Consistent Conjectural Variations in the Differentiated Goods Market

Hiroshi ONO
Professor
Faculty of Economics
Hokkaido University

1. Introduction

When the number of firms in the market, is few, each competitor naturally anticipates the strategies of rival firms. In this context, it is of great significance to illustrate the way in which firms form conjectural strategies against rival moves. While there have been several types of conjectural variations such as Cournot, Bertrand, and the collusive type, traditional treatment on these has never seriously made allowance for rational conjecture of firms. However, Bresnahan(1981) has recently presented an idea of consistent conjectural variations, which requires that the conjectural variation be supported by the corresponding competitors' reactions in the market. If the demand function for their commodity is empirically stable and the cost function of producing them is technologically given, firms are expected to be able to conjecture rival moves. In the duopoly case, Bresnahan has shown that traditional types of conjectural variations do not necessarily hold under general forms of the demand and cost functions. Perry (1982) has extended it to the case of symmetric oligopoly and obtained a surprising result that in the long-run where each firm is engaged in production at zero profit the fully consistent conjectural variation is competitive. That is, irrespective of the number of firms in the market (even if it is small), if each firm behaves rationally, the marginal-cost pricing rule would result. The implication of this result is that if each firm, with the same demand and cost functions, can conjecture rival firms'response rationally, it will realise the best reasonable conjecture as setting commodity price equal to marginal cost. An essential factor for this result is not the number of firms in the market, but rationality of the behavior of firms which makes the market competitive. In other words, in symmetric equilibrium each firm can calculate his rival's moves by supposing it were a competitor. Since there is no asymmetry of information among participators in the market, each firm is confident that his prediction based on conjectural variation would be commonly shared. However, the problem that is still unsolved is why the oligopolistic state implies the competitive solution in the long-run.

The purpose of this paper is to provide a plausible answer to the above question. In fact, we can show that homogeneity of the commodity concerned would cause competitiveness in the oligopolistic market. In the case of differentiated products where each local monopolist can exercise his monopolistic power over determination of the commodity price, we can prove that the marginal-cost pricing rule does not hold in general. In doing so, we employ the Dixit-Stiglitz-Horn model(hereafter the DSH model, for short)*. While this model explicitly specifies the form of the utility function taken there, it can include the homogeneous commodity example as a special case. Therefore, we can link our results with Perry's. Furthermore, we can supply a complete answer to the consistent conjectural variations assumed in the DSH model, which are left intact. The paper consists of four sections. Section 2 explains the DSH model and the symmetric equilibrium. All the basic equations will be presented there. Section 3 discusses the short-run consistency in the conjectural variation. We can extend the Perry thesis to a more general case for differentiated products. Then Section 4 examines the long-run consistency where zero-profit condition is imposed on firms in the market. Section 5 briefly

* Horn (1984) introduces the diea of conjectural variations (not necessarily consistent) into the Dixit-Stiglitz model (1977).

summarises some of conclusions derived in our paper. Finally, in the Appendix we shall discuss two types of stability conditions.

1. The Model

Consider the DSH model. A typical consumer can choose two kinds of commodities: differentiated goods(x_j , j=1, 2, ..., n) and homogeneous goods(Y). Equation (1) specifies the type of his utility function generated from consumption of the above commodities.

$$(1) \quad U=\left(\sum_{j=1}^{n} x_j^{\beta}\right)^{\alpha/\beta} Y^{1-\alpha} \qquad 0<\alpha<1,\ 0<\beta<1$$

Utility-maximizing behaviour would then result in the inverse demand function given by equation (2).

$$(2) \quad p_j=\frac{x_j^{\beta-1}}{\Sigma x_i^{\beta}}\,\alpha\, I\ ,\quad p_y=\frac{1}{Y}(1-\alpha)\, I$$

It should be noted that when β takes unity, differentiated goods are grouped as composite homogeneous goods. That is, by imposing the condition of $\beta=1$, our inverse demand function would correspond to a special case of Perry's thesis where $P'(X)<0$ and $P''(X)>0$ for $X=\sum_{i=1}^{n} x_i$. Suppose that the producer of commodity j knows the inverse demand function. Then he will maximize his profit by manipulating quantities produced. Equation(3) indicates the profit of commodity j*.

$$(3) \quad \Pi_j = p_j x_j - c_j(x_j)\ ,$$

where $c_j(x_j)$ stands for the cost function in producing x_j .

