the U-shaped marginal cost curve cannot necessarily have an inducement to enter the market, even if there is the monopolistically competitive equilibrium. In other words, some causes which make the monopolist's marginal cost curve U-shaped might play a part of "barriers to new entry" when price fluctuations are relatively small in the competitive market.

4. BEYOND THE MONOKERNEL ASSUMPTION

In the presence of the monokernel assumption, thus far, our discussion reveals that Negishi's general equilibrium theory of monopolistic competition is actually one which explains the monopolistic firm's entry into the purely competitive market. The firm is "monopolistic" in two ways. First, it has its own subjective demand curves which are negatively inclined. Second, as a result of the monokernel assumption, it is in a monopolistic position with respect to the opportunity for entering the market. This is not what Negishi intended to incorporate into his model. In the following, we shall try to relax his unintended restriction.

For simplicity, suppose that the firm which we have considered has only one rival monopolistic firm before it attains its domination of the market. Then, the discussion will be reduced essentially to that of duopoly. The excess demand of the purely competitive group will be shared between the firm (entrant 1) and its rival (entrant 2) selling at the same price, while it was monopolized by one monopolistic firm thus far.

Suppose that each entrant i ($i=1$ or 2) produces his output y_i at constant marginal cost MC_i. The manners in which the competitive excess demand is shared between y_1 and y_2 depend upon what is assumed about each entrant's reaction to the other's action. We shall assume, as in the simplest duopoly problem, that each entrant expects the other to make no change in the other's current output no matter what changes it makes in his own output. This assumption will be a little more reasonable in our theory than in the ordinary duopoly theory, because we have, in addition to both entrants, a group of purely competitive firms which may serve as a buffer slacking the direct interaction between both monopolistic entrants. Of course, the monokernel assumption does not exclude a case in which there are no purely competitive firms so that our theory is identical with that of duopoly.

Let y denote the excess demand of the purely competitive group, then we have:

$$y = y_1 + y_2 . \tag{24}$$

The competitive excess demand function is identical with the objective demand function (2) in the previous section, and it is given here again by:

$$p = \alpha y + p_0 . \tag{25}$$

Given the market data, each entrant constructs his own subjective demand curve:

$$p_i = a_i y_i + (\alpha - a_i) y + p_0 \tag{26}$$

which is the same equation as (3).

Thus, the expected total revenue TR_i of each entrant is:

$$TR_i = a_i y_i^2 + (\alpha - a_i)(y_i + y_j) y_i + p_0 y_i \qquad \text{for } i \neq j. \tag{27}$$

On the occasion of calculating the expected marginal revenue MR_i, both entrants expect the other to make no changes in its current output no matter what changes they make in their own output: i.e.,

$$\frac{dy_j}{dy_i} = 0 \qquad \text{for } i \neq j . \tag{28}$$

In view of this condition, differentiate (27) with respect to y_i. Then, we obtain:

$$MR_i = 2\alpha y_i + (\alpha - a_i) y_j + p_0 \qquad \text{for } i \neq j. \tag{29}$$

For any given value of y_j, each entrant i fixes his output y_i so that $MR_i = MC_i$. The conditions for the equilibrium outputs are written in full as follows:

$$
(30) \qquad \begin{aligned} -2\alpha y_1 + (a_1-\alpha) y_0 &= p_0 - MC_1 \\ (a_2-\alpha) y_1 - 2\alpha y_2 &= p_0 - MC_2 \end{aligned}
$$

We shall say that entrant i is "qualified" if his marginal cost is lower than the competitive equilibrium price ($MC_i < p_0$). Now, let us assume that both entrants 1 and 2 are qualified. Then, the right hand sides of both equations (30) are positive.

4–1. *Duopolistic Case Where Both Entrants Have Pessimistic Subjective Demand Curves*

First, let us consider a case where both entrants are pessimistic: i. e.,

$$
(31) \qquad a_i < \alpha \qquad \text{for } i=1 \text{ and } 2.
$$

Then, the off-diagonal elements of the coefficient matrix of the system of equations (30) are negative: i. e.,

$$
(a_i - \alpha) < 0, \qquad \text{for } i=1 \text{ and } 2.
$$

In view of both diagonal elements -2α of the matrix being positive, the system (30) generates positive values of outputs y_1 and y_2, if and only if the determinant of the coefficient matrix is positive (the Hawkins-Simon condition)[17):

$$
(32) \qquad |D| = \begin{vmatrix} -2\alpha & a_1-\alpha \\ a_2-\alpha & -2\alpha \end{vmatrix} > 0 .
$$

This condition is acceptable, because it is always satisfied if the following condition holds:

$$
(33) \qquad 3\alpha < a_i, \quad \text{or} \quad \alpha - a_i < -2\alpha, \qquad \text{for } i=1, 2.
$$

This implies that their expectation should not be so extremely pessimistic that the deviations of the slopes of their subjective demand curves from the slope of the market demand curve become much larger than -2α.

