Public Investment Criterion and Lifetime Equity

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Abstract

This paper extends the problem of public investment criteria in an overlapping generations economy distorted with the corporation income tax to the case of heterogeneous individuals. First, we can confirm that the appropriate discount rate for public investment should be not only the Sandmo-Drèze weighted average rate but also the population growth rate if the government enjoys complete freedom in its choice of lump-sum taxes. Second, it is shown that, if these are restricted to a uniform poll tax or subsidy which may be different between younger and older generations, the latter continues to hold while the former must be modified by an additional term representing the element of intragenerational lifetime equity.

I. *Introduction*

In my previous paper [10], the Sandmo-Drèze model of public investment was extended to an overlapping generations economy and it was shown that the *population growth rate* plays normatively an important role in the determination of the discount rate for public investment. That is, in a (non) tax-distorted overlapping generations economy with (without) the corporation income tax, if government surplus can be redistributed between younger and older generations, then the second-best (optimal) discount rate for public investment in steady states should not only be the weighted average formula (the market rate of interest) derived by Sandmo and Drèze [9] but also the population growth rate[1)].

The assumption of homogeneous individuals that all people are alike with respect to both preference and income stream was taken because analysis in the form of a representative individual was intended. However, this assumption is restrictive since the problem of income distribution within each generation was completely ignored. In general, there may be a trade-off between distribution and optimum growth policies or, put differently, between intragenerational and intergenerational equity[2)]. Therefore, the optimal public investment criterion should also be derived in consideration of intragenerational lifetime equity[3)]. The purpose of this paper is to tackle this problem by incorporating the assumption of heterogeneous individuals into the model developed in [10] and to examine how the above-mentioned two results must be modified.

Before proceeding, the conclusions of the paper will be summarized. First we can confirm that both the above results continue to hold if government surplus can be redistributed individually among all people by means of an ideal lump-sum tax-subsidy policy. However, this conclusion may be of little interest because we could also appeal to Samuelson's result [8] for social indifference curves and argue that with perfect lump-sum transfers each generation can be treated as one individual. Second, such an ideal lump-sum tax-subsidy policy, because of its infeasibility in practice, should be confined to a uniform poll tax-subsidy in each of the younger and older generations. With this confined policy, we can derive a *version* of the Sandmo-Drèze weighted average formula, which consists of a weighted average term of the rate facing consumers and the tax-distorted rate used by firms, the weights being different from those in the original formula, plus a modified term representing the *intragenerational lifetime equity*. It may follow from this that the second-best discount rate for public investment under such a confined policy is distinct from that under an ideal redistribution policy. However, this assertion is not correct because it can be shown that the population growth rate remains the appropriate discount rate. Thus, we can conclude that, while the *items* of opportunity cost per unit of public investment under the poll tax-subsidy policy differ from those under the ideal lump-sum tax-subsidy, the opportunity cost per unit *itself* is the same and is equal to the population growth factor.

II. *Optimal Discount Rate in a Centralized Planned Economy*

In this section, we will derive the optimal (first-best) discount rate for public investment in a centralized planned economy where the central authorities have command over both the intertemporal resource allocation and the intragenerational income distribution.

As in [10], let us consider an economy where (a) each individual lives for two periods, working in the first and retired in the second, (b) population is growing at a constant rate n, (c) there is a single goods which can endure for a single period alone and can be either consumed or invested, and (d) the usual neo-classical assumptions of decreasing returns to scale technology, convexity of preferences, and so on are satisfied.

Adding to such assumptions, it is now assumed that there are heterogeneous individuals of types M ($m=1,\dots,M$) in this economy, who are distinguished by the productivity of their labors[4)], the efficiency of their private investments, and their preferences denoted by w_m, $f_m(y_m)$, and $U_m(c_m^1, c_m^2)$ respectively, and that the relative proportion of people of each type m denoted by h_m is constant over time.

