Saving Uncertainty and Social Security

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I. INTRODUCTION

Today, each western advanced country has its public pension system which provides one of the major financial sources of consumption for old people during their period of retirement. The reasons why these systems exist are:
(1) As people usually do not have vision enough to see the whole of life, they need some system which is designed to make provision for them in their old age.
(2) Even if individuals are willing to prepare for their own retirement, we have to pay attention to the fact that there is a long interval between making a deposit and withdrawing it. During a very long period of time, there may occur either periods of inflation or slump. These prevent individuals from gaining economic security against their own old age by themselves.

These reasons raise the question whether we need a public pension system if we live in a full employment economy without inflation, and if we are not near-sighted and are willing to save for our retired days. It is true that people can set aside in advance a reserve for their own consumption in old age if they live in an economy as mentioned above. This implies that we don't need any public old-age pension systems in such an economy. However, the former "near-sightedness" of these two reasons may mean that people make their decision without allowing for factors which should be taken into account.

As for the latter of the two ("economic instability"), it may be suggested that they must decide how much to save for their period of retirement without correct information about the period. If people decide how much to consume taking no account of factors which should be considered, and/or in the face of some uncertainty, it is highly questionable whether they can afford to dispense with a public pension system in a full employment economy without inflation even if they make their decision having sufficiently long view to see their whole life span.

Introducing new factors which may affect people in a full employment economy without inflation as if those are "near-sightedness" or "economic instability," this paper is intended to examine some effects of the new factors on decisions to save. As a result, we may be able to understand the role played by our public old age pension systems.

II. A FRAMEWORK FOR DECISIONS TO SAVE

We assume the following hypothetical economy. People live for two periods, working in the first while being retired in the second. In the working period, each individual supplies labor and receives wage income Y which is equal to the marginal productivity of labor. As he does not work in retirement, he has to make provision for the needs of old age by the end of his working period. He divides his wage income Y between consumption in his working period C_1 and savings S. His savings constitute a part of the capital stock K of the next period when he is retired and contribute to production in that period. Production activity is put into practice at the beginning of each period. The level of output in period t is determined by the linear-homogeneous production function

(1) $Q_t = F(K_t, L_t)$,

where, Q_t, K_t, and L_t denotes output, capital stock, and labor in period t, respectively. Therefore, the interest rate r to be paid for the saving made in period $t-1$ is determined by the marginal productivity of capital at the beginning of period t. An individual who is retired in period t covers his old age consumption C_2 with both his savings S made in period $t-1$ and interest rS on the savings determined at the beginning of period t. Each individual's lifetime income is equal to his lifetime consumption. He neither receive nor leave any inheritance. Therefore, his lifetime income Y has already been determined when he decides how much to save. If the population of the generation which he belongs to and the population growth rate are given, the next period's labor L is also known. In the economy considered here, there is no unemployment; factors of production are utilized as far as they are supplied. Moreover, there is no inflation. How does an individual allocate his income between savings and consumption in the above-mentioned framework? As a starting point, we will take the most simple Fisherian theory of an individual's decision to save. According to the theory, an individual determines the volume of saving so as to maximize his utility depending on his present and future consumption under the constraint of his lifetime income. We may employ as an individual's utility function

(2) $U = U(C_1, C_2)$.

We can write the relation among variables as follows:

(3) $Y = C_1 + S, \quad C_2 = S(1+r)$.

Therefore, his savings is determined by maximizing (4) with respect to S:

$$(4)\ U = U\{Y-S,\ S(1+r)\}.$$

It is well known that we obtain an equilibrium condition (5) where $\partial U/\partial C_i = U_{iT}$:

$$(5)\ (U_{1T}/U_{2T}) - 1 = r.$$

The left hand side of (5) is the marginal rate of time preference. An individual's utility is maximized if he saves up to the level where that rate is equal to the interest rate. We will modify in some respects, by what follows, this simple theory of an individual's determination to save.