Solving the system of equations (30) with respect to y_i, we obtain:

$$
(34) \qquad y_1 = \frac{1}{|D|} \begin{vmatrix} p_0 - MC_1 & a_1-\alpha \\ p_0 - MC_2 & -2\alpha \end{vmatrix}, \text{ and}
$$

$$
(35) \qquad y_2 = \frac{1}{|D|} \begin{vmatrix} -2\alpha & p_0 - MC_1 \\ a_2-\alpha & p_0 - MC_2 \end{vmatrix}.
$$

From (34) and (35), it follows that each entrant's equilibrium output y_i is an increasing function of both profit margin ($p - MC_1$) and ($p_0 - MC_2$). In

17) D. Hawkins and H. A. Simon: "Note: Some Conditions of Macroeconomic Stability," *Econometrica*, Vol. 17, 1949, pp. 245–248.

other words, the lower the marginal cost of any entrant is, the greater the equilibrium levels of both entrants' outputs are.

From (34) and (35), moreover, it follows that the duopolistic equilibrium outputs of both entrants are positive even when one of them is a marginal entrant (i. e., $p_0 = MC_i$, for $i=1$ or 2), provided that the other is qualified (i. e., $p_0 > MC_j$, for $i \neq j$). In view of (24) and (25), we have:

$$p_0 - p = -\alpha(y_1 + y_2) > 0\,,$$

for $y_1 + y_2 > 0$. In other words, the duopolistic equilibrium price is lower than the competitive equilibrium price. Therefore, the marginal entrant must sell at a loss (i. e., $p < MC_i$, for the marginal entrant i). Such a situation cannot be equilibrium. Thus, we see that the system of equations (30) cannot determine the duopolistic equilibrium outputs without referring to any other restrictions: i. e., we must take account of the entrants' profitabilities.

The conditions that both entrants do not sell at a loss are given by the inequalities:

$$p = \alpha(y_1 + y_2) + p_0 \geqq MC_i\,\text{`} \qquad \text{for } i = 1, 2.$$

Rearranging these inequalities, we obtain:

(36) $$p_0 - MC_i \geqq -\alpha(y_1 + y_2)\,, \qquad \text{for } i = 1, 2.$$

Together with inequalities (36), the system of equations (30) gives:

(37) $$\begin{aligned} -\alpha y_1 + a_1 y_2 &\geqq 0\,, \quad \text{and} \\ a_2 y_1 - \alpha y_2 &\geqq 0\,. \end{aligned}$$

These profitabilities conditions (37) hold true for positive values of y_1 and y_2, if and only if

(38) $$\begin{vmatrix} -\alpha & a_1 \\ a_2 & -\alpha \end{vmatrix} > 0\,.$$

As is easily proven, (38) is impossible when both entrants have pessimistic subjective demand functions (i. e., when inequalities (31) hold). Hence, we conclude that there exist no duopolistic equilibrium positions in which each entrant does not sell at a loss, if both entrants have pessimistic subjective demand curves.

4–2. *Duopolistic Case Where Both Entrants Have Optimistic Subjective Demand Curves*

Next, let us consider a case where both entrants are optimistic: i. e.,

(39) $$a_i > \alpha \qquad \text{for } i = 1 \text{ and } 2.$$

In this case, the condition (38) holds true: hence, a region where inequalities (37) are satisfied exists on the positive quadrant of the y_1-y_2 plane.

