RESEARCH EFFICIENCY AND MARKET SHARES IN A COURNOT DUOPOLY MODEL*

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One aspect of market structure in an industry is the distribution of market shares among firms in that industry. Various determinants of market share have been discussed in the industrial organization literature; prominent among them are: economies of scale, other barriers to entry, mergers, and growth of industry size (see references (1, 4) for discussion of these factors). One aspect neglected in these discussion is the effect of research efficiency on changes in market shares. To the extent that research efficiency arises out of certain characteristics of the firms, e.g., size, level of diversification, age, or length of research experience, testable propositions can be formed to see how significantly these characteristics are related to market share changes. This paper lays a theoretical foundation for such possible inquiry by showing that, in the framework of a Cournot duopoly model of firms involved in R & D, an increase in research efficiency will lead to a higher market share for a duopolist under certain weak conditions.

The Model

A Cournot duopoly model of two firms engaged in research and development (R & D) is developed in this section. Firm No. 1 faces the following demand, cost and research production functions:

$$p_1 = p_1(\underset{(-)}{x_1}, \underset{(+)}{R_1}, \underset{(-)}{x_2}, \underset{(-)}{R_2})$$

$$C_1^T = C_1(\underset{(+)}{x_1}, \underset{(-)}{R_1}) + \Omega_1$$

$$R_1 = F_1(\underset{(+)}{\Omega_1}, \underset{(+)}{E_1}),$$

where the subscripts 1 and 2 refer to the first and second firms respectively,

* This paper is based partly on my Ph. D. dissertation, "Diversification, Research Productivity, Research Intensity, and Market Shares", Brown University, June, 1979. I am indebted to Hiroshi Ono and Harl Ryder for help in the analysis of the model presented here.

and

p=price, x=quantity of the product,
R=output of research, C^T=total cost,
C=cost of production, Ω=expenditure on R & D, and
E=level of research efficiency

The second duopolist similarly faces functions of the same form, with subscripts 1 and 2 interchanged.

Both firms maximize profit subject to X and Ω. Each duopolist assumes that the rival would maintain his levels of these two instrument variables.

The two firms face different (inverse) demand functions instead of sharing a common demand function as in usual Cournot models. The price that a duopolist can charge for his product depends on the quality that goes in the product. This quality is generated by the output of research and development. The price varies directly with quality of his own product and inversely with his own quantity and the rival's quantity and quality of product.

The total cost C^T consists of production cost C and expenditure Ω, measured in dollar units, on research and development. The production cost increases with output and decreases as a result of increased output of research. It is the same index of research output, R, that is being used both in the demand and the cost functions. The result of R & D in fact consists of many different findings in R & D laboratories. Some of these findings are relevant for production improvement and some for cost reduction. The aggregate of all these findings is given a single cardinal index, R, of research output. An increase in R can be due to an increase in either cost-reducing findings or product-improving findings or both. As a matter of trivial simplification it is assumed here that both $\frac{\partial p}{\partial R}$ and $\frac{\partial c}{\partial R}$ are positive. All the subsequent results would be unaltered if one of these two partials were allowed to be zero while the other is positive.

The research production function shows research output as a function of expenditure on research, and the level of research efficiency E. Research efficiency or productivity can be higher for a variety of reasons. Two prominent hypotheses in this regard link (1) the firm size, and (2) the level of diversification of products, to the efficiency of R & D. Discussions of these hypotheses and empirical tests can be found in references (5, 3, 2). Our formulation of the research production function is a general one, incorporating an efficiency parameter, whether the efficiency arises out of the "Schumpeterian hypothesis" (effect of firm size), the "Nelson hypothesis" (effect of diversification), or any other reason.

As a result of higher research efficiency (larger E), the research production function with respect to research expenditure shifts upward. When the efficiency is due to large size of the firm, the larger firm will have higher research efficiency. A distinction is made here between the size of output that is endogenously determined, and the firm size that is causing the exogenous shift of the research production function. A large difference in firm size is considered exogenous, being historically determined from factors outside the scope of the model. The parameter of research efficiency is affected by this size-class of a firm, whereas the firm's dollar value of sales is determined within the model. The level of E affects this value of sales, but the effect of a change in E on itself through the effect on firm size is of second-order magnitude and is neglected in this formulation.