Each income w_m in one period can be allocated to present consumptions ($c_1^1,\dots, c_M^1$), private investments ($y_1,\dots, y_M$), and public investment z, and then these invested goods can in turn produce respectively the outputs $[f_1(y_1),\dots, f_M(y_M)]$ and $g(z)$ available for future consumptions ($c_1^2,\dots, c_M^2$) in the next period. In the analysis to follow we will be concerned with steady states where the vectors (c_m^1), (c_m^2), (y_m), and z are all independent of time. Since aggregate demand must be equal to aggregate supply, the feasibility condition is given by the following equation:

$$(1)\quad \sum_{m=1}^{M}[(1+n)(c_m^1+y_m+z-w_m)+c_m^2-f_m(y_m)-g(z)]\,h_m=0.$$

Let us now assume that the social objective can be expressed by the additive social welfare function:

(2) $\sum_{m=1}^{M} U_m (c_m^1, c_m^2) h_m$,

and consider a problem that maximizes the function (2) subject to Eq.(1) with respect to (c_m^1), (c_m^2), (y_m), and z. This is the problem that seeks for optimal stationary states which achieve the following two objectives in a centralized planned economy: (i) the optimal allocation of available goods in one period to public investment, private investments, and per capita consumption, and (ii) the optimal distribution of per capita consumption among all people within two generations living in the period concerned. From the first-order conditions for this problem we can obtain the familiar relations:

(3) $U_m^1 (c_m^1, c_m^2)/U_m^2 (c_m^1, c_m^2) = 1+n,$ $\qquad m=1, \ldots, M,$

(4) $f'_m (y_m) - 1 = g'(z) - 1 = n,$ $\qquad m=1, \ldots, M,$

where U_m^i is the partial derivative of U_m with respect to c_m^i ($i=1$ and 2), and f'_m and g' are the marginal productivity of private and public investments respectively.

Thus, we find that the optimal discount rate for public investment in a centralized planned economy is the population growth rate, which is equal not only to that for private investments but also to the marginal rates of time preference for heterogeneous individuals.

III. *Second-Best Discount Rate in a Decentralized Market Economy*

In this section, we will derive the second-best discount rate for public investment in a decentralized market economy, distorted due to the corporation income tax, where individuals and firms in each generation are free to maximize their own objectives subject to their own private constraints.

To begin with, let us enumerate the main assumptions we employ in the subsequent analysis:

(i) Each consumer allocates his wealth between the first and second period consumptions and invests his saving in equities of the private firm and / or in government bonds so as to maximize his lifetime utility subject to his budget constraint.

(ii) There is a proportional tax rate on gross profit. Each firm is financed entirely by equities and maximizes its net profit after tax, the proceeds of which are entirely distributed to consumers. For simplicity, it is assumed that there is one private firm for each consumer, which implies that firms are regarded solely as agencies of individuals, so that we are not distinguishing between consumers and firms explicitly.

(iii) The government separates its capital and current budgets. That is, public investment is financed by borrowing on the capital market, while repayment with interest on public debt and transfers to both younger and older generations are paid out of public production and tax. The government, having *fixed* the value of profit tax rate, determines the level of public investment and the amounts of redistributional transfer so as to achieve a constrained optimum for the economy[5).

(iv) The device for resource allocation is the capital market which is cleared through the market rate of interest.

(v) Finally, there exist no public goods, external (dis-)economies, or uncertainty.

Since public investment in this model has both direct and indirect impact on income distribution within each generation through change of the market rate of interest and the redistributional way of government surplus respectively, it is convenient for us to classify the redistribution policy into an ideal lump-sum tax-subsidy and a uniform poll tax-subsidy.

1. *Ideal Lump-Sum Tax-Subsidy Policy*

The second-best optimizing problem for the government under such an ideal redistribution policy can be formulated as follows[6]:

$$\underset{[z\ ,\ (a_m^1)\ ,\ (a_m^2)]}{\text{Maximize}} \quad \sum_{m=1}^{M} U_m\ (c_m^1\ ,\ c_m^2)\ h_m$$

subject to

$$(5)\quad c_m^1 = w_m + a_m^1 - y_m - s_m\ , \qquad m=1\ ,\ldots,\ M,$$

$$(6)\quad c_m^2 = f_m\ (y_m) - t\ [\ f_m\ (y_m) - y_m] + a_m^2 + (1+r)\ s_m\ ,^{7)} \qquad m=1\ ,\ldots,\ M,$$