As for the system (30), there exists a unique solution (y_1, y_2), if and only if

$$|D| = \begin{vmatrix} -2\alpha & a_1-\alpha \\ a_2-\alpha & -2\alpha \end{vmatrix} \neq 0 . \tag{40}$$

4–2–1. *Stable Case*

First, let us examine a case where the determinant is positive: i.e.,

$$|D| > 0 . \tag{41}$$

Figure 11 illustrates one possible situation of this case. Here, the line $L_1 y_1^0$ is a graph of the first equation of (30). We shall call it the reaction line of entrant 1. It intersects the horizontal axis at the point y_1^0 whose abscissa is given by:

$$\frac{p_0 - MC_1}{-2\alpha} \tag{42}$$

This value is equal to the equilibrium output of the monopolistic firm in the presence of the monokernel assumption, when the coefficient of data-adjustment γ is unity (See the abscissa of point Q_e given by (9) and put

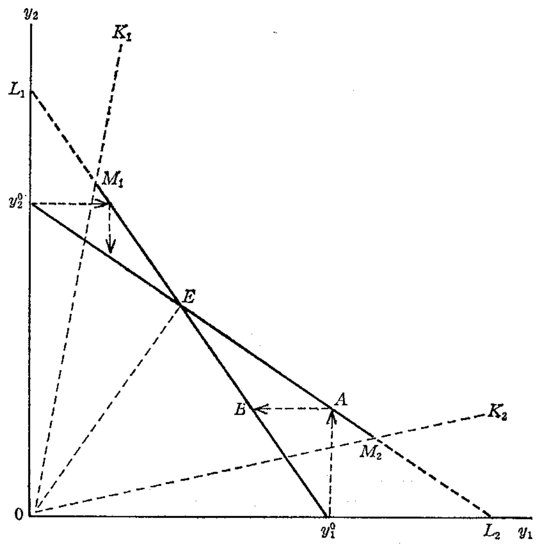

Figure 11. Duopolistic Equilibrium of Two Monpolistic Entrants Who Are Optimistic

$\gamma=1$ there). Thus, we shall call point y_1^0 in Figure 11 the first entrant's equilibrium point in the presence of the monokernel assumption.

The reaction line of the first entrant intersects the vertical axis at point L_1 whose ordinate is given by:

$$\frac{p_0-MC_1}{a_1-\alpha}. \tag{43}$$

The dotted line OK_1 shows the boundary of the first inequality of the profitability conditions (37). It intersects the reaction line $L_1y_1^0$ at point M_1, which divides the reaction line into two segments: L_1M_1 and $M_1y_1^0$. We shall call the former broken-line segment L_1M_1 the infeasible segment, and the latter $M_1y_1^0$ the feasible, because the first inequality of (37) tells us that entrant 1 sells at a loss when both entrants' outputs are given by a point on the former, and that he sell at a profit when their outputs are given by a point on the latter.

Similarly $y_2^0L_2$ is the reaction line of the second entrant. It intersects the horizontal axis at point L_2 whose abscissa is given by:

$$\frac{p_0-MC_2}{a_2-\alpha}. \tag{44}$$

It also intersects the vertical axis at point y_2^0 whose ordinate is:

$$\frac{p_0-MC_2}{-2\alpha}. \tag{45}$$

This is equal to the equilibrium level of output of entrant 2 in the presence of the monokernel assumption when his coefficient of data-adjustment is unity. Thus, point y_2^0 is called the second entrant's equilibrium position in the presence of the monokernel assumption.

The dotted line OK_2 is the boundary of the second entrant's profitability. The segment $y_2^0M_2$ is the feasible segment, and the broken-line segment M_2L_2 is the infeasible segment of the second entrant's reaction line.

As is shown in Figure 11, there exists an economically meaningful equilibrium, if and only if both entrants' feasible segments cross each other at E. And this happens, if and only if the abscissa of point y_1^0 is smaller than that of point L_2, and the ordinate of point y_2^0 is smaller than that of point L_1, simultaneously: i.e.,

$$\begin{aligned} &\frac{p_0-MC_1}{-2\alpha}<\frac{p_0-MC_2}{a_2-\alpha}, \quad \text{and} \\ &\frac{p_0-MC_2}{-2\alpha}<\frac{p_0-MC_1}{a_1-\alpha}. \end{aligned} \tag{46}$$

It follows from (46) that:

$$\frac{a_2-\alpha}{-2\alpha}<\frac{p_0-MC_2}{p_0-MC_1}<\frac{-2\alpha}{a_1-\alpha} \tag{47}$$

This implies that there exists an economically meaningful equilibrium, if and only if the ratio between both entrants' profit margins at the competitive equilibrium price falls into the interval between $(a_2-\alpha)/(-2\alpha)$ and $(-2\alpha)/(a_1-\alpha)$. Incidentally, the existence of the interval follows from the condition (41).