Equilibrium Conditions

Each firm acts on the assumption that the rival would maintain his levels of x and Ω. Duopolist #1 would thus maximize profit:

$$\text{Profit} = \pi_1 = p_1(x_1, R_1, x_2, R_2) - C_1(x_1, R_1) - \Omega_1 .$$

The first order conditions of profit maximization are:

$$(1) \qquad \phi_1 \equiv \frac{\partial \pi_1}{\partial x_1} = p_1 + x_1 \frac{\partial p_1}{\partial x_1} - \frac{\partial C_1}{\partial x_1} = 0$$

$$(2) \qquad \psi_1 \equiv \frac{\partial \pi_1}{\partial \Omega_1} = x_1 \frac{\partial p_1}{\partial R_1} \frac{\partial F_1}{\partial \Omega_1} - \frac{\partial C_1}{\partial R_1} \frac{\partial F_1}{\partial \Omega_1} - 1 = 0 .$$

The second order conditions are:

$$\text{(i) } \phi_{1x_1} < 0, \text{ (ii) } \psi_{1\Omega_1} < 0, \text{ and (iii) } |A_1| > 0 ,$$

where ϕ_1 and ψ_1 are taking subscripts x_1 and Ω_1 to represent partial derivatives with respect to these quantities:

$$(3) \qquad \phi_{1x_1} \equiv \frac{\partial^2 \pi_1}{\partial x_1^2} = 2\frac{\partial p_1}{\partial x_1} + x_1 \frac{\partial^2 p_1}{\partial x_1^2} - \frac{\partial^2 C_1}{\partial x_1^2}$$

$$(4) \qquad \psi_{1\Omega_1} \equiv \frac{\partial^2 \pi_1}{\partial \Omega_1^2} = x_1 \frac{\partial^2 p_1}{\partial R_1^2}\left(\frac{\partial F_1}{\partial \Omega_1}\right)^2 + x_1 \frac{\partial p_1}{\partial R_1} \frac{\partial^2 F_1}{\partial \Omega_1^2} - \frac{\partial^2 C_1}{\partial R_1^2} \frac{\partial F_1}{\partial \Omega_1} - \frac{\partial C_1}{\partial R_1} \frac{\partial^2 F_1}{\partial \Omega_1^2}$$

$$(5) \qquad \phi_{1\Omega_1} = \psi_{1x_1} \equiv \frac{\partial^2 \pi_1}{\partial x_1 \partial \Omega_1} = \frac{\partial p_1}{\partial R_1} \frac{\partial F_1}{\partial \Omega_1} + x_1 \frac{\partial^2 p_1}{\partial x_1 \partial R_1} \frac{\partial F_1}{\partial \Omega_1} - \frac{\partial^2 C_1}{\partial x_1 \partial R_1} \frac{\partial F_1}{\partial \Omega_1}$$

and

$$(6) \qquad |A_1| \equiv \begin{vmatrix} \phi_{1x_1} & \phi_{1\Omega_1} \\ \psi_{1x_1} & \psi_{1\Omega_1} \end{vmatrix} .$$

Duopolist ♯2 similarly has his profit-maximizing conditions which are identical to the expressions above after replacing subscript 1 by 2.

This equilibrium for each firm is partial equilibrium or equilibrium-in-process in the sense that x_2 and Ω_2 as assumed by firm 1 may not be the equilibrium quantities of firm 2, and vice versa. The equilibrium of the system is achieved when the assumed quantities of the rivals are actually their equilibrium quantities.

The individual equilibrium of firm 1 is rewritten here as

$$(7) \qquad \phi_1(x_1^*, \Omega_1^*\ ;\ x_2, \Omega_2) = 0\,, \qquad \text{and}$$

$$(8) \qquad \psi_1(x_1^*, \Omega_1^*\ ;\ x_2, \Omega_2) = 0\,,$$

where the asterisks denote equilibrium values. Since $|A_1| \neq 0$ by second order condition, we have by implicit function theorem (see reference 6).

$$(9) \qquad x_1^* = f_1(x_2, \Omega_2)$$

$$(10) \qquad \Omega_1^* = g_1(x_2, \Omega_2)\,.$$

Equations (7) and (8) are called the reaction functions of firm 1. Similarly the reaction functions of firm 2 are:

$$(11) \qquad x_2^* = f_2(x_1, \Omega_1)$$

$$(12) \qquad \Omega_2^* = g_2(x_1, \Omega_1)$$

The system as a whole is in equilibrium when equations (9) through (12) hold simultaneously, generating the set of equilibrium solutions $(\bar{x}_1, \bar{\Omega}_1, \bar{x}_2, \bar{\Omega}_2)$.

