Risk Aversion and Minsky's Crisis Model

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In a recent paper [2] , L. Taylor and S. O'Connell develop an interesting model to illustrate Hymann Minsky's financial crisis theory. With two key assumptions, they demonstrate the negative relationship of expected profits and the rate of interest, which is the key mechanism in Minsky's financial crisis theory. One of their assumptions is that the level of wealth in the economy is determined macroeconomically. The second assumption is that there is high substitutability between liabilities of firms and money in the public's portfolio. Though their model building is significant, it is hard to say that their attempt is successful. First, it is not made clear what they mean by the term of "high" substitutability. Second, nowhere in the model do they set forth the possibility of substitution between money and other assets caused by changes in the level of wealth determined endogenously.

In this paper, we shall reformulate their model by introducing the notion of risk aversion in the theory of portfolio selection under uncertainty. We will consider in our model the substitutability in portfolio through the risk aversion effect of wealth as well as the ordinary price effect of the interest rate. We assume that the level of wealth in the economy is determined macroeconomically, and the wealth elasticity of money holding's share in portfolio approximates to unity. Then, a downward shift in anticipated profits leads wealth to contract and, by increasing the public's risk aversion, causes the public to shift portfolio preferences toward money, money being a secure asset. Interest rates rise, leading to further damping of expected profits, and a debt–deflation crisis can occur.

I

Before developing our model, we shall show that there is a misleading part in that of Taylor and O'Connell. They conclude that a high substitutability between money and equity is required to derive a negative relationship of expected profits and the rate of interest, which is the key mechanism in Minsky's crisis theory. However, they make clear nowhere the definition of high substitutability. They show that there is an additional condition for explaining Minsky's mechanism, but they fail to give a detailed examination of it. The condition is that α — the share of fiscal debt issued as money — is a small enough fraction. It will be shown shortly that this condition is not additional, but more crucial than the high substitutability in their model.

This is the passage in question (pp.877–878, [2]).

$$(19) \quad \eta_i \, d_i + \eta_r \, d_r = -\eta_r \, d\rho + (1 - \varepsilon) \, d\alpha \,,$$

where

$$\eta_i = \mu_i + \alpha \varepsilon_i$$

and

$$\eta_r = \mu_r + \alpha\varepsilon_r .$$

A higher bond interest rate cuts back on demand for money, so that μ_i is negative. Since demand for equity also falls, ε_i is negative, making $\eta_i < 0$. The partial derivative μ_r is negative, but an increase in r or ρ raises the demand for nominal equity. From the standard assumption that assets are gross substitutes, $\varepsilon_r > |\mu_r|$. However, if money and equity are close substitutes in asset demand, the magnitudes of the two partial derivatives will be close to each other. If, further, α is a small enough fraction, then $\eta_r < 0$. For reasons to be made clear shortly, we shall assume high substitutability between money and equity, so that the portmanteau derivative η_r is indeed negative.

It is evidence that there need to be two conditions for the derivative η_r to be negative. One is that money and equity are close substitutes in asset demand, i.e. the magnitudes of the two partial derivatives, ε_r and $|\mu_r|$ are close to each other. The other condition is that α is a small enough fraction. The former condition seems to be represented in terms of high substitutability between money and equity.

We should notice that $\mu+\varepsilon+\beta = 1$ (p.876, [2]), then $\mu_r+\varepsilon_r+\beta_r = 0$. Therefore, β_r will be zero if ε_r and $|\mu_r|$ are close to each other. This is incompatible with the condition of substitutability between equity and bonds which is the usual supposition in the theory of portfolio selection. Thus we are unable to assert the condition of high substitutability between only money and equity, along the lines of standard price theory.

The other condition will play a key role in the derivation of $\eta_r < 0$. α indicates a relative situation on the supply side in the money market to bonds market. Suppose that a decrease of returns to equity has the same degree of positive effects on each demand in both the money and bonds markets. Then, a condition where α is a small enough fraction implies that the width of demand–supply disequilibrium in the money market is more extensive than the width of disequilibrium in the bonds market. A decrease in the expected profit rate will raise the rate of interest, since a rise in the rate of interest caused by an increase of demand in the money market will dominate a fall in the rate of interest caused by an increase of demand in the bonds market. This is the reasoning by which a development of the key mechanism in Minsky's crisis theory is made possible in the Taylor and O'Connell model.

It is hard for us to look at the high substitutability between money and equity, without neglecting the high substitutability between equity and bonds, along just the lines of standard price theory. This difficulty in what follows would be overcome by introducing the notion of risk aversion in the theory of portfolio selection under uncertainty.

II

Before we formally develop our model, let us show its basic concepts. First, we assume that the level of wealth is determined endogenously. It can be shown from the model that a decrease in the rate of expected profit will reduce the value of wealth. Second, we suppose that any wealth owner (rentier) is a risk avertor, and his risk aversion is a decreasing function of wealth. According to the standard theory of portfolio selection, an increase of risk aversion will lead the wealth owner to shift portfolio preferences toward money, money being a secure asset, from equity and bonds which are risky assets. It is then shown that a decrease of the expected profit rate will raise the demand for money. Thus we will be in a better position to indicate the negative relationship of expected profits and the rate of interest, which is the key mechanism in Minsky's crisis theory.