First of all, in this paper, we are dealing with the so-called life-cycle savings which is made to finance old-age consumption C_2. We may notice that C_2 can be divided into basic consumption which is enough to maintain his dignity as a human being and added consumption which allows him to enjoy a comfortable life. The former can correspond to civil-minimum. We call the former as minimum required consumption C_{2m} and the latter as affluent consumption C_{2A}. An individual's consumption during his period of retirement C_2 is written as follows:

$$(6)\ C_2 = C_{2m} + C_{2A}.$$

Dividing C_2 in this way, we may consider that C_{2A} can be consumed only after C_{2m} is fulfilled. This means that savings which finance C_{2m} are deducted from income before the residual is distributed to C_1 and savings for C_{2A}. It is needless to say that C_1 can be divided into minimum required consumption C_{1m} and affluent consumption C_{1A}. We should deal with C_{1m} and C_{1A} in the same way as C_{2m} and C_{2A}. The level of C_{im} (i=1,2) may be determined by economic conditions or circumstances in a society where it is consumed. It follows that C_{im} is given as a restrictive condition to an individual's consumption-saving decision. Hence, an individual can not choose the size of C_{im} but C_{iA}. His utility function is defined on the (C_{1A}, C_{2A}) plane which consists of a part of the (C_1, C_2) plane. The function is modified to

$$(7)\quad U = U(C_{1A}, C_{2A}) = U(Y-S-C_{1m}, S(1+r)-C_{2m}).$$

As the second point, we should not forget the Fisherian hypothesis that an individual takes the interest rate r as given when he decides how much to save. On the contrary, the interest rate on his savings made in this period is assumed to be determined at the beginning of the next period in the model of our economy. In other words, an individual has to decide the size of his savings without knowing the exact value of the interest rate to be consulted in making his savings decision. Therefore, he should determine how much to save depending upon the expected value of the interest rate no matter how it is estimated. This implies that we should rewrite the interest rate as a stochastic variable.

$$(8)\quad r = \bar{r} + \gamma_r \cdot \varepsilon_r,$$

where ε_r is a random variable and its expected value $E(\varepsilon_r)$ is assumed to be zero. $\bar{r}$ denotes the expected value of r, and γ_r the degree of uncertainty of r. Obviously, C_{2m} is also to be dealt with as a stochastic variable. We write

$$(9)\quad C_{2m} = \bar{C}_{2m} + \gamma_c \cdot \varepsilon_c,$$

where ε_c is a random variable and it is also assumed that the expected value of ε_c is equal to zero. $\bar{C}_{2m}$ denotes the expected value of C_{2m} and γ_c the degree of uncertainty of C_{2m}.

As a result, our utility function of the individual has been modified as follows:

$$(10)\ U= U\{Y-S-C_{1m},\ S(1+\bar{r}+\gamma_r\cdot\varepsilon_r)-\bar{C}_{2m}-\gamma_c\cdot\varepsilon_c\}\ .$$

These two points revise the Fisherian theory of the decision to save within the framework of partial equilibrium. The third modification makes an individual's savings decision go beyond the framework.

In this paper, we assume an economy where production is practiced under a certain production function. The function defines the clear relation between the wage rate, the interest rate, and so on. For example, keeping labor L constant, a higher level of production means a lower interest rate and a higher wage rate. Turning to the decision to save, we have to refer to some expected values of variables which our economy haven't yet determined. If some relation can be established between these expected values, it may be desirable to take it into account. The third modification is to take account of relations between variables directed by our production function. We will explain these relations one by one.

As we have assumed a linear homogeneous production function, it can be rewritten as

$$(11)\ Q= L\cdot f(k) \qquad (k= K/L)$$

where we omit time suffix t. If labor L in the next period is given, the output level Q of the next period is determined by the next period's capital stock K which is equal to total savings nS in this period. We postulate that saving is made only by individuals, and that n represents the population of a generation which people who save in this period belong to. On the other hand, marginal productivity of capital stock r in the next period is determined as follows:

(12) $r = \partial Q/\partial K = f' > 0$, where $\partial r/\partial K = f'' < 0$

Equation (12) means that larger savings in this period reduce the expected value of the interest rate in the next period through larger capital stock. This leads us to the following relation:

(13) $\partial \bar{r}/\partial S = n(\partial \bar{r}/\partial K) < 0$.