The individual (partial) equilibrium of a duopolist would change when rival's quantities change, as follows:

By totally differentiating equations (7) and (8) we get

$$\phi_{1x_1} dx_1^* + \phi_{1\Omega_1} d\Omega_1^* + \phi_{1x_2} dx_2 + \phi_{1\Omega_2} d\Omega_2 = 0$$

and

$$\psi_{1x_1} dx_1^* + \psi_{1\Omega_1} d\Omega_1^* + \psi_{1x_2} dx_2 + \psi_{1\Omega_2} d\Omega_2 = 0$$

or,

$$\begin{bmatrix} \phi_{1x_1} & \phi_{1\Omega_1} \\ \psi_{1x_1} & \psi_{1\Omega_1} \end{bmatrix} \begin{bmatrix} dx_1^* \\ d\Omega_1^* \end{bmatrix} = - \begin{bmatrix} \phi_{1x_2} & \phi_{1\Omega_2} \\ \psi_{1x_2} & \psi_{1\Omega_2} \end{bmatrix} \begin{bmatrix} dx_2 \\ d\Omega_2 \end{bmatrix}$$

which is rewritten as

$$A_1 dz_1^* = -B_1 dz_2\,.$$

Thus,

$$dz_1^* = -A_1^{-1} B_1 dz_2$$

which gives

$$(13)\qquad -A_1^{-1}B_1=\begin{bmatrix}\dfrac{\partial x_1^*}{\partial x_2} & \dfrac{\partial x_1^*}{\partial \Omega_2}\\[2mm] \dfrac{\partial \Omega_1^*}{\partial x_2} & \dfrac{\partial \Omega_1^*}{\partial \Omega_2}\end{bmatrix}$$

$$=\frac{1}{|A_1|}\begin{bmatrix}-\psi_{1\Omega_1}\phi_{1x_2}+\phi_{1\Omega_1}\psi_1 & -\psi_{1\Omega_1}\phi_{1\Omega_2}+\phi_{1\Omega_1}\psi_{1\Omega_2}\\ \psi_{1x_1}\phi_{1x_2}-\phi_{1x_1}\psi_{1x_2} & \psi_{1x_1}\phi_{1\Omega_2}-\phi_{1x_1}\psi_{1\Omega_2}\end{bmatrix}$$

Similarly, as firm 1's quantities change, firm 2 adjusts :

$$(14)\qquad -A_2^{-1}B_2=\begin{bmatrix}\dfrac{\partial x_2^*}{\partial x_1} & \dfrac{\partial x_2^*}{\partial \Omega_1}\\[2mm] \dfrac{\partial \Omega_1^*}{\partial x_1} & \dfrac{\partial \Omega_1^*}{\partial \Omega_1}\end{bmatrix}$$

$$=\frac{1}{|A_2|}\begin{bmatrix}-\psi_{2\Omega_2}\phi_{2x_1}+\phi_{2\Omega_2}\psi_{2x_1} & -\psi_{2\Omega_2}\phi_{2\Omega_1}+\phi_{2\Omega_2}\psi_{2\Omega_1}\\ \psi_{2x_2}\phi_{2x_1}-\phi_{2x_2}\psi_{2x_1} & \psi_{2x_2}\phi_{2\Omega_1}-\phi_{2x_2}\psi_{2\Omega_1}\end{bmatrix}.$$

When the system is in equilibrium, the following four functions hold simultaneously :

$$(15)\qquad \bar{x}_1=f_1(x_2,\Omega_2)$$

$$(16)\qquad \bar{\Omega}_1=g_1(x_2,\Omega_2)$$

$$(17)\qquad \bar{x}_2=f_2(x_1,\Omega_1)$$

$$(18)\qquad \bar{\Omega}_2=g_2(x_1,\Omega_1)\,.$$

If the actual quantities $x_1, \cdots, \Omega_2$ locally deviate from the equilibrium quantities $\bar{x}_1, \cdots, \bar{\Omega}_2$, the firms are assumed to respond as follows :

