INVESTMENT BEHAVIOR OF THE FIRM AND DISEQUILIBRIUM THEORY*

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INTRODUCTION

In the microeconomic theory of investment behavior, the derivation of an optimal adjustment path for capital which is approximated by the flexible accelerator depends critically on the market-clearing condition, that is to say, on the perfect ability to reach an equilibrium. Then any transaction is free of external restriction, so that agents are able to sell or buy any amount they want to at the going equilibrium price. If we assume that agents are subject to static price expectation, convex internal adjustment cost and diminishing returns to scale, the optimal path for capital, labor employment and output supply converges to each optimal long-run point which represents the *notional* terminal point.

When output is in excess supply (such a case occurs as a result of general over-production), however, the *effective* demands for inputs are smaller than the notional demands for them in the long-run as well as in the short-run. This is the essential point of Patinkin's analysis [9]. Similarly, according to Clower [3], the effective demand for outputs is less than the notional one when the labor is in excess supply. Their analyses are integrated to form a general disequilibrium model by Barro and Grossman [1]. Further, Grossman has used an analogous approach to study the capital adjustment path for the firm in [6]. Their common emphasis was that the classical demand schedule for labor had no role in the market disequilibrium. In this respect, the conventional models of investment behavior have been relieved to include the effective adjustment by Grossman. In this paper, we shall present an expository note to Grossman's disequilibrium model with the aid of a simple graphical manner, and we shall point out a trickiness associated with the procedure that Grossman has relied upon to incorporate the "fixity" of capital into the formulation which "is consistent with the horizontal supply curve for the investment goods of perfect competition".[1] In the spirit of adjustment cost approach, the amount of

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1) Grossman, [6] p. 633.

output is governed by the production function and by the adjustment cost whether the latter is separable from the former or not. In this point of view, Grossman's treatment of the "fixity" of capital is not the same as the usual approach though he may not agree with our comment. Indeed, his method to build the "fixity" of capital explicitly into the formulation is nothing but to introduce the rising investment cost by postulating monopsonistic capital goods markets rather than by postulating internal cost of investment in the form of output foregone.

I. A CONVENTIONAL NEOCLASSICAL MODEL: UNCONSTRAINED VERSION

The essential difference between capital and labor as productive resources is that capital is imperfectly variable in a sense discussed in the several papers while labor is perfectly variable.[1] The firm, as an agent, is conceived as a collection of productive resources together with the motive of maximizing the value of this collection. All the activities that are intended and realized within the firm as a collection of productive resources are mutually dependent. If we wish to build the "fixity" of capital explicitly into the formulation, we must introduce the rising investment cost either by postulating monopsonistic capital goods markets or by introducing internal cost of investment in the form of output foregone. The latter postulation is most suitable for the above internal dependence. Taking the "fixity" and the internal dependence into account, the "general" production function has to be expressed by

$$y = f(K, L, I),$$

where y, K, L and I denote the total amount of output, the capital stock, the labor employment, and the gross investment rate, respectively. The fixity is expressed by $f_I < 0$ and $f_{II} < 0$. Among the authors there have been various postulates about the content of the adjustment cost as a separable term from $f(.)$.[2]

Alternatively, there has been an approach which would neglect the internal dependence and the internal adjustment cost by postulating monopsonistic capital goods markets. It is well-known that Keynes [7] has assumed that the expected return would diminish with the investment rate and that the firm would face a rising schedule for the investment goods.[3]

1) See especially Lucas [8] and Treadway [12].

2) For example the following technical relationship has been postulated by Gould [5].

$$y = F(K, L) - a(I), \quad a'(I) > 0, \quad a''(I) > 0$$

3) Keynes [7] p. 136.

These assumptions are enough to assure an unique intersection of the interest rate with the declining marginal efficiency of capital. The similar monopsonistic case has been considered by Eisner and Strotz [4].[4] There exists another reason that the monopsonistic capital goods markets are considered. To my inference, most literature uses continuous time analysis on the assumption that it is possible for the firm to shorten the delivery lags into zero, if it would like to pay a higher delivery price.[5] That is to say, there remains a reason that we incorporate the external adjustment cost for shortening the delivery lags. Indeed Grossman's formulation should be justified on these bases.