In addition, from (11) and (12), we know that the larger Q/L brings about the smaller r. Therefore, L being constant, we can identify the following relation between expected values of output and the interest rate:

(14) $\partial \bar{r}/\partial \bar{Q} < 0$.

Moreover, as $\bar{r}$ and $\bar{Q}$ go in opposite directions on (11), we can expect the following relation between the two during the next period:

(15) $\partial \bar{Q}/\partial \bar{r} < 0$.

Although it is not directly connected with a production function, we can assume that, in general, the minimum required level of consumption goes up along with per capita output. It follows that we assume C_{2m} increases together with per capita output in the same period. Therefore, if the population and its growth rate are known, the following relation between expected values of Q and C_{2m} can be established:

(16) $\partial \bar{C}_{2m}/\partial \bar{Q} > 0$, $\partial \bar{Q}/\partial \bar{C}_{2m} > 0$.

From (14), (15), and (16), it can be deduced that $\bar{C}_{2m}$ and $\bar{r}$ move in opposite directions. That is,

(17) $\partial\overline{C}_{2m}/\partial\overline{r}<0$, $\quad \partial\overline{r}/\partial\overline{C}_{2m}<0$.

Recalling $\partial\overline{Q}/\partial S>0$, (16) leads us to the following relation:

(18) $\partial\overline{C}_{2m}/\partial S=(\partial\overline{C}_{2m}/\partial\overline{Q})\cdot(\partial\overline{Q}/\partial S)>0$.

We have established some relationship, as noted above, prescribed by our production function among variables and expected values. However, these types of relations should not be expanded without limit. For instance, the expected interest rate is considered to have no effect on saving in spite of the fact that we have accepted relation (13) which is just the opposite. Similarly, r_c and γ_r have not such a relation as mentioned above, because these things are thought not to be involved in adjustments through the production function.

III. DECISION TO SAVE

Now we can tackle the problem of an individual's decision to save in a full employment economy without inflation. The individual chooses S so as to maximize E[U], the expected value of (10), which depends on present and future consumption. As $\overline{r}$ and $\overline{C}_{2m}$ are functions of S, we can write the following:

(19) $d\overline{r}=(\partial\overline{r}/\partial S)dS$, $\quad d\overline{C}_{2m}=(\partial\overline{C}_{2m}/\partial S)dS$.

Differentiating E[U], we get

(20) $dE[U]/dS=E[-U_1+U_2\{1+\overline{r}+\gamma_r\cdot\varepsilon_r+S(\partial\overline{r}/\partial S)-(\partial\overline{C}_{2m}/\partial S)\}]=0$,

where $\partial U/\partial C_{iA}=U_i$. Assuming that $\partial\overline{r}/\partial S$ and $\partial\overline{C}_{2m}/\partial S$ are approximately constant, the following equilibrium condition can be derived:

(21) $\{E[U_1]/E[U_2]\}-1=\bar{r}-\{(\partial\bar{C}_{2m}/\partial S)-S(\partial\bar{r}/\partial S)\}\equiv\bar{r}-Z.$

The left hand side of (21) is the marginal rate of time preference θ_p based on the relation given by a production function. From (13) and (18), we get

(22) $Z>0$.

S does not affect on either $\bar{r}$ or $\bar{C}_{2m}$ when we make no allowance for the relation due to the production function. Then, the equilibrium condition changes to

(23) $\{E[U_1]/E[U_2]\}-1=\bar{r}$.

The left hand side of (23) is the marginal rate of time preference θ without taking account of the relation prescribed by a production function. Comparing (21) and (23), we see that the introduced relation reduces the expected earnings rate of savings by Z.

The value of S which satisfies (21) is the optimal saving S_p* when the production function is taken into consideration. We will examine the shape of the rate of marginal time preference θ_p as a function of S. θ_p can be written as follows:

