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ECONOMIC GROWTH AND INTERNATIONAL TRADE
— Rybczynski Line and Engel Line —

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I.

The purpose of this paper, by concepts of Rybczynski line and Engel line, is to consider the effect of economic growth on the pattern of international trade. Using these analytical tools, we can briefly treat such a knotty topic as the relation of the economic expansion and foreign trade pattern, with relatively small efforts. But, we will not explicitly deal with causes of economic growth in this paper. We largely depend upon Professor H. G. Johnson\(^\circ\) in regard to the basic approach.

II.

Let's begin to prepare assumptions and to clarify the framework of analysis.

1. Two countries: we suppose the world consists of two countries, \(A\) and \(B\), only. Country \(A\) is “a small one” in the economic sense and country \(B\) is “the rest of the world”. Let’s further assume that economic growth occurs in country \(A\) alone and nothing happens in country \(B\). This permits economic growth of country \(A\) to have no effect on the equilibrium commodity terms of trade given in the international market and, therefore, to behave as “Price-taker”.

2. Two commodities: there are only two goods, \(X\) and \(Y\), in this model. Both \(X\) and \(Y\) are finished goods and are neither inferior goods nor Giffen goods. Commodity \(X\) is always labor intensive in comparison with commodity \(Y\) and there are no reversals in the relationship of factor intensity (i.e., capital-labor ratio).

3. Two factors of production: there are only two kinds of production factors, \(K\) (capital) and \(L\) (labor). In the economy we assume that both factors are always employed and utilized fully, and that they can freely move between industries \(X\) and \(Y\), but they cannot be entirely removed from one country to another.

(4) Factor endowment: country A is relatively abundant in labor and country B in capital. Therefore, provided that both countries have no large differences in respect to the social preference system of goods, labor is relatively cheap in country A and capital is relatively cheap in country B. Owing to Heckscher-Ohlin theorem, then, country A has "comparative advantage" in commodity X, and country B has "comparative advantage" in commodity Y.

(5) Real economy: we will also assume, by the traditional theory of international trade, that the "neutrality of money" holds throughout the present discussion. Accordingly, in both the commodity market and the factor market, we should treat only the "relative price", and then we need not deal with the "absolute price".

(6) Competitive markets: all of the markets satisfy the condition of a perfect competition. There are no differences in price factors between industries, prices of all kinds are adequately flexible, and therefore, the equality of the marginal substitution rate and the price ratio (relative price) is always maintained. This means there is no "internal economy" in a special firm as well as gaps between the private and the social cost owing to "external economy and diseconomy".

(7) Production function: the production functions of both commodities are linear homogeneous (i.e, constant return to scale) and they are common to two countries in each commodity. That is to say, the production method of the same commodity is also the same in both countries. Because the marginal productivity of each factor is decreasing, the production possibility curve (i.e, opportunity cost curve) is concave to the origin.

(8) Social indifference map: it is assumed that the social indifference curves are given and are exactly the same or approximately the same in both countries. And furthermore, their shape is assumed to remain the same after economic growth.

(9) No barriers to trade: we will assume throughout that there are no barriers to trade such as traffic costs, tariffs, import restrictions and so on.

III.

Economic growth, whether it owes to increase of factors (capital formation and population growth) or to technological progress, makes the production possibilities frontier shift outward, and then, permits the consumption possibilities frontier to remove outward. The resultant change of trade pattern depends upon the overall effect, the synthesis of production effect and consumption effect. The most convenient tools to grasp these two effects are "Rybczynski line" and "Engel line" (i.e, the income
consumption curve) respectively.

In Figure 1, Rybczynski line (ray OR) and Engel line (ray OE) are shown in solid line, when the neutral economic growth takes place and then the production possibilities frontier shifts outward in parallel (\(AA \rightarrow A'A'\)), if and only if the equilibrium terms of trade remain constant, and how to change "the triangle of foreign trade" (\(QCP \rightarrow C'Q'P'\)) are depicted.

Commodity \(X\) is measured in the horizontal axis and commodity \(Y\) in the vertical axis. Since the equilibrium production and consumption point of pregrowth are point \(P\) and \(C\) respectively, this country exports \(QP\) of commodity \(X\) and imports \(CQ\) of commodity \(Y\). Line \(MN\) is the consumption possibilities frontier of pregrowth, and the slope is the equilibrium terms of trade. By the assumption, because the relative price remains changeless in postgrowth, the equilibrium production and consumption point remove from \(P\) to \(P'\) and from \(C\) to \(C'\) severally, and this country exports \(Q'P'\) of \(X\) and imports \(C'Q'\) of \(Y\). The trace of these equilibrium production points and consumption points is defined as "Rybczynski line" and "Engel line" respectively. Figure 1 shows a neutral change when both export and import increase in proportion to the rate of growing output (i.e., the rate of economic growth). If the overall effect of growth in country \(B\) were also a neutral type or totally unchangeable, the pattern of international trade would remain unchanged and the volume of trade would be sure to expand.