Maximizing Π_j with respect to x_j yields equation (4).

$$(4) \quad \frac{d\Pi_j}{dx_j}=p_j + x_j\left[\frac{(\beta-1)x_j^{\beta-2}}{\Sigma x_i^{\beta}}-\frac{x_j^{\beta-1}}{(\Sigma x_i^{\beta})^2}\left(\Sigma \beta x_i^{\beta-1}\frac{dx_i}{dx_j}\right)\right]-c_j'(x_j),$$

where $c_j'(x_j)$ denotes the marginal cost. When competitive-

* Since we assume that each commodity j(j 1, 2, ... , n) is produced in the region of increasing returns, it is produced by only one firm.

ness in the industry is characterised as symmetric, it is natural to assume that each firm there has the same expectation on conjectural variations about rival response. Then each firm treats the other competitors as a group, rather than regarding them as distinct and independent objectives. Therefore, the conjectural variation about firm i's response by firm j is identical with the rest of the group. That is*,

$$(5) \quad \frac{dx_i}{dx_j} = \frac{1}{n-1}\,\delta \qquad\qquad i \neq j$$

where $-1 < \delta < n-1$. Substituting equation (5) into equation (4), we obtain equation (6).

$$(6) \quad \frac{d\Pi_j}{dx_j} = p_j\left[1 - \frac{x_j}{\Sigma x_i^{\beta}}\left(x_j^{\beta-1} + \frac{\delta}{n-1}\sum_{i\neq j} x_i^{\beta-1}\right)\right] - c_j'(x_j) = 0$$

In the symmetric case where $x_i = x$ for all i, equation (6) would reduce to equation (6)':

$$(6)' \quad \frac{d\Pi_j}{dx_j} = \beta p\left(1 - \frac{1+\delta}{n}\right) - c'(x) = 0$$

At this point, it is of significance to note that whatever firms conjecture their rival's response in changing the supply equation (7) would hold for a relatively large n.

$$(7) \quad \beta p_j = c'(x_j)$$

Then, we may predict that the consistent conjectural variation should be realized at $\delta = -\beta$, since in the homogeneous case($\beta=1$) consistency requires that $\delta = -1$. This may suggest that when product differentiation exists in the market, the marginal-cost pricing rule would not hold in a strict sense.

* This form of the conjectural variation is also assumed in Horn (1984). Our explanation on conjectural variation is the same as Perry's in essence. However, since each firm produces a differentiated product, we can not specify the inverse demand as a function of simply added gross output level produced in the differentiated goods industry.

2. Short-Run Consistency

Consider the symmetric short-run equilibrium where each firm possibly earns a positive profit and the number of firms in the industry is for some reason or other fixed. Once subjective conjecture is formed, the reaction function of each firm will be derived by solving equation (6).

$$x_j = h_j(x^j),$$

where $x^j = (x_1, \ldots, x_{j-1}, x_{j+1}, \ldots x_n)$. Suppose firm $k(\neq j)$ changes its supply. Then the other firms would show the same response against it. Therefore, for computational convenience, equation (6) might be rewritten as equation (8).

$$(8)\quad p_j^\beta\left[1 - \frac{x_j}{x_k^\beta + (n-1)x_j^\beta}\left(x_j^{\beta-1} + \frac{n-2}{n-1}\delta x_j^{\beta-1} + \frac{\delta}{n-1}x_k^{\beta-1}\right)\right] - c'(x_j) = 0$$

In fact, the reaction function of firm $j(\neq k)$ could be represented by

$$x_j = g_j(x_k), \quad \text{for all } j \neq k.$$

The consistent conjectural variation requires equation (9) to hold.

$$(9)\quad \frac{dg_j}{dx_k} = \frac{1}{n-1}\delta$$

In the case of constant marginal cost, we can establish Theorem 1.

Theorem 1 ($c''=0$)

Suppose product differentiation exist ($0 < \beta < 1$).

(a) When $n=2$, any conjectural variation is consistent.

(b) When $n > 2$, the consistent conjectural variation δ either takes a negative value-- $\delta = -\beta(n-1)/(n-\beta)$, or implies a collusive behavior--$\delta = n-1$. However, the latter is excluded from sufficient conditions for maximizing profits.