$$(19)\qquad \dot{x}_1=\lambda_1(\bar{x}_1-x_1)$$

$$(20)\qquad \dot{\Omega}_1=\lambda_2(\bar{\Omega}_1-\Omega_1)$$

$$(21)\qquad \dot{x}_2=\lambda_3(\bar{x}_2-x_2)$$

$$(22)\qquad \dot{\Omega}_2=\lambda_4(\bar{\Omega}_2-\Omega_2)\,,$$

with $0<\lambda_1, \lambda_2, \lambda_3, \lambda_4\leq 1$, $\dot{x}_1, \cdots, \dot{\Omega}_2$ are the time-rates of change: $\dfrac{dx_1}{dt}, \cdots \dfrac{d\Omega_2}{dt}$. The values of the λ's determine the speeds of adjustment. When the λ's are 1, the system is adjusted instantaneously. With the λ's as positive, the quantity of a variable increases when it falls below the equilibrium quantity and decreases when it is above.

Taking Taylor expansion around the equilibrium in the system (19) through (22), we get

$$(23)\qquad \begin{bmatrix} \dot{x}_1 \\ \dot{\Omega}_1 \\ \dot{x}_2 \\ \dot{\Omega}_2 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & \lambda_1 \dfrac{\partial \bar{x}_1}{\partial x_2} & \lambda_1 \dfrac{\partial \bar{x}_1}{\partial \Omega_2} \\ 0 & -\lambda_2 & \lambda_2 \dfrac{\partial \bar{\Omega}_1}{\partial x_2} & \lambda_2 \dfrac{\partial \bar{\Omega}_1}{\partial \Omega_2} \\ \lambda_3 \dfrac{\partial \bar{x}_2}{\partial x_1} & \lambda_3 \dfrac{\partial \bar{x}_2}{\partial \Omega_1} & -\lambda_3 & 0 \\ \lambda_4 \dfrac{\partial \bar{\Omega}_2}{\partial x_1} & \lambda_4 \dfrac{\partial \bar{\Omega}_2}{\partial \Omega_1} & 0 & -\lambda_4 \end{bmatrix} \begin{bmatrix} x_1 - \bar{x}_1 \\ \Omega_1 - \bar{\Omega}_1 \\ x_2 - \bar{x}_2 \\ \Omega_2 - \bar{\Omega}_2 \end{bmatrix}$$

The Jacobian Matrix in (23) can be written as

$$(\lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4) \cdot D$$

where, from the expressions of equations (13) and (14),

$$D = \begin{bmatrix} -I & -A_1^{-1} B_1 \\ -A_2^{-1} B_2 & -I \end{bmatrix}.$$

The necessary and sufficient conditions for the system to be stable are provided by the Routh-Hurwitz conditions. Of these, one necessary condition is that the determinant of the Jacobian matrix in (23) is positive. Therefore,

(24) If the equilibrium $(\bar{x}_1, \cdots, \bar{\Omega}_2)$ in (23) is stable, $|D| > 0$.

This result (24) will be used in comparative static analysis in the next section. We shall use condition (24) under the assumption that the system is stable. A justification for this assumption is that an unstable system would not be able to survive and therefore would not be observed.

Change in E and Market Shares

In this section the effect of a change in the level of R&D efficiency of firm 1 on the market shares of the two duopolists is investigated. It is shown that as E_1 increases, leading to the improvement of research productivity of the firm, his share of the market increases under certain conditions.

By total differentiation of the equilibrium solutions (15) through (18),

$$\begin{bmatrix} -1 & 0 & \dfrac{\partial \bar{x}_1}{\partial x_2} & \dfrac{\partial \bar{x}_1}{\partial \Omega_2} \\ 0 & -1 & \dfrac{\partial \bar{\Omega}_1}{\partial x_2} & \dfrac{\partial \bar{\Omega}_1}{\partial \Omega_2} \\ \dfrac{\partial \bar{x}_2}{\partial x_1} & \dfrac{\partial \bar{x}_2}{\partial \Omega_1} & -1 & 0 \\ \dfrac{\partial \bar{\Omega}_2}{\partial x_1} & \dfrac{\partial \bar{\Omega}_2}{\partial \Omega_1} & 0 & -1 \end{bmatrix} \begin{bmatrix} d\bar{x}_1 \\ d\bar{\Omega}_1 \\ d\bar{x}_2 \\ d\bar{\Omega}_2 \end{bmatrix} = - \begin{bmatrix} \dfrac{\partial \bar{x}_1}{\partial E_1} \\ \dfrac{\partial \bar{\Omega}_1}{\partial E_1} \\ \dfrac{\partial \bar{x}_2}{\partial E_1} \\ \dfrac{\partial \bar{\Omega}_2}{\partial E_1} \end{bmatrix} dE_1$$