We can guess at the feasibility of the equilibrium as follows. Note that the slope of the boundary lines OK_1 and OK_2 are α/a_1 and a/α_2, respectively; and that the former is larger than the latter by the condition (38). ((38) holds true when both entrants are optimistic.) On the other hand, the slope of the dotted-line OE is given by the ratio of both entrants' equilibrium outputs:

$$\frac{y_2}{y_1}=\begin{vmatrix} -2\alpha & p_0-MC_1 \\ a_2-\alpha & p_0-MC_2 \end{vmatrix} \div \begin{vmatrix} p_0-MC_1 & a_1-\alpha \\ p_0-MC_2 & -2\alpha \end{vmatrix} \tag{48}$$

which, if $MC_1=MC_2$, reduces to

$$\frac{y_2}{y_1}=\frac{a_2+\alpha}{a_1+\alpha}. \tag{49}$$

In view of the condition (38), it holds that:

$$\frac{\alpha}{a_1}>\frac{a_2+\alpha}{a_1+\alpha}>\frac{a_2}{\alpha} \tag{50}$$

This implies that the equilibrium point E lies in the feasible region.

Now, we are in a position to discuss the stability of the duopolistic equilibrium point E in Figure 11. Suppose that the initial position is given at the first entrant's monopolistic equilibrium point y_1^0 in the presence of the monokernel assumption. Then, remove the monokernel assumption. Given the market data at point y_1^0, the second entrant will plan to supply his output shown by the ordinate of point A on his reaction line (strictly speaking, on his feasible segment $y_2^0M_2$) if appropriate conditions are satisfied. At point A, the first entrant does not maximize his profit, because A is not a point on his reaction line. He will, therefore, alter his plan and move to point B on his reaction line. This process of adjustment will continue until the final equilibrium is attained at E. Similarly, the dynamic adjustment process which starts from the initial point y_2^0 is also stable.

Incidentally, the second entrant cannot enter the market from the initial position y_1^0, if point A lies on the infeasible segment M_2L_2. Thus, the condition for the second entrant to be able to enter the market from the initial position y_1^0 (i.e., from the situation where the first entrant has already established his domination) is given by:

(51)
$$\frac{\alpha+a_2}{2\alpha}<\frac{p_0-MC_2}{p_0-MC_1}$$

which states that the abscissa of point M_2 is larger than that of point y_1^0.

Similarly, the condition for the first entrant to be able to enter the market from the initial position y_2^0 (i. e., from the situation where the second entrant has already established his domination) is given by:

(52)
$$\frac{p_0-MC_0}{p_0-MC_1}<\frac{2\alpha}{\alpha+a_1}.$$

Clearly, the condition (47) is narrower than (51) and (52). This implies that each entrant can always enter the market from the initial situation where the other has already established his domination, if there exists an economically meaningful duopolistic equilibrium.

4–2–2. *Unstable Case*

Next, let us consider a case where the determinant (40) is negative: i. e.,

(53)
$$|D|<0.$$

This implies that there exists the interval between two values:

(54)
$$\frac{-2\alpha}{a_1-\alpha}<\frac{a_2-\alpha}{-2\alpha}$$

If the ratio between both entrants' profit margins at the competitive equilibrium price falls into this interval: i. e.,

$$\frac{-2\alpha}{a_1-\alpha}<\frac{p_0-MC_2}{p_0-MC_1}<\frac{a_2-\alpha}{-2\alpha},$$

the abscissa of point y_1^0 is larger than that of L_2, as is drawn in Figure 12, and the ordinate of point y_2^0 is larger than that of L_1. Hence, both entrants' reaction lines $y_1^0L_1$ and $y_2^0L_2$ cross each other at point E.

Since both entrants are optimistic, the boundary line OK_1 is steeper than the boundary OK_2: i. e., there exists a feasible region. The feasibility of the equilibrium point E will be confirmed under the not so unreasonable condition that the difference between both entrants' marginal cost is not so great.