(Proof)

Rewrite equation (8) as follows.

$$(10)\quad p_j \beta T - c'(x_j) = 0 ,$$

where

$$T = 1 - \frac{x_j}{x_k^\beta + (n-1)x_j^\beta}\left(x_j^{\beta-1} + \frac{\delta}{n-1} x_k^{\beta-1} + \frac{n-2}{n-1}\delta x_j^{\beta-1}\right) .$$

Then

$$\frac{1}{p_j}\frac{dp_j}{dx_k} = (\beta-1)\frac{1}{x_j}\frac{dx_j}{dx_k} - \frac{1}{x_k^\beta + (n-1)x_j^\beta}\left[\beta x_k^{\beta-1} + (n-1)\beta x_j^{\beta-1}\frac{dx_j}{dx_k}\right] .$$

Under the symmetric equilibrium, $x_j = x_k = x$.

$$(11)\quad \frac{1}{p_j}\frac{dp_j}{dx_k} = \frac{\beta-n}{nx}\frac{dx_j}{dx_k} - \frac{\beta}{nx}$$

Similarly,

$$(12)\quad \frac{dT}{dx_k} = \frac{\beta(n-1) + (n-\beta)\delta}{n^2(n-1)}\left[-\frac{dx_j}{dx_k} + 1\right]$$

Differentiating equation (10) with respect to x_k and using equations (11) and (12), we obtain:

$$(13)\quad \frac{\beta p}{n^2 x}\left\{\left[(n-1-\delta)(\beta-n) - \frac{\beta(n-1)+(n-\beta)\delta}{n-1}\right]\frac{dx_j}{dx_k}\right.$$

$$\left. -\beta(n-1-\delta) + \frac{\beta(n-1)+(n-\beta)\delta}{n-1}\right\} - c''\frac{dx_j}{dx_k} = 0$$

Consistency in conjecture requires the following to hold:

$$(14)\quad \frac{dx_j}{dx_k} = \frac{1}{n-1}\delta$$

Substituting equation (14) into (13), we can arrange equation (13) as follows:

$$(15)\ \frac{n-2}{n-1}\ [(n-\beta)\delta+\beta(n-1)][\delta-(n-1)]-\ c''\frac{n^2x}{\beta p}\delta\ =\ 0$$

When $c''=0$, it can be seen that any value of δ satisfies equation (15), for $n=2$.

On the other hand, when the number of firms in the market is greater than two, we have two distinct conjectural variations which satisfy equation (15).

$$\delta = -\ \beta\frac{n-1}{n-\beta},\quad \text{and } n - 1$$

However, in order for the consistent conjectural variation really to meet the condition for maximizing profit, it is required that $d^2\Pi_j/dx_j^2 < 0$. The sufficient condition for profit-maximization is given in Appendix as equation (A1).

$$(A1)\ \frac{d^2\Pi_j}{dx_j^2} = -\beta p_j\ (n-1-\delta)[\frac{1-\beta}{nx} + \frac{\delta(1-\beta)}{n(n-1)x} + \frac{2\beta(1+\delta)}{n^2x}]\ -c'' < 0$$

If $\delta=n-1$ and $c''=0$, then $\frac{d^2\Pi_j}{dx_j^2}=0$, which violates sufficiency for maximizing profits. (Q.E.D.)

It should be noted that for sufficiently large n, $\delta\cong-\beta$. When the marginal cost is not necessarily constant, we obtain Theorem 2.

Theorem 2 (Variable Marginal Cost)

(1) $c''>0$ (Increasing Marginal Cost)

The consistent conjectural variation is negative, but less than competitive. That is, $-\beta\frac{n-1}{n-\beta}<\delta<0$

(2) $c''<0$ (Declining Marginal Cost)

Consistent conjectural variation either takes a positive value ($\delta>0$) or lies in a negative range ($-1<\delta<-\beta\frac{n-1}{n-\beta}$).

(Proof)

Rewrite equation (15) as follows:

$$(15)'\ \frac{n-2}{n-1}\ [(n-\beta)\delta + \beta(n-1)][\delta - (n-1)]= c''\frac{n^2x}{\beta p}\ \delta$$

Suppose $n \geqq 3$. Then, if $\delta = n+1$, the LHS of equation (15)' vanishes, while the RHS of the equation would take a non-zero value. Therefore, we can exclude the case of $\delta = n+1$. That is, $\delta-(n-1)<0$ always.