which, by virtue of equations (13) and (14), can be rewritten as

$$\begin{bmatrix} -I & -A_1^{-1}B_1 \\ -A_2^{-1}B_2 & -I \end{bmatrix} \begin{bmatrix} d\bar{z}_1 \\ d\bar{z}_2 \end{bmatrix} = -\begin{bmatrix} \Gamma_1 \\ \Gamma_1 \end{bmatrix} dE_1, \tag{25}$$

$$\Gamma_1 \equiv \begin{bmatrix} \dfrac{\partial \bar{x}_1}{\partial E_1} \\ \dfrac{\partial \bar{\Omega}_1}{\partial E_1} \end{bmatrix} \qquad \Gamma_2 \equiv \begin{bmatrix} \dfrac{\partial \bar{x}_2}{\partial E_1} \\ \dfrac{\partial \bar{\Omega}_2}{\partial E_1} \end{bmatrix}$$

By differentiating the equilibrium conditions of the two duopolists totally when E_1 changes but E_2 is fixed it can be shown that

$$\Gamma_1 = -A_1^{-1} V_1 \quad \text{and } \Gamma_2 = -A_2^{-1} V_2 \tag{26}$$

where $$V_1 = \begin{bmatrix} \phi_{1E_1} \\ \psi_{1E_1} \end{bmatrix} \quad \text{and } V_2 = \begin{bmatrix} \phi_{2E_1} \\ \psi_{2E_1} \end{bmatrix}$$

The equations (25) are rewritten as

$$D d\bar{z} = -\Gamma dE_1, \tag{27}$$

with solutions

$$\frac{d\bar{z}}{dE_1} = -D^{-1}\Gamma. \tag{28}$$

The inverse does exist if we assume that the system is stable, because of (24).

Inverting the matrix D by partitioning,

$$\begin{bmatrix} -I & -A_1^{-1}B_1 \\ -A_2^{-1}B_2 & -I \end{bmatrix}^{-1} \equiv \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} \tag{29}$$

where

$$\begin{aligned} M_1 &= -\left[I-(A_1^{-1}B_1)(A_2^{-1}B_2)\right]^{-1} \\ M_2 &= -M_1(A_1^{-1}B_1) \\ M_3 &= -(A_2^{-1}B_2)\, M_1 \\ M_4 &= -(I+A_2^{-1}B_2 M_2). \end{aligned}$$

M_1 exists because $|I-(A_1^{-1}B_1)(A_2^{-1}B_2)|$ is equal to $|D|>0$ as shown below: It is true that

$$\begin{bmatrix} -I & 0 \\ A_2^{-1}B_2 & -I \end{bmatrix} D = \begin{bmatrix} I & A_1^{-1}B_1 \\ 0 & I-(A_1^{-1}B_1)(A_2^{-1}B_2) \end{bmatrix}.$$

Evaluating determinants of the two sides of the equations, and since the determinant of the product of two matrices is equal to the product of the determinants, we have

$$|I|\cdot|D| = |D| = \left|I-(A_1^{-1}B_1)(A_2^{-1}B_2)\right|.$$

The signs of $\frac{d\bar{z}}{dE_1}$ then depends on the signs of M_1, M_2, M_3, M_4, T_1, and T_2.

We can generate a set of sufficient conditions such that $\frac{d\bar{z}}{dE_1}\geq 0$ and $\frac{d\bar{z}}{dE_1}\leq 0$. The conditions are that the following assumptions hold:

(A 1) $\phi_{1x_2}\leq 0$; similarly for the other firm, $\phi_{2x_1}\leq 0$.

(A 2) $\phi_{1\Omega_1}\geq 0$; $\phi_{2\Omega_2}\geq 0$.

(A 3) $\phi_{1\Omega_2}\leq 0$; $\phi_{2\Omega_1}\leq 0$.

(A 4) $\psi_{1E_1}\geq 0$.

(A 5) $\psi_{1x_2}\leq 0$; $\psi_{2x_1}\leq 0$.

(A 6) $\psi_{1\Omega_2}\leq 0$; $\psi_{2\Omega_1}\leq 0$.