In order to discuss the stability of equilibrium, let point A be the initial position where the second entrant maximizes his profit. The first entrant, however, will not stay at A, because he does not maximize his profit here. He will increase his output to the level which point B on his reaction line gives. When the market position changes from A to B, the

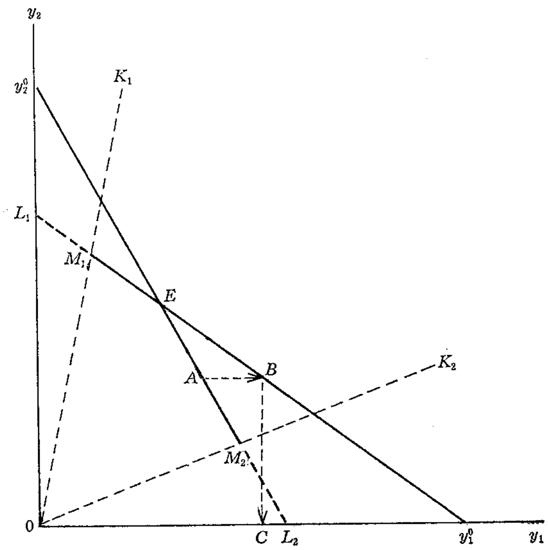

Figure 12. Unstable Duopolistic Equilibrium of Two Optimistic Entrants

second entrant will find that he cannot stay in the market because he must sell at a loss: therefore, he will drop out of the market. The result is shown by point C. In the absence of his rival, the first entrant will attain his monopolistic equilibrium at point y_1^0. Thus, the duopolistic equilibrium is unstable. This implies that each entrant cannot enter the market even if there exists the duopolistic equilibrium, when the other entrant has already established his domination of the market under the condition (53).

4–3. *Duopolistic Case Where One Entrant is Optimistic and the Other Pessimistic*

Finally, let us discuss a case where one entrant has an optimistic subjective demand curve and the other has a pessimistic one. Without a loss of generality, we can assume:

$$a_1 < \alpha\,, \quad \text{and} \quad a_2 > \alpha\,. \tag{55}$$

Figure 13 illustrates one possible situation. Here, a positively sloping line $y_1^0L_1$ is a reaction line of the first entrant who is assumed to be pessimistic. Since it is assumed that the condition (38) holds, we have an economically meaningful duopolistic equilibrium point E at which the feasible segments of both reaction lines cross each other.

Let us suppose that the first entrant has already established his monopolistic equilibrium at point y_1^0. In the absence of the monokernel assumption,

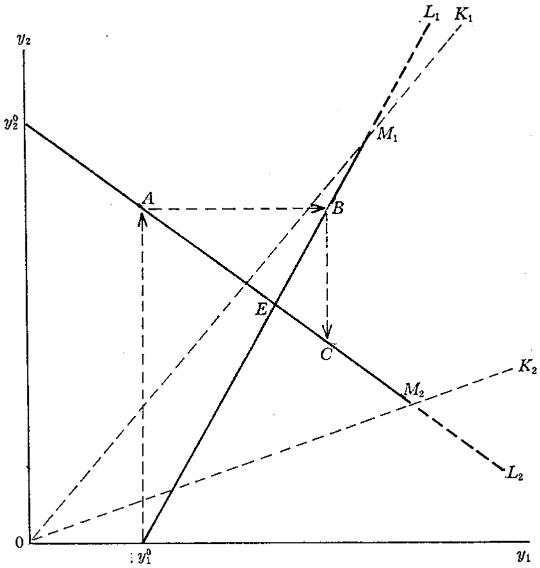

Figure 13. Duopolistic Equilibrium of Two Monopolistic Entrants: One Pessimistic and the Other Optimistic

the second entrant will plan to supply his output to the level which is equal to the ordinate of point A; and A will be attained sooner or later. The first entrant does not maximize his profit at A, so that he will alter his output to the level which is equal to the abscissa of point B. Given the market situation at B, the second entrant will change his supply to the level which is equal to the ordinate of point C. This process of adjustment will continue until the final equilibrium is attained at point E.

Clearly, this dynamic process is of the same nature as the cobweb process. Hence, it depends upon the relation between slopes of both entrants' reaction lines whether the duopolistic equilibrium is stable or unstable: i.e., the duopolistic equilibrium under the assumption (55) is stable, neutral, or unstable according as:

$$\frac{2\alpha}{a_1-\alpha} \gtreqless \frac{\alpha-a_2}{2\alpha}. \tag{56}$$

4-3-1. *Stable Case*

We have already examined the dynamic process started from the initial position in which the first entrant (pessimist) has established his monopolistic equilibrium in the market.

