A RECONSIDERATION OF THE NEOCLASSICAL THEORY OF ECONOMIC GROWTH*

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The development of economic theory for past three decades has been extremely remarkable especially in the study of the static working of decentralized system of resource allocation and of the balanced growth path together with dynamic asymptotic property towards it. Various types of the model of economic growth which have been developed so far can be classified into the following three categories. The first is the multi-sectoral theory of economic growth. This includes both of the dynamic Leontief model and the von Neumann growth model. These two models are common in that the description of the rational behavior of individual units (e.g. household and firm) is neglected. It seems to me that the multi-sectoral dynamic model belongs to the field of "economic engineering", provided such a nomenclature is permitted. The second is the neoclassical theory of economic growth. In this theory the existence of the business corporation as an economic unit institutionally different from the household is not taken into account. The third consists of the Keynesian theory of economic growth and the dynamic general equilibrium theory.[1] In this category it is supposed that the household and the firm are two different units and the national economy is an interdependent organic entity in which each unit is closely connected with one another through markets. In the Keynesian theory of economic growth the direct analysis of the behavior of individual units is not carried out and the model consists of aggregative behavioristic functions and equations specifying the price response in markets. On the other hand in the dynamic general equilibrium theory the intertemporal optimization behavior of the household and the firm is examined in terms of calculus of variations (or Pontryagin's control theory). In this analysis such individual behavioristic functions as consumption function, investment function, labor demand function and product supply function are derived and on the basis of this microeconomic consideration the procedure of aggregation is conducted.

In the present paper we shall reconsider the logical structure of the neoclassical theory of economic growth with special reference to its microeconomic foundation to which Uzawa [14], [17] and Cass=Yaari [3] have recently paid attention. Before proceeding to a mathematical analysis of the model, it is useful to elucidate some characteristics of the framework of the neoclassical world.[2] In what follows we are concerned with the so-called "representative" individual who reflects an average figure of many individuals when the word individual is employed. Further we introduce a simplifying assumption that there is not the government sector. As has been described above, the household and the firm are two different aspects of the same individual and an individual is an entrepreneur as well as a worker in the neoclassical theory. An individual is engaged in productive activity utilizing the services of capital stock and his own labor and makes decision regarding division of products into consumption and capital formation which is optimal in view of his intertemporal preference ordering. So that the savings function and the investment function are completely identical. Although the view that the Keynesian growth theory presupposes the production function with fixed factor proportion and, on the other hand, the neoclassical growth theory presupposes the production function with variable factor proportion prevails widely, this is an inadequate view.

The technological property of the production function is not necessarily an essential point for dividing the Keynesian theory from the neoclassical theory. In fact J. L. Stein and H. Rose have recently developed the Keynesian growth model which incorporates the substitutable production function.[3] In the neoclassical world the individual produces output which he needs for consumption and capital accumulation for himself and does not rely upon others for input of labor service. So that the neoclassical world is a mere set of individuals who are autarkic (Robinson Crusoe!). There are no markets of products and labor services. There is no financial borrowing and lending, since individuals are not divided into two groups, namely, a group who saves (lends) and another group who invests (borrows). Under our assumption it follows that there are not prices, wage rate and rates of interest. Thus, strictly speaking, we can not study the modern capitalist market economy by means of the neoclassical theory of economic growth (of course

we can not study the socialist economy either.).
In other words, the decentralized market economy can be described in terms of the static general equilibrium theory concerning the static aspect and can be described in terms of both of the dynamic general equilibrium theory and the Keynesian theory of economic growth concerning the dynamic aspect, provided we ignore the problems which emerge outside the market system (e.g. the necessity of the supply of public goods and accumulation of social overhead capital, externality, pollution and environmental disruption).