$$(7)\quad U_m^1\ (c_m^1\ ,\ c_m^2)/U_m^2\ (c_m^1\ ,\ c_m^2) = 1+r\ , \qquad m=1\ ,\ldots,\ M,$$

$$(8)\quad f'_m\ (y_m) = 1+r\ /\ (1-t)\ , \qquad m=1\ ,\ldots,\ M,$$

$$(9)\quad \sum_{m=1}^{M} (c_m^1 + y_m + z - w_m - a_m^1)\ h_m = 0\ ,$$

$$(10)\quad z = b\ ,$$

$$(11)\quad g(z) + t \sum_{m=1}^{M} [\ f_m\ (y_m) - y_m]\ h_m = \sum_{m=1}^{M} [(1+n)\ a_m^1 + a_m^2]\ h_m + (1+r)\ b\ ,$$

where s_m is saving of the individual of type m; a_m^1 is lump-sum transfer from the government to him in the first period; a_m^2 is lump-sum transfer to him in the second period; t is proportional tax rate on profit; b is an issue of public debt per capita; and r is the market rate of interest. Note that in this formulation, we distinguish the saving s_m of each individual from his private investment y_m, because the former takes the form of public bond or pure consumption loans among heterogeneous individuals in the same generation.

Eqs.(5) and (6) stand for the budget constraints for the individual of type m in the first and second periods of his lifetime respectively, and Eqs.(7) and (8) are the utility maximization conditions for him with respect to s_m and y_m respectively. Eq.(9) represents the market equilibrium condition for goods at the first period in steady states[8]. Finally, Eqs.(10) and (11) are the government capital and current budgets respectively. Given a government policy $[z,\ (a_m^1),\ (a_m^2)]$, Eqs. (5)$-$(9)

determine an equilibrium in the private sector in relation to $[(c_m^1), (c_m^2), (s_m), (y_m), r]$.

The market equilibrium condition for goods at the second period

$$(12) \quad \sum_{m=1}^{M} [c_m^2+(1+n)\, a_m^1 - f_m\,(y_m) - g\,(z)]\, h_m = 0$$

and the capital market equilibrium condition

$$(13) \quad \sum_{m=1}^{M} s_m h_m = b$$

have not been introduced in the above formulation, which can be derived from Eqs.(5), (6), and (9)−(11). It is of course understood that the feasibility condition (1) is automatically satisfied by Eqs.(9) and (12).

Eliminating z, b, (c_m^2), and (s_m) from the above problem, our second-best problem can then be formulated as the maximization of the following Lagrange function with respect to (a_m^1), (a_m^2), and z:[9]

$$(14) \quad L = \sum_{m=1}^{M} U_m\{c_m^1,\ (1+r)\,(w_m+a_m^1-y_m-c_m^1)+f_m\,(y_m)-t\,[f_m\,(y_m)-y_m]+a_m^2\}h_m$$
$$-\lambda\{\,g\,[\sum_{m=1}^{M}(w_m+a_m^1-y_m-c_m^1)\,h_m]+t\sum_{m=1}^{M}[f_m\,(y_m)-y_m]\,h_m$$
$$-\sum_{m=1}^{M}[(1+n)\,a_m^1+a_m^2+(1+r)\,(w_m+a_m^1-y_m-c_m^1)]\,h_m\},$$

where (i) for each type m, y_m depends on r, (ii) for each type m, c_m^1 and c_m^2 depend on r, a_m^1, and a_m^2, and (iii) r depends on z, (a_m^1), and (a_m^2). Using Eqs.(7), (8), and (9), the first-order conditions for this problem are given by the following equations:

$$(15) \quad (1+r)U_m^2/\lambda = [-g'+(1+r)]\,(\partial c_m^1/\partial a_m^1)+g'-2-n-r, \qquad m=1,\ldots,M,$$

$$(16) \quad (1+r)U_m^2/\lambda = (1+r)\,[-g'+(1+r)]\,(\partial c_m^1/\partial a_m^2)-1-r, \qquad m=1,\ldots,M,$$

$$(17) \quad \sum_{m=1}^{M}(U_m^2/\lambda)\ s_m h_m = -g'\sum_{m=1}^{M}[(\partial y_m/\partial r)+(\partial c_m^1/\partial r)]\ h_m$$
$$+[1+r/(1-t)]\sum_{m=1}^{M}(\partial y_m/\partial r)\ h_m+(1+r)\sum_{m=1}^{M}(\partial c_m^1/\partial r)\ h_m-z.$$