(24) $\theta_p=\theta_p(S;\ Y,\ C_{1m}\ ,\ \bar{r},\ \gamma_r,\ \bar{C}_{2m},\ \gamma_c)$,

where $\bar{r}$ and $\bar{C}_{2m}$ also depend on S. We give adequate values to parameters $(Y,\ C_{1m},\ \bar{r},\ \gamma_r,\ \bar{C}_{2m},\ \gamma_c)$ of θ_p. $\bar{r}$ and $\bar{C}_{2m}$ must be consistent with each other when a production function is allowed for. We denote this specific set of values as $(\bar{r}_o,\ \bar{C}_{2mo})$. On the other hand, as these two parameters are single-valued functions of S, the value of S is also determined at the same time. We denote it as S*. These parameters and S* determine the value of θ_p that, we assume, is equal to θ_p*. That is to say,

$$\theta_p{}^* = \theta_p(S^*;\ Y,\ C_{1m},\ \gamma_r,\ \gamma_c,\ \bar{r}_o,\ \bar{C}_{2mo}).$$

If we take parameters ($\bar{r}$, $\bar{C}_{2m}$) to be constant instead of varying with S, the θ_p function is the same as the θ function. Therefore, the set of parameters which satisfy $\theta_p{}^* = \theta_p(S^*)$ also fulfil the θ function, that is

$$\theta_p{}^* = \theta(S^*;\ Y,\ C_{1m},\ \gamma_r,\ \gamma_c,\ \bar{r}_o,\ \bar{C}_{2mo})$$

In other words, these two functions, θ and θ_p, go exactly through point (S^*, $\theta_p{}^*$) on the (S, θ (or θ_p)) plane. However, when S diverts from S^*, the θ_p function is no longer the same as the θ function because parameters ($\bar{r}$, $\bar{C}_{2m}$) change from ($\bar{r}_o$, $\bar{C}_{2mo}$) to a certain set of values. On the contrary, the values of parameters of the θ function are constant in spite of the change in S. Therefore, the values of θ and θ_p are different when S diverts from S^*. That is to say, these two functions cross at (S^*, $\theta_p{}^*$) on the (S, θ(or θ_p)) plane and have different slopes. Then, we have $\theta_p{}^* = \theta(S^*)$ when $S = S^*$ and $\bar{r} = \bar{r}_o$. These satisfy the equilibrium condition (23). We get:

$$\bar{r}_o = \theta(S^*;\ Y,\ C_{1m},\ \gamma_r,\ \gamma_c,\ \bar{r}_o,\ \bar{C}_{2mo}) = \theta_p{}^*.$$

Obviously, this equation shows that S^* is the optimal saving when a production function is not allowed for. Differentiating θ_p, we get

$$(25)\quad d\theta_p/dS = d\theta/dS - Z\{E[U_2]E[U_{12}] - E\{[U_1]E[U_{22}]\}/\{[E[U_2]\}^2 .$$

θ_p is less steep than θ because (22) shows Z to be positive and, assuming $U_{12} \geqq 0$ and $U_{22} < 0$, we get

$$(26)\quad \{E[U_2]\cdot E[U_{12}] - E[U_1]\cdot E[U_{22}]\}/\{E[U_2]\}^2 > 0 .$$

Moreover, as the indifference curves are convex to the origin,

(27) $d\theta_p/dS>0$.

This means that θ_p is upward sloping when we measure S on the horizontal axis and θ, θ_p, and $\bar{r}$ on the vertical axis.

On the other hand, as for the right hand side of the equilibrium condition (21), we get

(28) $\partial(\bar{r}-Z)/\partial S=2(\partial\bar{r}/\partial S)<0$,

because we assume that $\partial\bar{r}/\partial S$ and $\partial\bar{C}_{2m}/\partial S$ are approximately constant. (28) indicates that $(\bar{r}-Z)$ is downward sloping and is twice as steep as $\bar{r}$.

Thus, taking the production function into account, the equilibrium condition changes from

$\theta=\bar{r}$

to

$\theta_p=\bar{r}-Z$.

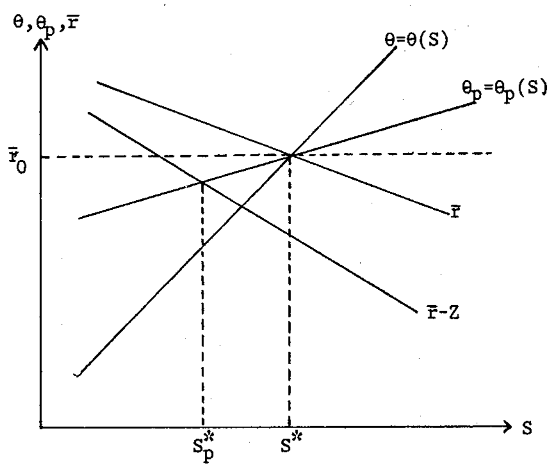

Figure 1.