Hence we can start from the conventional neoclassical production function which is different from the one that the adjustment cost approach has used:

$$y = F(K, L), \tag{1}$$

where it is assumed that $F(.)$ has convex isoquant and has the limiting properties given by

$$F_K(K, L), \qquad F_L(K, L) > 0,$$

$$F_{KK}(K, L), \qquad F_{LL}(K, L) < 0, \qquad \text{for } K, L > 0,$$

$$D = \begin{vmatrix} F_{KK} & F_{KL} \\ F_{LK} & F_{LL} \end{vmatrix} > 0 \qquad \text{(diminishing returns to scale)},$$

$$F_{LK} = F_{KL} > 0.$$

The investment cost function $C(I)$ is assumed to have two continuous derivatives

$$C'(I) > 0, \qquad C''(I) > 0, \qquad \text{for } I \geqq 0.$$

At every point in time, the firm employs labor and sells output so as to maximize current profits subject to a given stock of capital. However, over a period of time, the firm invests so as to achieve the optimal long-run (target) quantity of capital. The time path of investment chosen to achieve the target capital stock is that which maximizes the present value of the entire future stream of returns to the firm. Thus the firm has to choose time paths for K and L to maximize:

$$V = \int_0^\infty e^{-rt}\big[py - wL - C(I)\big]\,dt$$

subject to

[4] Eisner and Strotz [4] p. 69–71, they have considered the "expansion cost" as the expenditure or the cost premium rather than the lost output.

[5] See Sakai [11].

$$\dot{K} = I - \beta K,$$
$$y = F(K, L),$$
$$y \leqq y^{\circ},$$

where r, p, w, $\dot{K}$, β and y° denote the interest rate, the product price, the nominal wage rate, the net increment of capital, the depreciation ratio, and the demand for the output, respectively. The conventional neoclassical approach has assumed that the firm was a price taker rather than a price searcher and behaved as if his current supply of output as well as his planned future supplies of output could in fact be sold. In this régime, the appropriate integrand the firm will maximize is the following Hamiltonian:

$$(2) \qquad \mathscr{H} = e^{-rt}\left[pF(K, L) - wL - C(I) + \lambda(I - \beta K)\right]$$

Here we have to note that the firm is facing a sufficiently adequate demand for the output and that this is equivalent to assume the demand constraint to be effectless. This is the primal character of the so-called *notional* model.

The optimality conditions are arranged to yield

$$(3) \qquad C''(I)\,\dot{I} - (r+\beta)\,C'(I) + pF_K(K, L) = 0,$$

$$(4) \qquad pF_L(K, L) = w.$$

In these equations we can find out the equilibrium price of output as a partial index of defining the marginal value products of capital and labor. Or, in other words, the equilibrium price, p, is the important key signal reflecting the market conditions, so that the firm will equate the marginal variable cost, w/F_L, with the marginal revenue realized in the market, p. From this classical equality, the following labor demand function is derived

$$(5) \qquad L = L(K, w/p)$$

such that

$$F_L(K, L) = w/p.$$

The output supply function is simply

$$(6) \qquad y = F(K, L).$$

The investment demand function may be separated into three components. The first component is the target (long-run optimal) capital stock, denoted by K^* and given by

$$(7) \qquad K^* = K^*(r, \beta, w/p),$$

such that

$$(8) \qquad pF_K\left(K^*, L(K^*, w/p)\right) = (r+\beta)\,C'(\beta K^*).$$

The second component is the optimal time path of the capital stock, which, if we consider quadratic approximations of the F and C functions, is given by the gradual adjustment relationship:

$$\dot{K} = \alpha(K - K^*), \tag{9}$$

where α is the notional adjustment coefficient, and is given by

$$\alpha = r/2 - \left[(r/2)^2 + (r+\beta)\beta - pD/F_{LL}C''(0)\right]^{1/2} < 0. \tag{10}$$

The last component is the replacement investment which is equal to βK by assumption. Hence the notional gross investment function is given by

$$I = \alpha(K - K^*) + \beta K. \tag{11}$$

Under the assumption that the expectations regarding w, p, r and β are static and that the production function $F(.)$ is a type of diminishing returns to scale, the firm, under the market-clearing conditions, plans for the capital stock to approach K^* asymptotically. Accordingly, the firm plans for labor demand and output supply to approach asymptotically $L^* = L(K^*, w/p)$ and $y^* = F(K^*, L^*)$, respectively. Hence all the quantities, whether they are observed in the short-run or in the long-run, are classified as the notional ones.

Figure 1, which is presented by Bowden [2], illustrates the optimal adjustment path for the firm. We start from the point A, where the demand is greater than the output supply because of a shift in demand. Since the adjustment of labor is assumed to be free of restriction, the new employment to the point B is instantaneous. On the other hand, the adjustment of capital is costly so that the approach to K^* is illustrated as a gradual movement from B to C. Here the movement from B to C satisfies the classical equality of the marginal product of labor with the

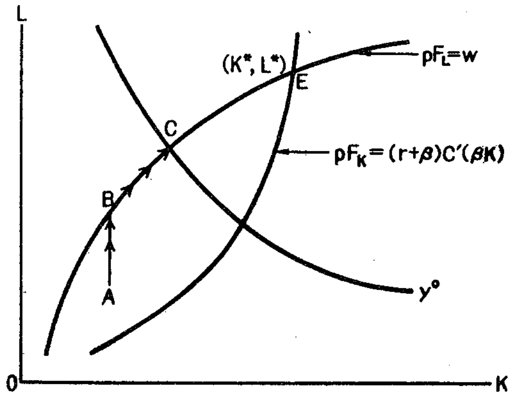

Figure 1.

real wage rate, so that the gradual movement is observed along the $pF_L=w$ curve. This gradual adjustment process will continue till the firm finds out the unexpected accumulation of inventory of the product because of excess supply. The point C is, therefore, a critical point that the firm can neglect the demand constraint continuously or not. In other words, this point makes the notional quantities vain and makes the firm a searcher for a new behavior rule of transactions in market disequilibrium.