(1) $c'' > 0$

With the same reasoning, δ can not take a specific value. Suppose

$$(n-\beta)\delta + \beta(n-1) > 0 \quad (\rightarrow \delta > -\beta\frac{n-1}{n-\beta}).$$

Then the RHS of equation(15)' would take a negative value. If consistent conjectural variation exists, then δ must be negative from the requirement that RHS in the equation is negative. Therefore,

$$-\beta\frac{n-1}{n-\beta} < \delta < 0 \ .$$

It should be noted that

$$-1 < -\beta\frac{n-1}{n-\beta} \ .$$

On the other hand, suppose

$$(n-\beta)\delta + \beta(n-1) < 0 \quad (\rightarrow \delta < -\beta\frac{n-1}{n-\beta}).$$

Then the LHS of the equation takes a positive value. In order for equality in equation(15)' to hold, δ must take a positive value. Therefore, there are no consistent conjectural variations which satisfy these two requirement simultaneously.

(2) $c'' < 0$

We can apply the same reasoning as discussed in the above. Suppose

$$(n-\beta)\delta + \beta(n-1) > 0 \quad (\rightarrow \delta > -\beta\frac{n-1}{n-\beta}) .$$

Then, the LHS of the equation becomes negative, which requires δ be positive. Therefore, the positive value of δ would satisfy equation (15)'.

On the other hand, suppose

$$(n-\beta)\delta + \beta(n-1) < 0 \quad (\rightarrow \delta < -\beta\frac{n-1}{n-\beta}) .$$

Then the LHS of the equation becomes positive. This implies that δ must be negative. Therefore,

$$-1 < \delta < -\beta\frac{n-1}{n-\beta} .$$

(Q.E.D.)

It should be noted in the above discussion that when the number of firms is two($n=2$) then the consistent conjectural variation must be of the Cournot type ($\delta=0$). However, this result seems to have originated from the assumption on the specific form of the utility function. Therefore, we implicitly assume away the case.

Let us compare our results with Perry's. Depending on the form of the marginal cost function, we can classify the range of consistent conjectural variations as given in Figure 1. It is easily seen in Figure 1 that when the commodity concerned is homogeneous (i.e., $\beta=1$), then $-\beta(n-1)/(n-\beta)$ reduces to -1. This implies Region III never appears for the homogeneous commodity case. In fact, the above argument has successfully derived from Perry's Propositions as a special case in our context. However, when product differentiation exists in the market, each firm can exploit the consumer. In particular, the difference between Perry's result and ours is crucial for the case of declining marginal cost. In Perry's case, consistent conjectural variation must take only a positive value. Since each firm can cheaply produce the commodity, it can rationally conjecture that even if every firm in the market increases its output enough demand would be generated due to the price effect. While this is true in our case, too, each firm, as being a local monopolist, can now exploit consumers. Therefore, some degree of competitiveness can be compatible with consistent conjectural variation.

3. Long-Run Consistency

In the previous section, we assumed the number of firms in the differentiated goods sector as being exogenous. That is, the relationship given by equation (6) holds for any given n. In the long-run, however, the number of firms itself would be endogenously determined. The equation to be added is the one which states any firm maximising profits can not enjoy positive profits. That is,

$$(16)\quad \Pi_j = \frac{x_j^\beta}{x_k^\beta + (n-1)x_j^\beta} - c(x_j)$$

By equations (6) and (16), both x_j and n are simultaneously determined as functions of x_k. We can compute the effects of a change in x_k on x_j and n.

$$(17)\quad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \dfrac{dx_j}{dx_k} \\ \dfrac{dn}{dx_k} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

where

$$a_{11} = \frac{\beta p}{n^2 x}\left[(n-1-\delta)(\beta-n) - \frac{\beta(n-1)+(n-\beta)\delta}{n-1}\right] - c''$$

$$a_{12} = \beta p\,\frac{2(1+\delta)-n}{n^2}$$

$$a_{21} = \frac{\beta}{nx} - \frac{c'}{c}$$

$$a_{22} = -\frac{1}{n}$$

$$b_1 = \frac{\beta P}{n^2 x} \{\beta(n-1-\delta) - \frac{\beta(n-1) + (n-\beta)\delta}{n-1}\}$$

$$b_2 = \frac{\beta}{nx}.$$

Since n changes in the long-run, we are interested in the effect of a change in gross output X_0 due to a change in x_k rather than that of a change in one firm's output. Note that using the relation that $\frac{dx_0}{dx_k} = \frac{d}{dx_k}\{(n-1)x_j\}$ $x_k\frac{dn}{dx_k} + (n-1)\frac{dx_j}{dx_k}$, we have the following matrix system:

$$\begin{bmatrix} \frac{a_{11}}{n-1} & a_{12} - a_{11}\frac{x}{n-1} \\ \frac{a_{21}}{n-1} & a_{22} - a_{21}\frac{x}{n-1} \end{bmatrix} \begin{bmatrix} \frac{dx_0}{dx_k} \\ \frac{dn}{dx_k} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Then, we can establish Theorem 3.