Our model is shown by the following system of equations:

(1) $W = Pe\,E + F$,
(2) $A(W)\,\mu(i, r + \rho)W = M$,
(3) $B(W)\,\varepsilon(i, r + \rho)W = Pe\,E$,

where all notations except $A(W)$ and $B(W)$ are the same as those used by Taylor and O'Connell. Equation (1) corresponds to (11) in their model, which indicates the definition of wealth. Equations (2) and (3) are each conditions for market balance of money and equity. As usual, we leave out a dependent condition for the market balance of bonds. Given M, E, F, and $r + \rho$, the system of equation (1) ~ (3) determines the level of wealth (W), the price of equity (Pe), and the rate of interest (i) endogenously.

The asset demand functions are made up of two component parts. One is the function of rates of return to each asset, which is the usual representative form of substitutability between each asset along the lines of standard price theory. The other is the function of wealth, which represents substitutability through the risk aversion effect along the lines of standard portfolio selection theory. Needless to say, $A\mu + B\varepsilon + C\beta = 1$.

Let us suppose that any wealth owner is a risk avertor, and his risk aversion is a decreasing function of wealth. According to the theory of portfolio selection, then, an increase of risk aversion will lead the wealth owner to shift portfolio preferences toward money, money being a secure asset, from equity and bonds which are risky assets. Thus we suppose that

(4) $A'(W) < 0$
and
(5) $B'(W) > 0$.

Assuming that neither asset is inferior with respect to wealth, and considering the wealth constraint $A\mu W + B\varepsilon W + C\beta W = W$, then we have

(6) $$0 < \frac{\partial A\mu W}{\partial W} = A'\mu W + A\mu < 1\ ,$$

(7) $$0 < \frac{\partial B\varepsilon W}{\partial W} = B'\varepsilon W + B\varepsilon < 1\ .$$

Substituting from (1) for $Pe\,E$ in (3), the system of equations (1) ~ (3) is reduced to the following system:

(8) $$\begin{array}{c} A(W)\,\mu(i, r+\rho)W = M \\ W - B(W)\,\varepsilon(i, r+\rho)W = F \end{array}$$

where i and W are equilibrating endogenous variables.

Differentiating (8), we obtain the following matrix form:

$$(9)\quad \begin{bmatrix} A\mu_i W & A'\,\mu W + A\mu \\ -B\varepsilon_i W & 1 - B'\,\varepsilon W - B\varepsilon \end{bmatrix} \begin{bmatrix} di \\ dW \end{bmatrix} = \begin{bmatrix} dM - A\mu_r W d\rho \\ dF + B\varepsilon_r W d\rho \end{bmatrix}.$$

This system will determine di and dW if and only if the matrix on the extreme left is non-singular. Taking the determinant of the matrix, we obtain

$$D = A\mu_i W(1 - B'\,\varepsilon W - B\varepsilon) + B\varepsilon_i W(A'\,\mu W + A\mu)\,.$$

It follows from (6) and (7) that D is negative.

Now we shall examine the effects of changes in expected profit on the level of wealth and the rate of interest. From (9) we obtain

$$(10)\quad \frac{dW}{d\rho} = \frac{1}{D}\,[A\mu_i W \cdot B\varepsilon_r W - A\mu_r W \cdot B\varepsilon_i W] > 0\,,$$

$$(11)\quad \frac{di}{d\rho} = -\frac{1}{D}\,[A\mu_r W \cdot (1 - B'\,\varepsilon W - B\varepsilon) + B\varepsilon_r W \cdot (A'\,\mu W + A\mu)]\,.$$

Equation (10) shows that an increase in ρ will raise financial wealth. In effect, the rentier's net worth is determined macroeconomically from the valuation of anticipated profits.

The sign on the right hand side in (11) is, in general, ambiguous. However, if the wealth elasticity of money holding's share in portfolio approximates to unity, the term $(A'\,\mu W + A\mu)$ will be close to zero:

$$\begin{aligned} A'\,\mu W + A\mu &= A\mu\,(A'\,\mu W / A\mu + 1) \\ &= A\mu\,(1 - e) \to 0 \qquad as \quad e \to 1 \end{aligned}$$

where $e = -\dfrac{\partial A\mu}{\partial W} \Big/ \dfrac{A\mu}{W}$, the wealth elasticity of money holding's share. In such case, therefore, we will obtain

$$\frac{di}{d\rho} < 0\,.$$

This is the key mechanism of Minsky's financial crisis theory.

In summary, our model determines the level of wealth endogenously. A decrease in the anticipated rate of profits will reduce the level of wealth. A decrease of wealth will enhance the risk aversion of wealth owners. If such an increase of risk aversion makes the wealth owner acquire a larger share of money holdings in his portfolio, the decrease of expected profits will raise the rate of interest.

References

[1] Minsky, Hyman P., *John Maynard Keynes* (New York: Columbia University Press, 1975).

[2] Taylor, Lance and Stephen A. O'Connell, "A Minsky Crisis" *Quarterly Journal of Economics*, C (1985), 871–885.