Substituting U_m^2/λ in Eq.(16) into Eq.(17), and then using Eq.(9), Eq.(17) reduces to

$$(18) \quad g'\,(z)\sum_{m=1}^{M}[(\partial y_m/\partial r)+(\partial c_m^1/\partial r)-s_m\,(\partial c_m^1/\partial a_m^2)]\,h_m =$$
$$[1+r/(1-t)]\sum_{m=1}^{M}(\partial y_m/\partial r)\ h_m+(1+r)\sum_{m=1}^{M}(\partial c_m^1/\partial r)-s_m\,(\partial c_m^1/\partial a_m^2)]\,h_m.$$

Here, since for each type m[10]

$$(19) \quad (\partial c_m^1/\partial r) = s_m\,(\partial c_m^1/\partial w_m)/(1+r)+(\partial c_m^1/\partial r)_{\bar{U}_m},$$

$$(20) \quad (\partial c_m^1/\partial a_m^1) = (\partial c_m^1/\partial w_m) = (1+r)\,(\partial c_m^1/\partial a_m^2),$$

we can derive the Sandmo-Drèze weighted average formula:

$$(21)\quad g'(z)=\frac{(1+r)\sum_{m=1}^{M}\left(\frac{\partial c_m^1}{\partial r}\right)_{\bar{U}_m}h_m+\left(1+\frac{r}{1-t}\right)\sum_{m=1}^{M}\left(\frac{\partial y_m}{\partial r}\right)h_m}{\sum_{m=1}^{M}\left(\frac{\partial c_m^1}{\partial r}\right)_{\bar{U}_m}h_m+\sum_{m=1}^{M}\left(\frac{\partial y_m}{\partial r}\right)h_m}$$

$$=\frac{(1+r)\overline{\left(\frac{\partial c_m^1}{\partial r}\right)_{\bar{U}_m}}+\left(1+\frac{r}{1-t}\right)\left(\frac{\partial \bar{y}_m}{\partial r}\right)}{\overline{\left(\frac{\partial c_m^1}{\partial r}\right)_{\bar{U}_m}}+\left(\frac{\partial \bar{y}_m}{\partial r}\right)},$$

where $\overline{(\partial c_m^1/\partial r)_{\bar{U}_m}}=\sum_{m=1}^{M}(\partial c_m^1/\partial r)_{\bar{U}_m}h_m$ and $\bar{y}_m=\sum_{m=1}^{M}y_m h_m$. Here, a bar denotes the variable per capita or the weighted average, the weights of which are the relative proportions (h_m) of people of each type m. Note that the discount rate determined by this formula can be interpreted in terms of the "opportunity cost principle".

On the other hand, from Eqs.(15) and (16), we can obtain

$$(22)\quad [-g'+(1+r)]\ [\partial c_m^1/\partial a_m^1)-(1+r)\ (\partial c_m^1/\partial a_m^2)]+g'-1-n=0.$$

Here, by Eq.(20), Eq.(22) becomes the "golden-rule" condition:

$$(23)\quad g'(z)-1=n.$$

Thus, we can confirm that even under the assumption of heterogeneous individuals, if government surplus can be redistributed by means of the ideal lump-sum tax or subsidy, the appropriate discount rate for public investment is the population growth rate, which is also equal to the Sandmo-Drèze weighted average formula.