The path of Rybczynski line is determined by the shift of the production possibilities frontier, and the course of Engel line is restricted by
the property of a social indifference map. In each line, we can identify five possibilities®; neutral, pro-trade bias, anti-trade bias, ultra-pro-trade bias, and ultra-anti-trade bias. How to change the triangle of foreign trade relies upon the range which Rybczynski line and Engel line pass through any point of the consumption possibilities frontier.

Using Figure 1, we can classify these five possibilities as follows. On Rybczynski line, the case that point \( P \) moves along the line \( PR \) is defined as neutral, and the case that point \( P \) passes through the range \( P''P''' \), the range \( P''N' \) and the range \( P'''M' \) is defined as, pro-trade bias, anti-trade bias, ultra-pro-trade bias, and ultra-anti-trade bias, respectively. Similarly, on Engel line, the case that point \( C \) moves along the line \( CE \) is defined as neutral, and the case that point \( C \) passes through the range \( C'C''' \), the range \( C'M' \) and the range \( C''N' \), is defined as pro-trade bias, anti-trade bias, ultra-pro-trade bias, and ultra-anti-trade bias, respectively. Note that except for the neutral case, the classification of bias type is a reverse between Rybczynski line and Engel line. This is caused by the opposite effect of economic growth on the expansion of foreign trade in the production and consumption side. But, in Engel line, we must exclude two paths, that is, ultra-pro-trade bias and ultra-anti-trade bias in the following sections, on the assumption that both commodity \( X \) and \( Y \) are not inferior goods.

IV.

We are now ready to study in detail the overall effect of economic growth on the pattern of foreign trade. To avoid complication, in following figures, we will omit both the production possibilities frontier and the social indifference curves, and the consumption possibilities frontiers are drawn only.

(1) Neutral case

Figure 2 is a concise form of Figure 1 and is a neutral case as stated already. Rybczynski line and Engel line intersect at the origin. In this neutral case, the production possibilities frontier shifts outward in parallel and the social indifference map is homothetic. The income (output of commodities) elasticity of demand of both goods, \( X \) and \( Y \), is unity. For example, the income elasticity of demand of \( Y \) (importables to this country) is shown as follows®:

\[
\frac{dX}{dx} = \frac{dY}{y} = \frac{MM'}{OM} = \frac{NN'}{ON}
\]

® We, in this paper, prefer Johnson's definition to Hick's definition for the classification of "bias type". In Hick's terms, "pro-trade bias" is named as "export biased" and "anti-trade bias" is named as "import biased".

® We can get the same result by using the income elasticity of demand of \( X \).
where $CC'/OC$ is the increasing rate of demand of $Y$ and $MM'/OM$ is the growth rate of income in terms of $Y$. Since these ratios are entirely equal, as proved in Figure 2, $\eta_Y$ is unity. Namely, the demand of imports increases in proportion to the growth of income. Therefore, if the overall effect of economic growth in country B were either neutral or zero (no economic growth), the pattern of international trade would remain constant.

There is a noticeable point in this context. It is the fact that both production effect and consumption effect need not necessarily be neutral in order that the overall effect of economic growth be neutral. For instance, even though the production effect is pro-trade bias and the consumption effect is anti-trade bias, the overall effect may really be neutral.

Figure 3 shows such a case. Although Rybczynski line and Engel line do not intersect at the origin, but cross at the point $H$ on the line $H'H''$ which passes through the origin and is parallel to the consumption possibilities frontier $MN$. Since Engel line $CE$ keeps to the right of line $CE'$, the consumption effect is anti-trade bias, and since Rybczynski line $PR$ keeps to the right of line $PR'$, the production effect is pro-trade bias. Such a situation is caused by the property of the social indifference map and the lopsided shift of production possibilities frontier. In the case of Figure 3, nevertheless, the overall effect of economic growth on the pattern of foreign trade is neutral, because the income elasticity of demand is unity. As the line $QQ''$ is parallel to the line $CC'$, the increase of
importables $Y$ of postgrowth can be shown as the segment of a line $Q'Q''$. Because $Q'Q''/QC$ is equal to $Q'Q''/C'Q''$ and $\Delta HCQ$ and $\Delta QQ''Q'$ are homological triangles, $QQ''/HC$ is equal to $CC'/HC$. That is, the ratio $CC'/HC$ is the increase rate of demand of commodity $Y$. Furthermore, the increase rate of income in terms of $Y$ is shown by the ratio $MM'/OM$, which is equal to $NN'/ON$. Thus, the income elasticity of demand of $Y$ can be expressed as follows:

$$\eta_Y = \frac{CC'}{HC} \cdot \frac{MM'}{OM} = \frac{CC'}{HC} \cdot \frac{NN'}{ON}$$

It is evident that $CC'/HC$ is equal to $NN'/ON$ in Figure 3. Therefore, $\eta_Y$ is unity. This means both exportation and importation increase in proportion to the growth rate of income.