II. A DISEQUILIBRIUM MODEL: CONSTRAINED VERSION

There may be several prices coexisting in the market at any moment when agents are in market disequilibrium, so that agents do not face a single known price. Or there may be only one price, but if it is not an equilibrium price, agents can not sell or buy any amount they want to. In these circumstances, agents would not reckon market prices as fair signals reflecting the market conditions, so that agents would remedy their behavior rules in order to open their transaction before the markets come to be in equilibrium again. Since the markets are unable to adjust the equilibrium state rapidly and the transmission as well as processing of information is costly, it is not unreasonable to suppose that there is only one price prevailing over each market even if it is not an equilibrium price. As shown later, this kind of price plays no role in demand constrained world. Instead the marginal variable cost is charged with guiding the firm to complement the market inability to adjust.

Hence the new rule is introduced into our model which is proposed by Patinkin, Clower, Barro and Grossman, being applied to study the capital accumulation for the firm by Grossman.

Assume, for the convenience of illustrating the essence, that the demand y° is constant over a period of time. Then the firm will always be able to sell output quantity y°, and no more. At the point C in Figure 1, the demand constraint is effective, so that the following integrand is appropriate as a consequence of the new behavior rule of the firm:

$$\hat{\mathscr{H}} = e^{-rt}\Big[py-wL-C(I)-\mu_1\big(y-F(K,L)\big)-\mu_2(y-y^\circ)+\lambda(I-\beta K)\Big] \tag{12}$$

The optmality conditions are

$$p-\mu_1-\mu_2=0\,, \tag{13}$$

$$\mu_1 F_L = w\,, \tag{14}$$

$$\mu_2 \geqq 0 \text{ and } \mu_2(y-y^\circ)=0\,, \tag{15}$$

$$C''(I)\,\dot{I}-(r+\beta)\,C'(I)+\mu_1 F_K=0\,. \tag{16}$$

On the y° isoquant including the point C, the demand constraint is effective, so that $\mu_2>0$. This means from (13) and (14)

$$pF_L(K, L)-w=\mu_2 F_L(K, L)>0\,, \tag{17}$$

that is to say, F_L is greater than w/p along the effective demand for labor, so that there exists no definite relationship between L and w/p. Then the effective demand for labor, $\hat{L}$, is given by

$$\hat{L}=\hat{L}(K, y^\circ) \tag{18}$$

such that

$$F\big(K, \hat{L}(K, y^\circ)\big)=y^\circ\,.$$

Now the condition (16) becomes in this régime

$$C''(I)\,\dot{I}-(r+\beta)\,C'(I)+wF_K\big(K, \hat{L}(K, y^\circ)\big)\big/F_L\big(K, \hat{L}(K, y^\circ)\big)=0 \tag{19}$$

Since the last term is evaluated on the isoquant

$$F\big(K, \hat{L}(K; y^\circ)\big)=y^\circ\,,$$

it becomes a function of K only, so that we shall define $M(K)$ such as

$$M(K)=F_K/F_L \tag{20}$$

$$M'(K)=-\hat{L}_{KK}=(F_{KK}F_L-F_{LK}F_K)/F_L^2<0\,. \tag{21}$$

In this régime, the variable $\hat{I}$, the effective gross investment demand, is separated into three components as before. The first component is the effective target capital denoted by $\hat{K}^*$, and given by

$$\hat{K}^*=\hat{K}^*(r, \beta, w, y^\circ) \tag{22}$$

such that

$$M(\hat{K}^*, y^\circ)\,w=(r+\beta)\,C'(\beta\hat{K}^*)\,. \tag{23}$$

The second component is the optimal path of capital stock, which is given by the gradual adjustment relationship along an isoquant:

$$\dot{K}=\hat{\alpha}(K-\hat{K}^*) \tag{24}$$

where $\hat{\alpha}$ may be denoted as the effective adjustment coefficient, and is given by

$$\hat{\alpha}=r/2-\big[(r/2)^2-M'(K)\,w/C''(0)+(r+\beta)\,\beta\big]^{1/2}<0\,. \tag{25}$$

If $K\geqq\hat{K}^*$, then $\dot{K}$ will be zero. Otherwise, $\dot{K}$ will be positive. The last component is the effective replacement investment. Hence the effective gross investment demand function is given by

$$\hat{I} = \hat{\alpha}(K - \hat{K}^*) + \beta K. \tag{26}$$

Apparently, we can insist as a general corollary that $\hat{\alpha}$ is not equal to α and that $\hat{K}^*$ is not equal to K^*.