Now, let us examine a case in which the second entrant (optimist) has established his monopolistic equilibrium in the initial situation. Given the market data at point y_2^0 (in Figure 13), the first entrant will try to adjust his supply to these data. There are two possibilities. One is a case in which he can sell at a profit, the other is a case in which he must sell at a loss. In the former case, the feasible segment $y_1^0M_1$ of the first entrant's reaction line has a point whose ordinate is equal to that of point y_1^0. In the latter case, it has no such a point: i.e., the point falls on the infeasible segment M_1L_1. In order that the process converges to point E, the ordinate of point M_1 must not be less than that of point y_1^0: i.e.,

$$\frac{p_0-MC_2}{-2\alpha} \leqq \frac{p_0-MC_1}{-(\alpha+a_1)}\,. \tag{57}$$

Suppose that the initial market position is given by point M_1. On the basis of M_1, the second entrant will plan his supply so as to maximize his profit. In order that his plan is realized on his feasible segment $y_2^0M_2$, the abscissa of point M_2 must not be less than that of M_1: i.e.,

$$\frac{-a_1(p_0-MC_1)}{\alpha(\alpha+a_1)} \leqq \frac{p_0-MC_2}{-(\alpha+a_2)}\,. \tag{58}$$

From (57) and (58), we have a sufficient condition for the dynamic process to converge to the duopolistic equilibrium from the monopolistic equilibrium of either entrant:

$$\frac{a_1(\alpha+a_2)}{\alpha(\alpha+a_1)} \leqq \frac{p_0-MC_2}{p_0-MC_1} \leqq \frac{2\alpha}{\alpha+a_1} \tag{59}$$

The right-hand side of this inequality is less than unity, because the first entrant is pessimistic ($a_1<\alpha<0$); this implies, therefore, that marginal cost of the pessimist must not be higher than that of the optimist ($MC_1 \leqq MC_2$). Incidentally, it is not unreasonable to consider the interval between the right-hand side and the left-hand side of (59) in which the ratio of both entrants' profit margins must fall, because the left-hand side is always smaller than the right-hand side for any value of α when the difference between slopes of both entrants' subjective demand curve is not so great as:

$$\frac{a_1}{a_2} < 8\,. \tag{60}$$

If the condition (57) is not satisfied (e.g., the pessimist's marginal cost is higher than the optimist's), the first entrant (pessimist) cannot enter the market in which the second entrant (optimist) has established his monopolistic equilibrium at y_2^0; and it is possible to happen that the first entrant drops out of the market in which he has once established his domination at y_1^0.

If the condition (58) is not satisfied (e.g., the optimist's marginal cost is too higher than the pessimist's), it is possible for the second entrant to drop out of the market in which he has already established his domination.

4-3-2. *Neutral Case*

Figure 14 shows a possible situation in which the duopolistic equilibrium is neutral. Let the initial position be point y_2^0 at which the second entrant has established his monopolistic equilibrium in the market. On the basis of the market position y_2^0, the first entrant adjust his supply so as to maximize his expected profit; and the market position shifts from point y_2^0 to point C on his feasible segment $y_1^0M_1$. In face of the new market position C, the second entrant will find that he has no other way to sell at a loss than to drop out of the market. Thus, the market position shifts from point C to point D. This is the same situation as that

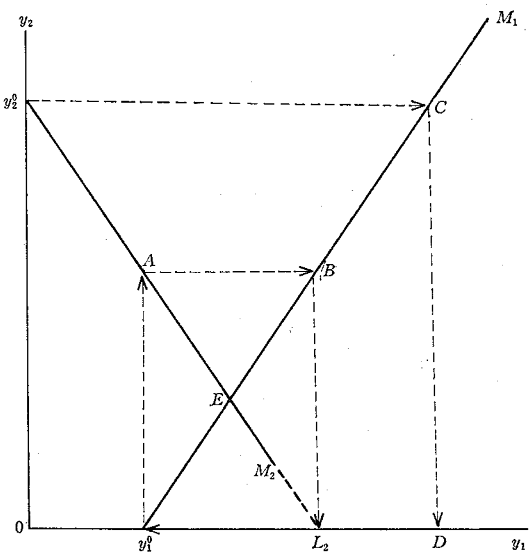

Figure 14. Neutral Case of Duopolistic Equilibrium

in which the monokernel assumption is assumed. The first entrant, therefore, will adjust his supply to the level of point y_1^0 at which he attains his monopolistic equilibrium.