This change brigns about the optimal saving S_p* based on the introduced relation which is smaller than S* without such relation. This is shown in Figure-1. In other words, an individual saves too much when the production function is not taken into his savings decision.

IV. CHANGES IN EXPECTED VALUES AND UNCERTAINTY

How is optimal saving affected by changes of expectation and uncertainty?

[4-1] The change in $\bar{C}_{2m}$

Noticing that $\bar{r}$ varies with the change of $\bar{C}_{2m}$, we differentiate θ_p with respect to $\bar{C}_{2m}$.

$$(29)\quad \partial\theta_p/\partial\bar{C}_{2m}=\{S(\partial\bar{r}/\partial\bar{C}_{2m})-1\}\cdot\{E[U_2]E[U_{12}]-E[U_1]E[U_{22}]\}/\{E[U_2]\}^2$$

As the first term on the right hand side is negative from (17) and the second term is positive from (26), (29) is negative. Therefore, the θ_p curve is shifted downwards when $\bar{C}_{2m}$ is increased. This raises the optimal saving $S_p{}^*$.

What effect does the change in $\bar{C}_{2m}$ give to $(\bar{r}-Z)$, the right hand side of the equilibrium condition (21)? Differentiating $(\bar{r}-Z)$ with respect to $\bar{C}_{2m}$, we get

$$(30)\quad \partial(\bar{r}-Z)/\partial\bar{C}_{2m}=\partial\bar{r}/\partial\bar{C}_{2m}<0\quad,$$

because $\partial\bar{C}_{2m}/\partial S$ and $\partial\bar{r}/\partial S$ are assumed to be approximately constant and $\partial S/\partial\bar{C}_{2m}=0$. The sign of (30) is determined from (17). Therefore, $(\bar{r}-Z)$ is shifted downwards, too, when $\bar{C}_{2m}$ is increased. This reduces the optimal saving $S_p{}^*$. Thus, we can not predict what effect the increasing $\bar{C}_{2m}$ has on $S_p{}^*$.

On the other hand, when we do not allow for a production function, we get the following equations:

$$(31)\quad \partial\theta/\partial\bar{C}_{2m}=\{E[U_1]E[U_{22}]-E[U_2]E[U_{12}]\}/\{E[U_2]\}^2<0$$

$$\partial\bar{r}/\partial\bar{C}_{2m}=0\quad.$$

Although $\bar{r}$ is not changed, the increased $\bar{C}_{2m}$ results in a downward shift of θ. In this case, it is clear that increasing $\bar{C}_{2m}$ makes the optimal saving S^* go up.

[4-2] The change in γ_c

Differentiating θ_p with respect to γ_c, we get

$$(32)\ \partial\theta_p/\partial\gamma_c=\{E[U_1]\,E[U_{22}\cdot\varepsilon_c]-E[U_2]\,E[U_{12}\cdot\varepsilon_c]\}/\{E[U_2]\}^2<0 \quad .$$

The sign of (32) is determined by the fact that (33) is easily proved by consulting the reference [2] and [4].

$$(33)\ E[U_{22}\cdot\varepsilon_c]<0,\ E[U_{12}\cdot\varepsilon_c]\geqq 0 \quad .$$

Therefore, from (32), θ_p is shifted downwards due to the increased γ_c. This causes the savings to rise.

From the right hand side of the equilibrium condition (21), we get

$$(34)\ \partial(\bar{r}-Z)/\partial\gamma_c = 0 \quad .$$

(34) shows that $(\bar{r}-Z)$ does not change. Consequently, the optimal saving S_p^* goes up due to the increment of γ_c.

On the other hand, we get (35) when a production function is not taken into consideration.

$$(35)\ \partial\theta/\partial\gamma_c= \partial\theta_p/\partial\gamma_c<0$$

$$\partial\bar{r}/\partial\gamma_c = 0 \quad .$$

Therefore, the increment of γ_c raises the optimal saving S^*. This is analogous to the case where a production function is taken into account.