Let us now construct a rigorous neoclassical dynamic model paying attention to its microeconomic foundation. For the sake of mathematical simplification it is assumed that there is only one kind of product and it can be used for both of consumption and investment. And it is assumed that each individual possesses a unit quantity of labor service and use it as an input to the productive activity. An individual divides his output produced into consumption and investment so as to maximize the utility integral

$$(1) \quad \int_0^{\infty} u_i(c_i)\exp(-\beta_i t)dt$$

where c_i is the consumption of the i-th individual, $u_i(\cdot)$ is the utility function which satisfies

$$(2) \quad \begin{array}{l} u_i(0)=0, \ u_i(\infty)=\infty, \ u_i' \geqq 0 \\ u_i''<0, \ u_i'(0)=\infty, \ u_i'(\infty)=0 \end{array}$$

The configuration of $u_i(\cdot)$ is kept to be invariant over time and β_i is the subjective rate of discount of the i-th individual. The budget restraint of the i-th individual is written as

$$(3)\quad c_i+Dk_i=q_i$$

where k_i is the amount of capital held by him and q_i is the output produced. The symbol D is an operator that means the derivative with respect to time. The relationship between q_i and k_i is described by the production function

$$(4)\quad q_i=f_i(k_i)$$

where $f_i(\cdot)$ has the following properties:

$$(5)\quad \begin{array}{l} f_i(0)=0,\ f_i(\infty)=\infty,\ f_i'\geqq 0, \\ f_i''<0,\ f_i'(0)=\infty,\ f_i'(\infty)=0 \end{array}$$

The dynamic optimization problem described above is a problem of the calculus of variations in which a state variable is k_i and a control variable is c_i.
By Pontryagin's control theory c_i must satisfy

$$(6)\quad \lambda_i=u_i'(c_i)$$

which is a necessary condition for maximum of the Hamiltonian function

$$(7)\quad H_i=\exp(-\beta_i t)[u_i(c_i)+\lambda_i Dk_i]$$

where λ_i is an auxiliary variable whose economic implication is the imputed price of capital on the stationary equilibrium. λ_i must satisfy the following differential equation by the maximum principle:

$$(8) \quad D\lambda_i = -[f_i'(k_i) - \beta_i]\lambda_i$$

where $\lambda_i \not\equiv 0$ at each point in time. We can depict a phase diagram of the solution from (3), (4), (6) and (8). It is typically illustrated in Fig. 1.

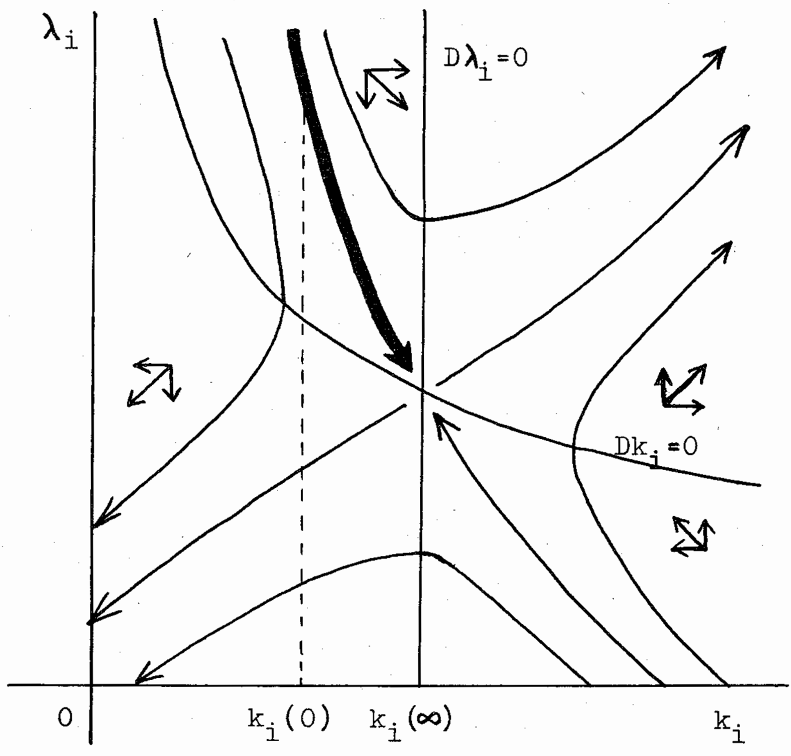


Fig. 1. Geometric illustration of the solution of the system of simultaneous differential equations, (3) and (8)