2. *Uniform Poll Tax-Subsidy Policy*

Finally, let us now consider the uniform poll tax-subsidy policy:

$$(24)\quad a_m^1=a^1 \text{ and } a_m^2=a^2, \qquad m=1,\dots,M,$$

Noting that under this confined redistribution policy (i) (c_m^1) and (c_m^2) depend on r, a^1 and a^2, and (ii) r depends on z, a^1 and a^2, the first-order conditions for our second-best problem are given by the equations:

$$(25)\quad (1+r)\sum_{m=1}^{M}(U_m^2/\lambda)\ h_m=(-g'+1+r)\sum_{m=1}^{M}(\partial c_m^1/\partial a^1)\ h_m+g'-2-n-r,$$

$$(26)\quad (1+r)\sum_{m=1}^{M}(U_m^2/\lambda)\ h_m=(1+r)\ (-g'+1+r)\sum_{m=1}^{M}(\partial c_m^1/\partial a^2)-1-r,$$

$$(27)\qquad \sum_{m=1}^{M}(U_m^2/\lambda)\, s_m h_m = -g' \sum_{m=1}^{M}[(\partial y_m/\partial r)+(\partial c_m^1/\partial r)]\, h_m + (1+r)\sum_{m=1}^{M}(\partial c_m^1/\partial r)\, h_m + \sum_{m=1}^{M}[1+r/(1-t)]\ (\partial y_m/\partial r) h_m - \sum_{m=1}^{M} s_m h_m.$$

First of all, from Eqs.(25) and (26) we can obtain

$$(28)\qquad (-g'+1+r)\sum_{m=1}^{M}[(\partial c_m^1/\partial a^1)-(1+r)\ (\partial c_m^1/\partial a^2)]\, h_m + g' - 1 - n = 0.$$

Here, similarly to Eq.(20), since $(\partial c_m^1/\partial a^1)=(1+r)\ (\partial c_m^1/\partial a^2)$ for all m, we can derive the "golden-rule" condition, Eq.(23). Thus, we find that the population growth rate remains the proper discount rate for public investment even if the ideal lump-sum tax-subsidy policy because of its infeasibility in practice is confined to that of the uniform poll.

Next, multiplying both sides of Eq.(26) by $\bar{s}_m = \sum_{m=1}^{M} s_m h_m$ and then subtracting the resulting equation from Eq.(27), we can obtain

$$(29)\qquad -g'\ [\partial \bar{y}_m/\partial r)+(\partial \bar{c}_m^1/\partial r)-\bar{s}_m\ (\partial \bar{c}_m^1/\partial a^2)] + [1+r/(1-t\ )](\partial \bar{y}_m/\partial r) + (1+r)[(\partial \bar{c}_m^1/\partial r)-\bar{s}_m(\partial \bar{c}_m^1/\partial a^2)] = \sum_{m=1}^{M} s_m\ (U_m^2/\lambda)\ h_m - \bar{s}_m \cdot \sum_{m=1}^{M}(U_m^2/\lambda)\ h_m,$$

where $\bar{c}_m^1 = \sum_{m=1}^{M} c_m^1 h_m$. Noting that the right-hand side of this equation can be also written in terms of the covariance, $\mathrm{cov}(s_m,\ U_m^2/\lambda)$, we can now derive the following equation:

$$(30)\qquad g'(z) = \frac{(1+r)\left(\frac{\partial \bar{c}_m^1}{\partial r}\right)\Big|_{\text{comp.}} + \left(1+\frac{r}{1-t}\right)\left(\frac{\partial \bar{y}_m}{\partial r}\right)}{\left(\frac{\partial \bar{c}_m^1}{\partial r}\right)\Big|_{\text{comp.}} + \left(\frac{\partial \bar{y}_m}{\partial r}\right)} - \frac{\mathrm{cov}\ (s_m,\ U_m^2\ /\ \lambda)}{\left(\frac{\partial \bar{c}_m^1}{\partial r}\right)\Big|_{\text{comp.}} + \left(\frac{\partial \bar{y}_m}{\partial r}\right)},$$

where $(\partial \bar{c}_m^1/\partial r)\,|_{\text{comp.}} = (\partial \bar{c}_m^1/\partial r) - \bar{s}_m(\partial \bar{c}_m^1/\partial a^2)$, an interpretation of which will be given subsequently. We call this the *modified* Sandmo-Drèze weighted average formula.