As we have seen above, depending upon the interrelation of the shifting of the production possibilities frontier and the shape of social indifference curves, there are possibilities that the overall effect of economic growth on the trade pattern is neutral, even though Rybczynski line and Engel line do not intersect at the origin. But there are two limitations in this argument. First, the movement of intersecting point $H$ is limited to the region $H'H''$ and moreover both point $H'$ and $H''$ must be excluded. This is the reason that Engel line can not pass through to the left of line $CE''$ and under the line $CE'''$, by our assumption that both $X$ and $Y$ are not inferior goods. Second, Rybczynski line and Engel line which pass through the quadrant except for the first quadrant are merely the supplementary line of geometry, so that, these lines do not have any economic meaning.
(2) Pro-trade bias case

As presented in Figure 4, Rybczynski line and Engel line intersect \textit{acutely} at the right of line \( H'H'' \), for instance point \( H \). In this case, while the income elasticity of demand of \( Y \) (importables of this country) is more than unity, the income elasticity of demand of \( X \) (exportables of this country) is less than unity. As shown in Figure 4, since \( \frac{CC'}{OC} > \frac{CC^*}{OC} \) in terms of \( X \), \( \eta_Y = \frac{CC'}{OC} \cdot \frac{MM'}{OM} < 1 \). That is to say, this country will export goods \( X \) more than, and import goods \( Y \) less than, the rate of income growth. Such a change of trade pattern is called “pro-trade bias growth” in the sense that the foreign trade is to be encouraged.

![Figure 4](image)

(3) Anti-trade bias case

In this case, like the point \( H_3 \) in Figure 5, Rybczynski line and Engel line intersect at the left of line \( H'H'' \). In this case, while the income elasticity of demand of \( Y \) is smaller than unity, that of \( X \) is larger than unity. Since \( \frac{CC'}{OC} < \frac{CC^*}{OC} \) in terms of \( Y \), \( \frac{CC'}{OC} < \frac{MM'}{OM} \) and then, \( \eta_Y = \frac{CC'}{OC} \cdot \frac{MM'}{OM} < 1 \). The reverse of this relation holds in the case of commodity \( X \). Thus \( \eta_X = \frac{CC'}{OC} \cdot \frac{MM'}{OM} > 1 \). This growing country \( A \) will export goods \( X \) and import goods \( Y \) less than the rate of economic growth. This type of economic growth will surely discourage international trade. This is the reason that this case is named “anti-trade bias growth”.

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**Figure 4.**
(4) Ultra-pro-trade bias case

This case is the one that Rybczynski line and Engel line intersect obtusely at the right of line $H'H''$, as the point $H_{0x}$ has shown in Figure 6. Such a situation is apt to occur when the production possibilities frontier shifts extremely bias to exportables $X$ and the social indifference system has fairly inclined property to importables $Y$. In this case, it is, of course, clear that $\eta_Y > 1$ and $\eta_X < 1$ like the case of pro-trade bias. The triangle of foreign trade will expand remarkably higher than the rate of income growth and, moreover, the path of point $Q$ (indicates trade volume) goes in the direction of the southeast. This means the absolute reduction in
the production of $Y$ in this growing country. And eventually, this country may reach the complete specialization of commodity $X$.

(5) Ultra-anti-trade bias case

This case is the one that the intersecting point of Rybczynski line and Engel line lies to the right of the consumption possibilities frontier of postgrowth $M'N'$. The point is $H_{RM}$ as shown in Figure 7. Such unusual case, in contrast to ultra-pro-trade bias case, is apt to occur where the production possibilities frontier expands extraordinarily in favor of importables $Y$ and when the social indifference system has the property of being inclined to exportables $X$. In this case, it is clear that $\eta_Y < 1$ and $\eta_X < 1$, like anti-trade bias. In contrast to anti-trade bias, the great difference is the \textit{absolute} reduction of the triangle of foreign trade, that is, the shrinkage of trade volume. As economic growth is promoted, the point $Q$ will gradually approach the point $H_{RM}$ with no foreign trade and when the point $Q$ goes beyond the point $H_{RM}$, the pattern of international trade will be reversed. That is to say, this growing country $A$ will export the goods $Y$ and import the goods $X$.

![Figure 7](image)

Figure 8 represents the extreme case that Rybczynski line has a negative slope. Since the equilibrium production and a consumption point exchange their location with each other on the consumption possibilities frontiers of postgrowth $M'N'$, this growing economy must export $C'Q'$ of $Y$ and import $P'Q'$ of $X$. Such a radical change is perhaps brought about with the drastic change in the production side in the short run and, as a result, the very lopsided shift to importables may occur in the produc-
tion possibilities frontier. By our assumption that the slope of Engel line cannot possibly be negative, the production effect mainly takes the initiative in this case.

V.

The preceding is the analysis mainly of the overall effect of economic growth on the pattern of foreign trade in country A only. In order to see the real effect of economic growth on the international trade in “two countries model”, we should consider the effect of economic growth on the trade pattern in country B simultaneously. For example, when the overall effect is pro-trade bias in country A, it is neutral in country B, and similarly when it is ultra-pro-trade bias in country A, it is ultra-anti-trade bias in country B. A convenient way to answer the question is to use “an offer curve”. We will take up this problem next time.

(To be continued in the next issue)

REFERENCES