In Fig. 1 a unique optimal trajectory starting with initial value of the stock of capital, $k_i(0)$, that satisfies the transversality condition

$$(9)\quad \lim_{t\to\infty} \exp(-\beta_i t)\lambda_i = \lim_{t\to\infty} \exp(-\beta_i t)u_i'(c_i) = 0$$

and is economically meaningful is indicated by a heavy arrowed curve. Converting the differential equation (8) into

$$(10)\quad Dc_i = -\frac{u_i'}{u_i''}[f_i'(k_i) - \beta_i]$$

known as the Euler equation we obtain Fig. 2 in which c_i is measured along the vertical axis instead of λ_i.[4]

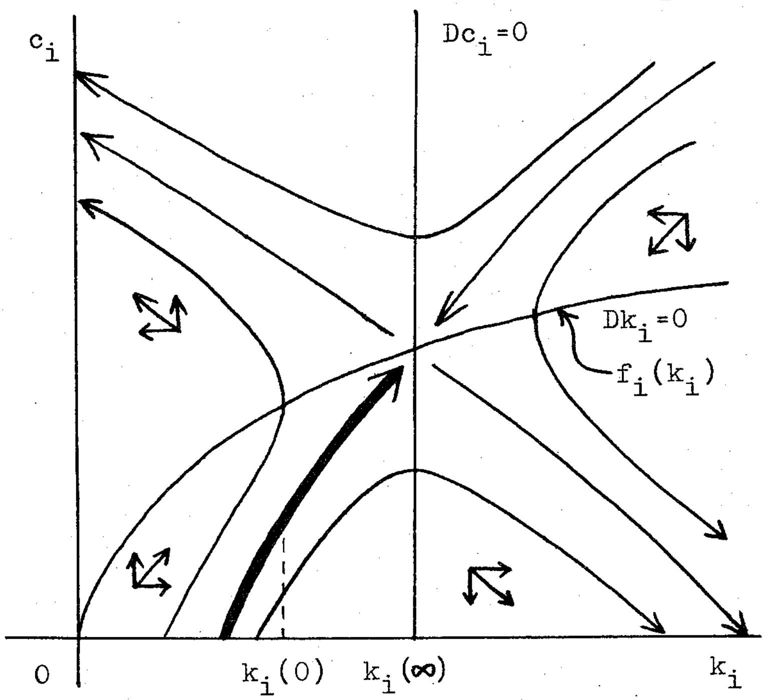


Fig. 2

Since the economic implications of the direction of movement of k_i and c_i(or λ_i) in the phase diagram are clear, the reader will not require the further explanation.

The long-run stationary equilibrium is an unstable saddle point[5] and the value of auxiliary variable (the marginal utility of consumption) is equal to the imputed price (or shadow price) of the capital asset

$$(11)\ \Psi_i = \int_t^{\infty} \Delta u_s^i \exp[-\beta_i(s-t)]ds, \quad t \leqq s \leqq \infty$$

on it as has been pointed out already. In (11) Ψ_i is the imputed price of capital of the i-th individual at time t and Δu_s^i is the increment of utility at time s due to the increase of a unit quantity of capital at time t.

By straightforward calculation we get

$$(12)\ \Delta u_s^i = \beta_i u_i'(c_i)$$

since $Dk_i = 0$ on the stationary equilibrium. Therefore

$$(13)\ \Psi_i = u_i'(c_i)$$

since c_i takes a constant value on the stationary equilibrium, thus resulting in $\lambda_i = \Psi_i$.