[4-3] The change in $\bar{r}$

Paying attention to the fact that a change in $\bar{r}$ is followed by a change in $\bar{C}_{2m}$, we differentiate θ_p and $(\bar{r}-Z)$ with respect to $\bar{r}$.

$$(36)\ \partial\theta_p/\partial\bar{r}=\{S-(\partial\bar{C}_{2m}/\partial\bar{r})\}\{E[U_2]\,E[U_{12}]-E[U_1]\,E[U_{22}]\}/\{E[U_2]\}^2 >0$$

$$\partial(\bar{r}-Z)/\partial\bar{r}=1>0 \quad .$$

The sign of the first equation (36) is determined from (17) and (26). That of the second is assured from our assumption that $\partial \bar{C}_{2m}/\partial S$ and $\partial \bar{r}/\partial S$ are approximately constant and that $\partial S/\partial \bar{r}= 0$. Thus, both θ_p and $(\bar{r}-Z)$ are shifted downwards by decreasing $\bar{r}$. While the downward shift of θ_p increases saving, that of $(\bar{r}-Z)$ decreases saving. Therefore, the net effect given to the optimal saving S_p* cannot be identified.

When the production function is taken into consideration, we get the following equations:

$$(37)\quad \partial\theta/\partial\bar{r}= S\{E[U_2]E[U_{12}]-E[U_1]E[U_{22}]\}/\{E[U_2]\}^2>0$$

$$\partial\bar{r}/\partial\bar{r}=1>0 \quad .$$

The declining $\bar{r}$ increases saving through θ and decreases it through $\bar{r}$. Therefore, the net effect of a falling $\bar{r}$ on the optimal saving S* can not be foreseen. This is also shown in the case where we allow for a production function.

[4-4] The change in γ_r

The effects of a change in the degree of uncertainty of the interest rate γ_r are as follows:

$$(38)\quad \partial\theta_p/\partial\gamma_r=S\{E[U_2]E[U_{12}\cdot\varepsilon_r]-E[U_1]E[U_{22}\cdot\varepsilon_r]\}/\{E[U_2]\}^2<0$$

$$\partial(\bar{r}-Z)/\partial\gamma_r = 0 \quad .$$

The sign of the first equation (38) is determined from (39) which is proved by consulting references [2] and [4].

$$(39)\quad E[U_{12}\cdot\varepsilon_r]\leqq 0, \qquad E[U_{22}\cdot\varepsilon_r]>0 \quad .$$

Thus, the increase in γ_r raises optimal savings S_p* because θ_p is shifted downwards while $(\bar{r}-Z)$ is not changed.

When we do not take the production function into consideration, the following equations are deduced:

$$(40)\quad \partial\theta/\partial\gamma_r = \partial\theta_p/\partial\gamma_r < 0$$

$$\partial\bar{r}/\partial\gamma_r = 0 \ .$$

Therefore, the increase in γ_r raises optimal saving S^* in this case, too.

V. COMPARISON BETWEEN OPTIMAL SAVING LEVELS

We have shown the effects of changes in the expected values and the degree of uncertainty on the level of optimal saving. Although it must be interesting to compare these effects with each other, we must specify those functions and parameters in order to reach any definite conclusions. Here, we confine ourselves to comparing roughly the difference between optimal saving levels in cases where the production function is taken into account ($S_p{*}'$ denotes the ex post optimal savings) and where it isn't ($S{*}'$). The comparison is made in these four types of changes: (a) increase in $\bar{C}_{2m}$, (b) increase in γ_c, (c) decrease in $\bar{r}$, and (d) increase in γ_r.

(a) Increase in $\bar{C}_{2m}$

The increase in $\bar{C}_{2m}$ brings about the saving increase effect through a downward shift in θ and θ_p and the saving decrease effect made by a shift in $(\bar{r}-Z)$. When we take account of the production function, the change in $S_p{*}$ is thought to be small because both increasing and decreasing effects work at one time. While the production function is not allowed for, S^* goes up because only the increasing effect works. In addition, we can notice the fact that $S_p{*}$ is smaller than S^* before $\bar{C}_{2m}$ is changed, as shown in Figure-1. Therefore, it may be reasonable to consider that $S_p{*}'$ is smaller than $S{*}'$ after $\bar{C}_{2m}$ is increased.