Now we shall find the effective target capital stock on the isoquant. From (20),

$$-dL/dK = M(K).$$

So at the long-run optimal point, from (23)

$$-\frac{dL}{dK}\bigg|_{\hat{L}^*}^{\hat{K}^*} = \frac{(r+\beta)(C'(0)+C''(0)\beta\hat{K}^*)}{w}.$$

This point exists on the intersection of the isoquant with the so-called expansion path which is illustrated as the rising curve in Figure 2. The effective target capital stock as well as the long-run effective demand for labor is shown by the point D. Thus the last stage of the optimal adjustment consists in movement along the isoquant, from C to D. As pointed out by Bowden, over the whole path, labor is "first hired, then fired".

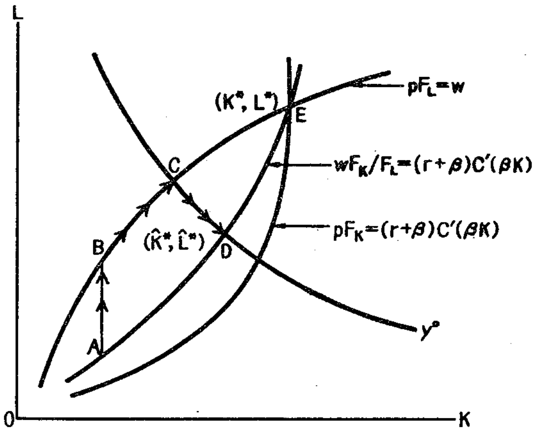

Figure 2.

CONCLUDING REMARKS

The adjustment process follows two stages, the first corresponding to the neoclassical analysis by Lucas, Gould and Treadway, the second a movement along a production isoquant.

Grossman's insistence that "investment is subject to increasing marginal cost" is expressed by the external adjustment cost which is an expenditure rather than lost output. This is true in spite of his emphasis that his analysis is according to the usual internal adjustment cost approach.

Finally, we shall make a remark that the usual internal adjustment approach is not able to yield meaningful and operational results. That is to say, postulating monopsonistic capital goods markets is the only successful way to analyze the transaction in the market disequilibrium. The reason is appended to this paper.

APPENDIX

This appendix presents a refutation of Grossman's use of the demand constraint coupled with the adjustment cost, which appears in Bowden (op. cit.) again, if the adjustment cost is the lost output with which the investment activity is accompanied. Then the constraints should be modified, for example, as

$$y = F(K, L) - a(I),$$

$$y \leqq y^{\circ},$$

where $a(I)$ denotes the internal adjustment cost in terms of lost output. Thus, as $a'(I)$ is strictly increasing, if and only if the demand constraint is not binding Grossman's derivation of the notional demands for both factors is quite right, letting $C(I) = pa(I) + qI$. So far as we are concerned with his disequilibrium model, however, his insistence that $C'(I)$ is strictly increasing "is consistent with the horizontal supply curve for investment goods of perfect competition", leaves room for a criticism.

If we regard his idea as relevant, then we have to form the following Hamiltonian with the modified constraint as the appropriate integrand:

$$\tilde{\mathscr{H}} = e^{-rt}\Big[py - wL - qI - \mu_1\big(y - F(K, L) + a(I)\big) - \mu_2(y - y^{\circ}) + \lambda(I - \beta K)\Big]$$

In this integrand we can find out that the total investment cost $C(I)$ is decomposed into the purchasing cost, qI, and the internal adjustment cost, $a(I)$, which plays a different role from the former. So far as the demand constraint is not binding, we can obtain the same result as the conventional neoclassical model or the notional model in Grossman's terms. Whereas the demand constraint is binding, however, we obtain a slightly different result which is represented by the equation:

$$0 = \left[a''(I) - \left(\frac{a'(I)}{F_L}\right)^2 F_{LL}\right]\dot{I} - \frac{a'(I)}{F_L}\left(F_{LK} - \frac{F_{LL}F_K}{F_L}\right)\dot{K} - (r+\beta)\left[a'(I) + q\frac{F_L}{w}\right] + F_K$$

Here we should note that the product isoquant is not shown by a sole curve because a positive net investment causes some output consumption so that it reduces the net output supply, which, in turn, causes shifts in the isoquant upwards. Furthermore, the existence of the internal adjustment cost may make the slope of the isoquant rising for some larger K.

In addition to this irrelevancy of usual isoquant conjecture, the calculation to derive the effective adjustment coefficient appears to be difficult.

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