Comparing Eq.(30) with Eq.(21), we can know how the items of opportunity cost per unit of public investment must be altered with change of the redistribution policy. First, the modified formula partly consists of a weighted average of the rate facing consumers and the tax-distorted rate used by firms [the first term in the right-hand side of Eq.(30)], while the weights are different from those in the original formula. We can see this as follows. Multiplying the Slutsky equation (19) by h_m and summing up, we obtain finally that

$$(31)\qquad \left(\frac{\partial \bar{c}_m^1}{\partial r}\right) - \bar{s}_m\left(\frac{\partial \bar{c}_m^1}{\partial a^2}\right) = \overline{\left(\frac{\partial c_m^1}{\partial r}\right)_{\bar{U}_m}} + \mathrm{cov}\left(s_m, \frac{\partial c_m^1}{\partial a^2}\right).$$

The left-hand side of the above equation is equal to $(\partial \bar{c}_m^1/\partial r)\,|_{\text{comp.}}$. According to Dixit and Sandmo [1], this can be thought of as a substitution effect on the first-period consumption *per capita* by purely formal analogy with the theory of individual choice. On the other hand, the covariance term in the right-hand side has a general interpretation as the aggregation error that would result if one tried to estimate the substitution effect *per capita* in terms of aggregates. Thus, the weights in the modified formula do not generally coincide with those in the original.

Second, adding to the weighted average term, the modified Sandmo-Drèze formula has an adjusted term containing the covariance term between s_m and U_m^2 [the second term in the right-hand side of Eq.(30)]. This covariance can be rewritten as follows:

$$(32) \quad \mathrm{cov}\,(s_m,\ U_m^2) = \bar{s}_m\,(R - \bar{U}_m^2),$$

where $R = \sum_{m=1}^{M} U_m^2(s_m h_m / \bar{s}_m)$ and $\bar{U}_m^2 = \sum_{m=1}^{M} U_m^2 h_m$. Let us note that R denotes the "distributional characteristic" introduced by Feldstein [2], which is the weighted average of marginal utilities of the second-period income (consumption), the weights being shares of the savings of people of each type. Thus, we find that the adjusted term represents an element of intragenerational lifetime equity resulting from change of the redistribution policy from ideal lump-sum tax-subsidy to the uniform poll tax-subsidy.

Substituting Eqs.(31) and (32) into Eq.(30), the modified Sandmo-Drèze formula can be also represented by the following equation:

$$(33) \quad g'(z) = \frac{(1+r)\left[\overline{\left(\frac{\partial c_m^1}{\partial r}\right)_{\bar{U}_m}} + \mathrm{cov}\left(s_m,\ \frac{\partial c_m^1}{\partial a^2}\right)\right] + \left(1 + \frac{r}{1-t}\right)\left(\frac{\partial \bar{y}_m}{\partial r}\right)}{\left[\overline{\left(\frac{\partial c_m^1}{\partial r}\right)_{\bar{U}_m}} + \mathrm{cov}\left(s_m,\ \frac{\partial c_m^1}{\partial a^2}\right)\right] + \left(\frac{\partial \bar{y}_m}{\partial r}\right)} + \frac{(\bar{s}_m/\lambda)\,(\bar{U}_m^2 - R)}{\left[\overline{\left(\frac{\partial c_m^1}{\partial r}\right)_{U_m}} + \mathrm{cov}\left(s_m,\ \frac{\partial c_m^1}{\partial a^2}\right)\right] + \left(\frac{\partial \bar{y}_m}{\partial r}\right)}.$$

It follows from Eq.(21) that, under the ideal lump-sum tax-subsidy policy, the market equilibrium rate of interest in a (non) tax-distorted economy with (without) the corporation income tax is always smaller than (equal to) the population gorwth rate. However, we cannot derive such evident relations from Eq.(33).

Summarizing now the above analytical results, we can conclude that, while the *items* of opportunity cost per unit in public investment under the uniform poll tax-subsidy policy differ from those under the ideal lump-sum tax-subsidy, the opportunity cost per unit *itself* is the same, being equal to the population growth factor.[11)]

Footnotes

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1. These results are similar to those obtained by Pestieau [6]. However, there are some important differences in the formulation on public production, labor, taxation, social time preference, and fiscal policy between Pestieau's model and mine. See Yoshida [10].
2. See, for example, Hamada [3] and Ordover and Phelps [5].
3. Nevertheless, this problem has not been treated in most of the literature on this field

except by Pestieau and Possen [7] and Okuno and Yakita [4]. The former showed that, employing the Kaldor two-class approach, the social discount rate for public investment in a Ramsey style model of optimum growth is unlikely to be equal to the social rate of time preference, but instead either lower or higher depending on whether public investment is redistributive or not. On the other hand, the latter showed that, employing the Mirrlees ability approach and taking the welfare of the least-favored individual as a social objective (the max-min), the appropriate discount rate is equal to the social rate of time preference, and that it is lower than that for private investment.