The consumption function of the i-th individual is a function of his rate of discount β_i

$$(14)\ c_i = c_i(\beta_i)$$

And the time shape of capital accumulation also depends upon β_i

(15) $k_i = k_i(\beta_i)$

Thus the supply function of the product is written as

(16) $q_i = f_i[k_i(\beta_i)] = q_i(\beta_i)$

Hence the investment function or savings function is described as

(17) $Dk_i = q_i(\beta_i) - c_i(\beta_i) = \varphi_i(\beta_i)$

Now we investigate the effects of a change in the rate of discount. In Fig. 2 a singular curve $Dc_i = 0$ shifts leftward if β_i increases. A new optimal path is illustrated as a broken arrowed curve in Fig. 3.
The effects of a change in β_i on the level of consumption and capital stock in the stationary equilibrium are shown as

(18) $\partial c_i(\infty)/\partial\beta_i < 0$

(19) $\partial k_i(\infty)/\partial\beta_i < 0$

respectively. Algebraically (18) and (19) are obtained by solving the simultaneous equations

$$(20)\quad \begin{bmatrix} 1 & -f_i'[k_i(\infty)] \\ 0 & f_i''[k_i(\infty)] \end{bmatrix} \begin{bmatrix} \partial c_i(\infty)/\partial\beta_i \\ \partial k_i(\infty)/\partial\beta_i \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

which is obtained by differentiating the simultaneous equations, $Dk_i = 0$ and $Dc_i = 0$.

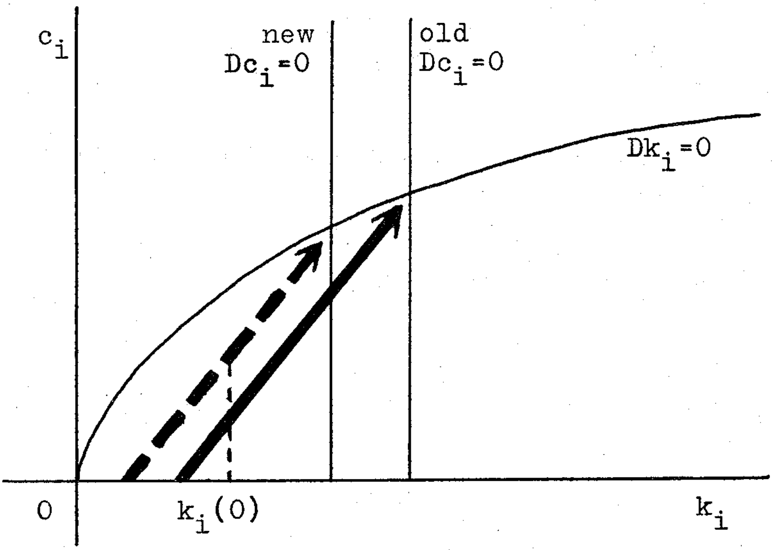


Fig. 3

Formally we can get the aggregative behavioristic functions by summing up (14), (16), (17) for each individual. Namely

$$(21)\quad c=\Sigma_{i=1}^{N} c_i(\beta_i)=c(\beta_1,\beta_2,\cdots,\beta_N)$$

$$(22)\quad q=\Sigma_i q_i(\beta_i)=q(\beta_1,\cdots,\beta_N)$$

$$(23)\quad Dk=\Sigma_i\varphi_i(\beta_i)=\varphi(\beta_1,\cdots,\beta_N)$$

where N is the population (number of individuals) at time t which is given by

$$(24)\quad N=N_0\exp(nt),\quad n\geqq 0$$

and of course c, q and Dk satisfy

$$(25)\quad q=c+Dk$$

at every moment in time. Thus it follows that the necessity of an aggregative production function is not essential to the neoclassical theory of economic growth.

As is well known the neoclassical theory of economic growth is divided into two fields: the theory of existence and stability of the balanced growth path as the positive analysis and the theory of stabilization and optimization policy as the normative analysis. I suppose that the reader has already noticed that the microeconomic analysis which has been described so far is similar to the ordinary optimum growth theory. My view on this point is as follows. The reason why the neoclassical growth theory is divided into positive theory and optimization theory is due to the method of study that the microeconomic foundation has been ignored. So that, strictly speaking, there is not the room of the optimization policy by the government in the neoclassical growth theory. This fact is similar to the fact that a competitive equilibrium is optimal in the sense of Pareto by the basic theorem of welfare economics in the static world. The reader, however, must not have the view that the ordinary neoclassical growth theory and optimum growth theory are insignificant. The rigorous neoclassical growth theory described in this paper is to the ordinary one what the dynamic general equilibrium theory is to the Keynesian theory of economic growth. Therefore the well known critique to the use of an aggregative production function by the Cambridge school applys not only to the ordinary neoclassical theory of growth but also to the Keynesian theory of growth.