(B) This last assumption is specified later in this section.

Assumption (A 1), (A 3), (A 5) and (A 6) together may be restated as follows:

A firm's marginal profit with respect to both his instrument variables decreases when his demand curve shifts down as a result of an increase in the levels of his rival's instrument variables.

(A 2) states that the marginal profit with respect to x increases when the demand curve shifts up and/or the total cost of production decreases as a result of increased research expenditure by the firm.

(A 4) states that the marginal profit with respect to Ω increases when his demand curve shifts up and/or total cost decreases as a result of increased research efficiency.

Using the assumptions, the signs of the following matrices are determined:

$$-A_1^{-1}B_1=\begin{bmatrix}\frac{\partial\bar{x}_1}{\partial x_2} & \frac{\partial\bar{x}_1}{\partial\Omega_2}\\ \frac{\partial\bar{\Omega}_1}{\partial x_2} & \frac{\partial\bar{\Omega}_1}{\partial\Omega_2}\end{bmatrix}\leq 0 \text{ (all elements nonpositive).}$$

Similarly, for firm # 2, $-A_2^{-1}B_2\leq 0$.

$$\Gamma_1=-A_1^{-1}V_1=\begin{bmatrix}\frac{\partial\bar{x}_1}{\partial E_1}\\ \frac{\partial\bar{\Omega}_1}{\partial E_1}\end{bmatrix}\geq 0 \text{ (all elements nonnegative).}$$

$$\Gamma_2 = -A_2^{-1}V_2 = \begin{bmatrix} \dfrac{\partial \bar{x}_2}{\partial E_1} \\ \dfrac{\partial \bar{\Omega}_2}{\partial E_1} \end{bmatrix} \leq 0 .$$

Using these results the sign of M_1 is determined below:

$$M_1 = \frac{-1}{|I-(A_1^{-1}B_1)(A_2^{-1}B_2)|} \begin{bmatrix} 1-t_4 & t_2 \\ t_3 & 1-t_1 \end{bmatrix}$$

where $\begin{bmatrix} t_1 & t_2 \\ t_3 & t_4 \end{bmatrix}$ is a notational representation of the matrix $(A_1^{-1}B_1)(A_2^{-1}B_2) \geq 0$. We have already noted that $|I-(A_1^{-1}B_1)(A_2^{-1}B_2)|>0$ from stability condition. Thus, if $t_1<1$ and $t_4<1$, then $M_1 \leq 0$. This requirement leads to our last assumption:

(B) $t_1<1$ and $t_4<1$,

where
$$t_1 = \frac{\partial \bar{x}_1}{\partial x_2}\frac{\partial \bar{x}_2}{\partial x_1} + \frac{\partial \bar{x}_1}{\partial \Omega_2}\frac{\partial \bar{\Omega}_2}{\partial x_1}$$

and
$$t_4 = \frac{\partial \bar{\Omega}_1}{\partial x_2}\frac{\partial \bar{x}_2}{\partial \Omega_1} + \frac{\partial \bar{\Omega}_1}{\partial \Omega_2}\frac{\partial \bar{\Omega}_2}{\partial \Omega_1} .$$

The interpretation of t_1 as follows: When x_1 changes (for whatever reason), it cannot stay there because rival's quantities would now change in reaction, and firm 1 has to adjust because of that change. If that amount of adjustment in x_1 is less than the original change in x_1 then $t_1<1$. The interpretation of t_4 is similar with respect to change in Ω_1.

As soon as the sign of M_1 is determined, the rest follows:

$M_1 \leq 0$

$M_2 \geq 0$

$M_3 \geq 0$

$M_4 \leq 0$. We already have $\Gamma_1 \geq 0$, $\Gamma_2 \leq 0$.

Thus,
$$\frac{d\bar{z}_1}{dE_1} \equiv \begin{bmatrix} \dfrac{d\bar{x}_1}{dE_1} \\ \dfrac{d\bar{\Omega}_1}{dE_1} \end{bmatrix} = -M_1\Gamma_1 - M_2\Gamma_2 \geq 0 .$$

$$\frac{d\bar{z}_2}{dE_1} \equiv \begin{bmatrix} \dfrac{d\bar{x}_2}{dE_1} \\ \dfrac{d\bar{\Omega}_2}{dE_1} \end{bmatrix} = -M_3\Gamma_1 - M_4\Gamma_2 \leq 0 .$$

If there are enough nonzero coefficients in the expressions above so that the final solutions come out as either positive or negative, then we conclude on the basis of assumptions (A 1) to (A 6) and (B) that when a duopolist's research efficiency increases his output and research expenditure would increase and those of his rival would decrease in the new final equilibrium.