4. In order to focus on the problem of public investment criteria, it is now assumed that all people have no preference as to leisure and devote the same amount of time to earning goods, and that their productivities at work differ from person to person according to their abilities.
5. It is this formulation on government policies that distinguishes the Sandmo-Drèze type's second-best problem from that of Diamond-Mirrlees.
6. The second-best problem under the assumption of homogeneous individuals in [10] is a special case: $c^1_m=c^1$, $c^2_m=c^2$, $w_m=w$, $y_m=y$, $s_m=b$, $a^1_m=a^1$, $a^2_m=a^2$, $f_m=f$, and $U_m=U$, $m=1 ,\dots, M$.
7. It is now assumed that interest on capital is not deductible for tax purposes as in Sandmo and Drèze [9].
8. Eqs. (9) and (12) correspond to Case (ii) in Theorem 1 of [10] and reflect a characteristic of the two-period overlapping generations economy that any *pure* consumption loan among all people in *different* generations is impossible, while among people of different types in the *same* generation it is possible. For example, let us now consider a case of two types of individuals: $M=2$. Then, Eq.(13) becomes $s_1h_1+s_2h_2=b$. Thus, we can suppose the following five possible cases in an equilibrium (see Fig. 1).

   Case 1 (2): $s_1h_1=b$ and $s_2h_2=0$ ($s_1h_1=0$ and $s_2h_2=b$).

   All public debts are held by individuals of type 1(2) and there are no pure consumption loans between them.

   Case 3 (4): $s_1h_1>b$ and $s_2h_2<0$ ($s_1h_1<0$ and $s_2h_2>b$).

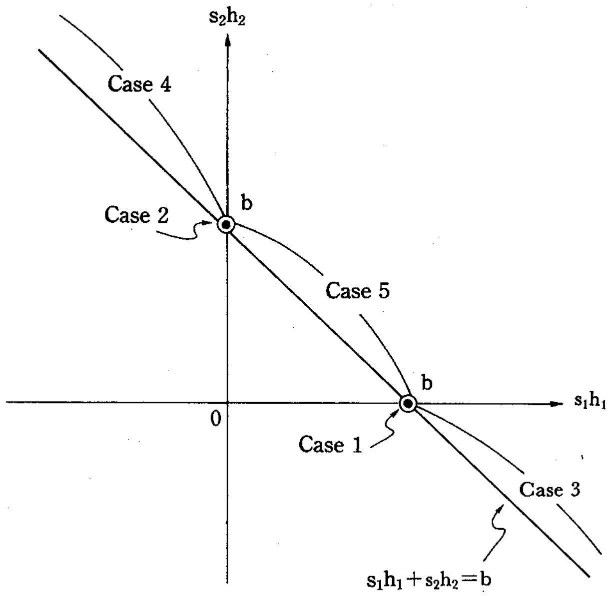
   In these cases, type 1(2) individuals hold all public debts and lend to type 2(1) individuals.

   Case 5: $0<s_1h_1<b$ and $0<s_2h_2<b$.

   The public debts are shared by both types of individuals and there are no pure consumption loans between them.

   However, it is *indeterminate* which of the five cases occurs in the second-best optimum. We can also confirm easily that Case (i) $r=n$ in Theorem 1 is one of the steady state equilibria in the private sector.
9. We could also formulate our second-best problem as the maximization of the objective function (2) subject to the constraints (5)—(11) with respect to endogenous variables as well as to policy variables. We thank Professor Yoshihiko Otani for this comment.
10. See Eqs.(15) and (16) in Sandmo and Drèze [9, p.399].
11. It goes without saying that if all government surplus is redistributed only to the *older* generation for some reason under both ideal lump-sum and uniform poll tax-subsidy policies, the latter part of this conclusion no longer holds.

Fig. 1



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