A NOTE ON THE NUMBER OF FIRMS IN A GROWING INDUSTRY*

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I INTRODUCTION

The purpose of this note is to investigate the stabilit [illegible] ib-rium of a growing industry. The word "equilibrium" mea [illegible] est and an industry is in equilibrium when none of its membe [illegible] ed to change his behavior. Then, a unique number of firms exists in that industry, because in the equilibrium state neither entry nor exit occurs. And we may treat the problem of entry and exit as the adjustment problem of the number of firms being necessary to attain the equilibrium of the industry. The adjustment of the number of firms in the industry is dominated by the industry's market conditions. And they depend largely upon two causes: one is an exogenous cause from the demand side, and the other endogenous from the supply side through the competitive process.

In view of this classification of causes, it is interesting to inquire what conditions the equilibrium number of firms in the industry and its stability depend upon. Howrey and Quandt [2] discussed the problem under the hypothesis that individual firms adjust their output level instantaneously through variations of an amount of capital as far as there is an excess (deficit) profit. Their discussion was followed by Myers and Weintraub [3] that analysed a more general case in which variability in output takes place through changes in not only the amount of capital but the amount of labour employed per capital. Both analyses were based on the same assumptions; homogeneous products with uniform price, equality in the scale of plant, downward sloping demand curve, inducement of entry (or exit), and so on.

Howrey and Quandt argued the adjustment process of the number of firms in the case of the growing market and derived the local and global stability of the process. However, their assumption that the rate of the growth of profit in that industry is exogenously given seems to be unrealistic, since in the competitive industry the level of profit is endogenously determined through the market forces.

In this note we examine the above process in terms of a more realistic

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model which incorporates such characteristics of the industry as the elasticity of demand and the growth of market size and treat the level of profit as an endogenous variable[1].

II MODEL

Our prime concern is whether the stability of the number of firms in a growing industry does or does not depend upon such characteristics of the industry as the price and income elasticities of demand (let them be η and $v>0$, respectively) and the growth rate of national income (let it be $g>0$), in addition to the firms' expectation on the future price. We use the extraporative expectation ;

$$p_t^e = p_t + a\dot{p}_t$$

where, p_t^e, p_t, a, and $\dot{p}_t$ are expected and current prices, a constant coefficient of adjustment, and rate of change in price, respectively.

We assume that an entrepreneur decides to enter into or to exit from the industry depending upon his expected profit being positive or negative. Hence the dynamic adjustment equation is (for simplicity we set $a=1$) ;

$$\dot{n}_t = k\left(p_t + \dot{p}_t - \frac{C(q_t)}{q_t}\right) \qquad k>0 \tag{1}$$

where q_t (constant over time) and $C(q_t)/q_t$ are output per firm and the average cost at time t, respectively. In our model (equally in Howrey and Quandt [2] and Myers and Weintraub [3]) the firm is assumed to accomplish its desire to grow by means of not expanding its plant scale itself but increasing the number of plants of the same scale. And it will be assumed that all firms, actual or potential, are identical with respect to cost[2]. Further, each firm faces the identical price of the products of other firms in the industry, $p_t = f(n_t q_t)$ and $f'(n_t q_t) < 0$ (here, n_t is the current number of firms in that industry). On the other hand, the quantity of demand of the industry (let it be $Q_t = n_t q_t$) can be written as the function of the unit price and national income (Y_t), i. e.,

1) The quantity which can be sold in the commodity market depends upon the elasticity of demand and also depends upon the factors over which the firm has no control, and the composite effect of changes in those factors on the quantity sold will be summarized in an 'income' elasticity v. Then it is common to think that the conditions of entry or exit depend upon such factors as the elasticity of demand, growth rate of national income, optimum scale of plant, and so on; see J. N. Bhagwati [1].

2) For the sake of this assumption, we can say nothing about whether new firms enter or old firms expand. The referee of *The Review of Economic Studies* pointed it out and suggested that relaxing this assumption would be a much more interesting than the route we take, but it is beyond the limits of this paper.

$$Q_t = Q(p_t, Y_t)^{3)} \tag{2}$$

Thus, the growth of demand of that industry depends upon the price change through the variation of the firms (i. e., quantity of supply) and upon the growth of national income, so we may presume that the process of competition among actual and potential firms and the stability of the equilibrium number of firms also depend upon the above factors, η, v, and g. In the following we investigate this problem. From the above demand function and the assumption that q_t is constant over time, we obtain[4)]

$$\frac{\dot{Q}}{Q} = -\eta\frac{\dot{p}}{p} + vg = \frac{\dot{n}}{n} \qquad \text{here,} \quad -\eta = \frac{\partial Q}{\partial p}\frac{p}{Q}, \quad v = \frac{\partial Q}{\partial Y}\frac{Y}{Q}, \quad g = \frac{\dot{Y}}{Y} \tag{3}$$

which is rewritten as

$$\dot{p} = -\frac{p}{\eta}\left(\frac{\dot{n}}{n} - vg\right) \tag{4}$$

Equation (4) implies that the price change is shown as the net effect between the effect resulting from the variation of the quantity of supply due to a change in the number of firms and the effect of the national income growth upon the demand for the industry's products. The behavior of the price is summarised as the following,

$$\begin{array}{lllll} & \dot{p} > 0 & & \frac{\dot{n}}{n} < vg \text{ i. e.,} & \text{the growth of supply} < \text{the growth of demand} \\ (4') & & \text{when} & & \\ & \dot{p} < 0 & & \frac{\dot{n}}{n} < vg \text{ i. e.,} & \text{the growth of supply} < \text{the growth of demand} \end{array}$$