(b) Increase in γ_c

The increase in γ_c results in only the saving increase effect due to a downward shift in θ and θ_p. As is clear from (35), θ shifts as much as θ_p at any S, $S_p{}^{*}{}'$ may be smaller than $S^{*}{}'$. However, it may be possible for $S_p{}^{*}{}'$ to be larger than $S^{*}{}'$ because θ and θ_p have different slopes. The higher the possibility, the steeper the slope of θ and the less that of θ_p and $(\bar{r}-Z)$.

(c) Decrease in $\bar{r}$

The fall in $\bar{r}$ yields both increase and decrease effects upon saving through downward shifts in θ, θ_p, $\bar{r}$, and $(\bar{r}-Z)$. Therefore, whether a production function is taken into consideration or not, we cannot conclude how much the net effect is. However, $S_p{}^{*}{}'$ can be smaller than $S^{*}{}'$ because $S_p{}^{*}$ is smaller than S^{*} before $\bar{r}$ is decreased.

(d) Increase in γ_r

The increase in γ_r results in only the increase effect on saving through downward shifts in θ and θ_p. We remember that $S_p{}^{*}$ is smaller than S^{*} before γ_r is changed. In addition, the first equation of (40) implies that 0 shifts as much as θ_p independent of S. These things lead us to expect that $S_p{}^{*}{}'$ is smaller than $S^{*}{}'$. However, as well as in the case of an increase in γ_c, the more it is possible for $S_p{}^{*}{}'$ to be larger than $S^{*}{}'$, the steeper the slope of θ and the less that of θ_p and $(\bar{r}-Z)$.

VI. CONCLUDING COMMENTS

The results of this paper can be summarized as follows:

(1) The level of an individual's optimal saving is decreased by taking account of the relation between variables and parameters prescribed by the production

function when expected values and uncertainty are assumed to be constant.

(2) It is difficult to predict the net effect of the change in expected values ($\bar{r}$, $\bar{C}_{2m}$) on optimal saving. The exception to that is the change in $\bar{C}_{2m}$ when the production function is not allowed for.

(3) An increase in uncertainty (γ_c, γ_r) always raises the optimal saving whether a production function is taken into consideration or not.

Therefore, it follows that the individual saves too much either when his attention is not paid to a production function or when he faces some uncertainty. This implies that his present consumption is reduced too much to provide for his future consumption. Hence, his utility is also decreased due to excessive provision. Removing these two preventing factors, he is able to enjoy maximum utility.

For one thing, the fact that he takes account of a production function means that he makes decisions in a wider framework than in a simple Fisherian type. Consequently, it is important to understand what our economic system is. Government may be requested to give exact recognition about the system to its citizens.

We can, in addition, point out reducing uncertainty which he can't help facing in making decisions. Uncertainty in future minimum required consumption γ_c may be eliminated by establishing such a social security system as an old-age pension and/or a retirement pension. If it is introduced, γ_c may be removed and uncertainty in the interest rate γ_r may have narrowly limited effects because we only have to decide the volume of affluent consumption C_{2A} to enjoy comfortable lives.

Even if we make decisions on saving and consumption having a view broad enough to see our whole lifetimes in a hypothetical full employment economy without inflation, our utility is reduced due to the uncertainty existing in such an economy. Our public pension systems may play an important role in eliminating such uncertainty.

REFERENCE

[1] Hayne E. Leland, "Saving and Uncertainty: The Precautionary Demand for Saving," *Quarterly Journal of Economics* , Vol.82, No.3 (August 1968).
[2] A. Sandmo, "The Effect of Uncertainty on Saving Decisions," *Review of Economic Studies*, Vol.37(July 1970).
[3] Jacques H. Drèze and Franco Modigliani, "Consumption Decisions under Uncertainty," *Journal of Economic Theory* , Vol.5(1972).
[4] Yasuhiro Sakai, "Economics of Uncertainty," Yuhikaku, 1982.