All we have to do next is let y_1^0 be the initial position and look at a cycle ($y_1^0 \rightarrow A \rightarrow B \rightarrow L_2 \rightarrow y_1^0$) in Figure 14. What we should note about this cycle is the fact that the second entrant (optimist) drops out of the market cyclically, while the first (pessimist) always stay in the market.

4-3-3. *Unstable Case*

Figure 15 shows a case in which the dynamic process is unstable.

Though the dynamic property is explosive in the neighborhood of the duopolistic equilibrium point E (as is shown by a series of arrows $F \to G \to H \to H \to I \to J \to K$), the result is completely the same as that of the neutral case (in Figure 14) when it comes to the problem of entry: i.e., the dynamic process started from the initial position y_2^0 will reach the position y_1^0 via points C and D; and the cycle started from point y_1^0 ($\to A \to B \to B' \to y_1^0$) will repeat itself.

Note that we have implicitly assumed that the condition (57) holds in Figure 15 as well as in Figure 14. If it is not satisfied, the first entrant's feasible segment will become shorter than what is as shown by the segment $y_1^0 M_1$, for example. Then, it is possible to happen that point C falls on the infeasible segment of the first entrant's reaction line: i.e., the first

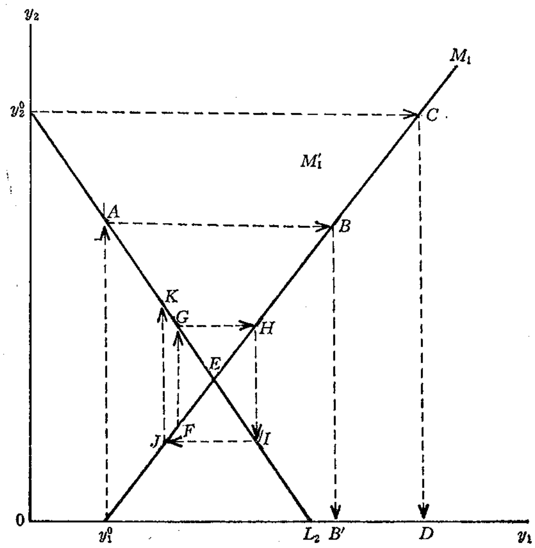

Figure 15. Unstable Case of Duopolistic Equilibrium

entrant cannot enter the market in which the second entrant has established his domination. This may happen when the first entrant's marginal cost is higher than the second entrant's (as we have mentioned it with respect to the inequality (59)). Of course, it is also possible for the first entrant to drop out of the market, even if he has already established his domination at point y_1^0.

5. CONCLUDING REMARKS

We mentioned, in *Section 2*, that Chamberlin's construction could not explain the number of monopolistic firms in the general market. In *Section*

3 where Negishi's theory was examined in terms of the partial equilibrium, we found that there existed a monopolistic equilibrium under the appropriate conditions; and that the stability of the equilibrium depended upon the monopolistic seller's mode of expectation. In particular, we had the unstable monopolistic equilibrium, when the monopolistic seller was so optimistic that his overestimation of the slope of the demand curve went far beyound a certain amount. In this case, it is highly probable that the number of monopolistic firms becomes zero: i.e., the monopolistic firm cannot enter the market. In other circumstances, the number of the monopolistic firms is one. This is a direct result of Negishi's monokernel assumption.

In *Section 4* where the monokernel assumption is relaxed, we found that there existed a duopolistic equilibrium under the appropriate conditions; and that the stability of the equilibrium depended upon the two entrants' modes of expectation. When both entrants have pessimistic expectations, there is no meaningful duopolistic equilibrium: i.e., the number of monopolistic firms is one in this case. When both entrants are optimistic to similar degrees, there exists a stable duopolistic equilibrium. In this case, therefore, it is highly probable that the number of monopolistic firms becomes two. When both entrants have optimistic expectations, but their expectations diverge from each other's very much, the duopolistic equilibrium is unstable. In this case, it is probable that the number of firms becomes one. Similar arguments apply to remaining cases.

This logic must be extended to the more general polipoly case. In this paper, however, we confine our discussion to the duopoly case. Hence, we leave much to be developed beyond the duopolistic case.

Incidentally, one more point will be added. In the same paper as that cited above, Negishi discussed the monopolistically competitive equilibrium when the monopolistic sellers have kinked subjective demand curves. We can easily examine this case in terms of the partial equilibrium. We must confine here, however, our remarks to the result that the equilibrium is not unique.