III ANALYSIS

Now, let us see the stability condition in our model. Substituting (4) into (1)

$$\dot{n} = k\left[f(nq) - \frac{f(nq)}{\eta}\left(\frac{\dot{n}}{n} - vg\right) - \frac{C(q)}{q}\right] \tag{1'}$$

or

$$\dot{n} = \frac{k\left[f(nq) + \dfrac{vg}{\eta}f(nq) - \dfrac{C(q)}{q}\right]}{1 + \dfrac{f(nq)}{\eta n}}$$

Then the local stability condition in the neighborhood of the equilibrium

3) Such a special type of the demand function (2) as $Q_t = Ap_t^{-\eta}Y_t$ is often used. We shall obtain the same results as those from the special case by using a more general function (2).

4) In the following discussion we abbreviate the time subscripts and $\dot{x}$ means dx/dt.

position is given by

$$\left.\frac{d\dot{n}}{dn}\right|_{\dot{n}=0} = k\frac{f'(nq)\left(1+\dfrac{vg}{\eta}\right)}{1+\dfrac{kf(nq)}{\eta n}}q < 0 \tag{5}$$

By virtue of the assumption that $f'(nq)$ is negative, it is clear that this condition is assured independently of parameters, η, v, and g.

Now, let us set $\dot{n}=0$ in (1′), i. e., at the equilibrium point ($n=n^*$), then we obtain

$$f(n^*q)+\frac{f(n^*q)}{\eta}vg-\frac{C(q)}{q}=0 \tag{6}$$

Solving this equation we get the equilibrium number of firms in a growing economy. This is equivalent to the result in Howrey and Quandt [2].

IV DISCUSSION

Further we shall examine the effects of changes in the parameters upon the equilibrium number of firms and upon the behavior of the price, and consider some economic implications of our model. From (1′) and (6) we can say that a rise of v and g makes the creation of a firm easier and makes the equilibriun number of firms increase. But the effects of η on the behavior of $\dot{n}$ and n are much more interesting than the effects of v and g. Differentiating (1′) and taking account of (4), we find that $\partial\dot{n}/\partial\eta>0$ if $\dot{p}<0$ and $\partial\dot{n}/\partial\eta<0$ if $\dot{p}>0$, and that $\partial n^*/\partial\eta<0$ from (6). In other words, an increase in the price elasticity of demand decreases the equilibrium number of firms, and the rate of growth of the number of firms is positively

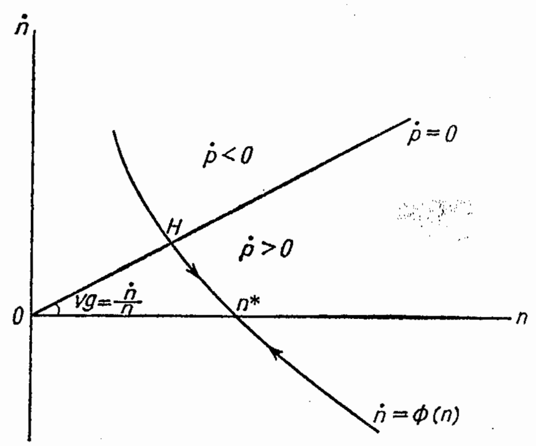

Figure 1

related to the increase in the price elasticity of demand when the growth of supply exceeds the growth of demand and *vice versa*. On the other hand, it is clear that the stability of the price largely depends upon the parameters v and g.

Next, it will be worthwhile to derive some economic implications from our model. From the discussion in the last section, the uniqueness of the equilibrium number of firms in a growing industry means that an increase in demand can not always be absorbed by the new creation of a firm. Accordingly it will be anticipated that after some point the exogenous growth of demand will raise the price alone. By using a diagram we can verify the noteworthy conclusion of our model. On the n–$\dot{n}$ plane, $\phi(n)$ curve which is defined as $\dot{n}=\phi(n)$ in the equation (1′) has a negative slope by virtue of the equation (5) and n^* is stable. From the equation (4) $\dot{p}=0$ line, that is to say, the relation between the growth of supply and the growth of demand holding p_t constant, is drawn as in Figure 1. By taking account of the relation (4′), it is clear that $\dot{p}$ is negative in the region above the $\dot{p}=0$ line and positive in the region under that line. At the intersection of the $\phi(n)$ curve and $\dot{p}=0$ line, not the change in the price of the products but the increase in the number of firms occurs. In the process of approaching to the equilibrium point n^*, an increase in the number of firms is accompanied with a fall in price untill the intersection H is attained, and it will be accompanied with a rise in the price beyond the intersection H.

REFERENCES

[1] Bhargwati, J. N. "Oligopoly, Theory Entry-Prevention, and Growth" *Oxford Economic Papers* 22 Nov. 1970; 297–310.

[2] Howrey, P. and Quandt, R. E. "The Dynamics of the Number of Firms in an Industry". *Review of Economic Studies* 35 July 1968; 349–353.

[3] Myers, M. G. and Weintraub, E. R. "A Dynamic Model of Firm Entry" *Review of Economic Studies* 38 June 1971; 127–129.