Theorem 3 (Long-run Consistency)

When product differentiation exists in the market, consistent conjectural variation is not competitive. Furthermore, if the number of firms is relatively large, then consistent conjectural variation would take the value of $(-\beta)$.

(Proof)

Note that

$$\Delta = \begin{vmatrix} \frac{a_{11}}{n-1} & a_{12} - a_{11}\frac{x}{n-1} \\ \frac{a_{21}}{n-1} & a_{22} - a_{21}\frac{x}{n-1} \end{vmatrix} = \frac{1}{n-1}(a_{11}a_{22} - a_{12}a_{21})$$

$$\Delta_1 = \begin{vmatrix} b_1 & a_{12} - a_{11}\frac{x}{n-1} \\ b_2 & a_{22} - a_{21}\frac{x}{n-1} \end{vmatrix}$$

$$= \begin{vmatrix} b_1 + \frac{1}{x}(a_{12} - \frac{a_{11}x}{n-1}) & a_{12} - \frac{a_{11}x}{n-1} \\ b_2 + \frac{1}{x}(a_{22} - \frac{a_{21}x}{n-1}) & a_{22} - \frac{a_{21}x}{n-1} \end{vmatrix}$$

$$= \begin{vmatrix} b_1 + \frac{1}{x} a_{12} & a_{12} - \frac{a_{11}x}{n-1} \\ b_2 + \frac{1}{x} a_{22} & a_{22} - \frac{a_{21}x}{n-1} \end{vmatrix} - \begin{vmatrix} \frac{a_{11}}{n-1} & a_{12} - \frac{a_{11}x}{n-1} \\ \frac{a_{21}}{n-1} & a_{22} - \frac{a_{21}x}{n-1} \end{vmatrix}$$

$$= \tilde{\Delta}_1 - \Delta .$$

It is easily computed that

$$(1)\quad b_1 + \frac{a_{12}}{x} = -\frac{\beta p(n-2)(n-1-\delta)(1-\beta)}{n^2(n-1)x}$$

$$(2)\quad b_2 + \frac{a_{22}}{x} = -\frac{(1-\beta)}{nx}$$

$$(3)\quad a_{12} - \frac{a_{11}x}{n-1} = \beta p \left\{ \frac{1+\delta}{n^2} + \frac{(1-\beta)(n-1-\delta)}{n^2(n-1)} \cdot + \frac{n(\beta+\delta)-\beta(1+\delta)}{n^2(n-1)^2} \right\} + \frac{x}{n-1} c''$$

$$(4)\quad a_{22} - \frac{a_{11}x}{n-1} = \frac{(n-1)(\beta-1)-\beta(1+\delta)}{n(n-1)} .$$

In deriving the fourth relation, we used equilibrium conditions that $c=1/n$, and that $c''=\beta(n-1-\delta)/n^2x$. When product differentiation exists ($0<\beta<1$), then $\tilde{\Delta}_1$ does not generally vanish. It is seen that in the homogeneous case ($\beta=1$), $\tilde{\Delta}_1=0$, which implies $dx_0/dx_k=-1$. (See Perry, Proposition 3).

In order to prove that $\frac{dx_0}{dx_k}$ approaches $-\beta$ for relatively large n, we must expand determinants Δ and $\tilde{\Delta}_1$.

$$\Delta = \frac{1}{n-1}(a_{11}a_{22} - a_{12}a_{21})$$

$$= \frac{\beta p}{n^3(n-1)x}\{n^2 - (1+\delta+\beta)n + \beta(1+\delta) + \frac{\beta(n-1)+(n-\beta)\delta}{n-1}$$

$$-\beta n^2 + \beta[2(1+\delta)+2+\delta]n - 2\beta(1+\delta)(2+\delta) + \frac{n^2 x}{\beta p}c''\}$$

$$= \frac{\beta p}{n^3(n-1)x}\{[(1-\beta)+\frac{xc''}{\beta p}]n^2 + \underbrace{\ldots\ldots}_{\text{lower degree of order parts}}\}$$

$$\tilde{\Delta}_1 = \frac{\beta p}{n^3(n-1)x}\{-\frac{(n-2)(n-1-\delta)(1-\beta)[(n-1)(\beta-1)-\beta(1+\delta)]}{(n-1)}$$

$$+ (1-\beta)(n-1)(1+\delta)+(1-\beta)^2(n-1-\delta)+\frac{(1-\beta)[n(\beta+\delta)-\beta(1+\delta)]}{n-1}$$

$$+ (1-\beta)\frac{n^2 xc''}{\beta p}$$

$$= \frac{\beta p}{n^3(n-1)x}\{(1-\beta)[(1-\beta)+\frac{xc''}{\beta p}]n^2 + \underbrace{\ldots\ldots}_{\text{lower degree of order parts}}\}$$