The total revenue of firm 1 increases too:

$$\frac{d(\bar{p}_1\bar{x}_1)}{dE_1}=p_1\frac{d\bar{x}_1}{dE_1}+\bar{x}_1\frac{d\bar{p}_1}{dE_1}$$

$$=\bar{p}_1\frac{d\bar{x}_1}{dE_1}+\bar{x}_1\frac{d\bar{p}_1}{d\bar{x}_1}\frac{d\bar{x}_1}{dE_1}+\bar{x}_1\Bigg[\underset{(+)}{\frac{d\bar{p}_1}{d\bar{R}_1}}\,\underset{(+)}{\frac{dF_1}{d\Omega_1}}\,\underset{(+)}{\frac{d\bar{\Omega}_1}{dE_1}}$$

$$+\underset{(-)}{\frac{d\bar{p}_1}{d\bar{x}_2}}\,\underset{(-)}{\frac{d\bar{x}_2}{dE_1}}+\underset{(-)}{\frac{d\bar{p}_1}{d\bar{R}_2}}\,\underset{(+)}{\frac{dF_2}{d\Omega_2}}\,\underset{(-)}{\frac{d\Omega_2}{dE_1}}\Bigg]$$

$$=\frac{d\bar{x}_1}{dE_1}\left(\bar{p}_1+\bar{x}_1\frac{d\bar{p}_1}{dx_1}\right)+x_1\ [\text{positive value}].$$

Since the expression in parenthesis is marginal revenue of x, it is positive, and therefore,

$$\frac{d(\bar{p}_1\bar{x}_1)}{dE_1}>0\,.$$

Thus, as firm 1 gets more research-efficient, his total revenue increases. Similarly, firm 2's total revenue decreases. This can be demonstrated following the procedure used above for firm 1. Thus firm 1's market share $\left(\text{which is } \dfrac{p_1x_1}{p_1x_1+p_2x_2}\right)$ increases, because the inverse of his market share $\left(1+\dfrac{p_2x_2}{p_1x_2}\right)$ clearly decreases.

The above result is based on assumptions which are only sufficient conditions. Even if these assumptions do not hold and the signs of changes in instrument variables in the reaction process of the two firms are not unambiguously determined, the end result may still come out the same: the firm with an increased research efficiency will have an increased market share. The merit of deriving this result through these assumptions is that unambiguous result is derived without having to determine relative magnitudes of changes in instrument variables by the two firms. The weakness of this derivation, however, is that some of the assumptions, (A 5) and (A 6) in particular, may not hold. Then it becomes a question of relative magnitudes.

The assumptions (A 1) to (A 6) generate a situation where either firm would respond to an increase in the level of any of the two instrument variables of the rival firm by reducing the levels of his own instrument variables. If (A 5) and (A 6) do not hold then this pattern of response may remain the same or it may be reversed in respect of either or both the instrument variables. For example, as a result of the rival's increased research activity in particular, a firm's demand curve, after shifting downward, may become so elastic with respect to his own research output that it may be profitable for him to engage in more research. This may or may not mean that in the next equilibrium the research expenditure of the firm would increase. And the final result in terms of market shares can still remain the same.

It should be noted that when research is only of the cost-reducing type, the assumptions (A 3), (A 5) and (A 6) are not needed because in this case $\phi_{1\Omega_2}=\psi_{1x_2}=\psi_{1\Omega_2}=0$.

Since this possibility was included in the now redundant assumptions the result of the model still remains valid.

Concluding Remarks

It has been shown in this paper that the market share of a duopolist goes up as a result of an increase in research efficiency. Research efficiency of a firm has been related to size (the Schumpeterian hypothesis) and the level of diversification (the Nelson hypothesis). If these hypotheses are valid, then in the periods of improved technological opportunity when industry-wide research expenditures increase, the larger and/or more diversified firm would enjoy an edge in research effort over a smaller or less diversified rival, and its share of the market would go up. This is a possible factor in the explanation of market share changes, and is an empirically testable proposition.

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