Therefore, for large n

$$\frac{\tilde{\Delta}_1}{\Delta} \cong \frac{(1-\beta)[(1-\beta)+\frac{xc''}{\beta p}]n^2}{[(1-\beta)+\frac{xc''}{\beta p}]n^2} = (1-\beta) .$$

Then

$$\frac{dx_0}{dx_k} = -1 + 1 - \beta = -\beta .$$

4. Concluding Remarks

We have examined whether Perry's results could be extended to the case of differentiated goods. While the model we used assumes a special type of the utility function, we have derived some important implications from our analysis.

1. The surprising result by Perry that fully consistent conjectural variation in the long-run is competitive seems mainly to come from homogeneity of the commodity concerned in the market. As long as consumers do not change their preference for differentiated commodities, consistent conjectural variation by firms is directly related to the degree of product differentiation.

2. When firms employ technology to decrease marginal cost, consistent conjectural variation takes a positive value in Perry's case. In our case, δ can take either sign. That is, each firm can exercise its monopolistic power over consumers. Therefore, a certain state of competitive- is consistent.

3. We assumed that the number of firms (n) and the degree of product differentiation are not directly related to each other. However, if an increase in n reduces distinctiveness in product differentiation, then β may approach unity. In that case, our result may imply Perry's thesis in the long-run.

Appendix: Stability Conditions

In the previous sections, we only stated the necessary conditions for maximizing profits. In order to guarantee a stable equilibrium, we briefly touch upon two stability conditions (see Perry (1982)). The first is a sufficient condition for maximizing profits when conjectural variations are given. That is,

$$\frac{d^2 \Pi_j}{dx_j^2} < 0 \ .$$

Since the necessary condition is given in equation (6), we can compute it as follows:

$$\text{(A1)} \quad \frac{d^2 \Pi_j}{dx_j^2} = -\ (n-1-\delta)\beta p_j \left\{ \frac{1-\beta}{nx} + \frac{\delta(1-\beta)}{n(n-1)x} + \frac{2\beta(1+\delta)}{n^2 x} \right\} - c''$$

It should be noted that in the collusive case a sufficient condition totally depends on the property of cost function.

The second stability condition is related to the one on conjectural variation. Suppose each firm has a conjecture on the behaviour of another firm as $\delta/(n-1)$. Using this conjecture, together with properties of demand and cost functions empirically known, each firm may compute another firm's response when it changes output, i.e., dx_j/dx_k in equation (13). Since this value generally depends on the number of firms existing in the market and on conjectural variation, we can express it as

$$\frac{dx_j}{dx_k} = \frac{1}{n-1}\,\zeta(n:\delta) \ .$$

Stability condition in the conjectural variation requires that around the point of equilibrium (that is, the consistent conjectural variation is satisfied at $\zeta(n;\delta)=\delta$), a displacement from the equilibrium will be forced back to its original place.

$$\text{(A2)}\quad \frac{\partial}{\partial\delta}\left(\frac{\zeta(n:\delta)}{\delta}\right)\Bigg|_{\delta=\zeta(n:\delta)} < 0$$

It turns out that the equivalent condition to equation(A2) is obtained by differentiating equation (15) with respect to δ. That is,

$$\text{(A2)}\quad \frac{n-2}{n-1}\,\{(n-\beta)[\delta-(n-1)]+(n-\beta)\delta+\beta(n-1)\} - c''\,\frac{n^2x}{\beta p} < 0 \ .$$

It can be seen that when consistent conjectural variation is approaching a collusive solution, a state of instability is likely.

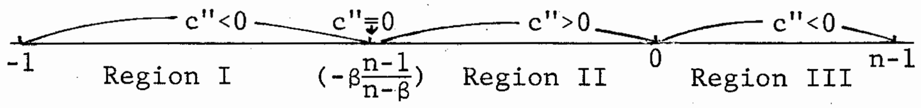

Figure 1. The Range of Consistent Conjectural Variation

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