Finally let us summarize the difference between the neoclassical growth theory and the Keynesian growth theory that has been developed by R. F. Harrod, E. D. Dormar, N. Kaldor, J. Robinson, A. W. Phillips, A. R. Bergstrom, J. Williamson, A. C. Enthoven, J. L. Stein, H. Rose, H. Uzawa and many other theorists.[6]

1. In the neoclassical growth theory it is not recognized that there are business firms as independent units different from households, while in the Keynesian theory such a fact which is the most important characteristic in the modern capitalist economy is fully recognized.
2. As consequences of this assumption there are not markets of products, labor services, and financial assets (provided there is not the government) in the neoclassical theory. So that there are not prices such as prices of products, wage rate and rates of interest and there are not disequilibrium of markets.

Namely each individual is in the autarkic state and the national economy is a mere set of individuals and does not form an organic entity characterized by interdependent relationships between each individual through transactions in markets.

3. It is not correct that there is not the investment function in the neoclassical theory. Correctly speaking, the form of the investment function and that of savings function are exactly identical. On the other hand, in the Keynesian theory the savings function of the household and the investment function of the firm take the different form.
4. The technological property of the production process characterized by the production function is not essential to the difference between the neoclassical theory and the Keynesian theory.
5. The view that the neoclassical theory of growth presupposes the price flexibility, especially concerning the factor price, is not correct. And the downward rigidity of money wage rate is not necessarily indispensable to the Keynesian system.
6. It is necessary to introduce the government sector and the national debt for the construction of the neoclassical theory of monetary growth, since there are not the primary securities issued by the private sector.
7. The crucial reason why the balanced growth path is stable grobally in the ordinary neoclassical growth theory is inexistence of the independent investment function. The substitutability between factors of

production and the price flexibility do not necessarily assure us the stability of the long-run dynamic equilibrium as has been clarified already by recent studies.

8. The view that microeconomic foundation concerning the behavior of individual units is ignored in both of the neoclassical growth theory and the Keynesian macrodynamic theory is misleading. It seems to me that the traditional methodology after Keynes that economic theories can be divided into two fields, namely, the macroeconomic theory represented by the Keynesian theory and the microeconomic theory represented by the general equilibrium theory is misleading.

NOTES

* This paper is a part of performances that have been obtained so far of my study in which the logical clarification of the relationships among various economic theories is intended. I have benefited from Prof. Uzawa's lecture and from discussions with Prof. Hayakawa, Prof. Kobayashi, Prof. Shirai, Mr. Sakai and Mr. Matsumoto. Needless to say remaining errors, if any, are due to me alone.

1. The dynamic general equilibrium theory was originated by Prof. Uzawa. See Uzawa [15], [16], [18].

The skelton of my own formulation will be exhibited in APPENDIX I. Kato [7] deals with the mathematical aspects of the model, the relations with the static general equilibrium theory—the ordinary general equilibrium theory—and other related topics in detail.

2. The word "neoclassical" is used in various senses in the literature. I, however, confine uses of the word "neoclassical" to the so-called neoclassical theory of economic growth in this paper. I never accept a view that the static general equilibrium theory after Hicks belongs to the neoclassical economics.
3. Stein [12], Rose [10]
4. The Euler equation (10) is equivalent to the following optimality condition derived by Uzawa.

$$\delta^i(c_i, Dc_i/c_i) = \nu_i$$

where

$$\delta^i(c_i, Dc_i/c_i) = \beta_i - \frac{u_i''(c_i)c_i}{u_i'(c_i)} \frac{Dc_i}{c_i}$$

is the Fisherian function representing the rate of time preference and

$$\nu_i = f_i'(k_i)$$

is the instantaneous marginal rate of transformation. Now we can obtain

$$Dq_i/q_i = f_i'(k_i)(1-x_i)$$

where $q_i = f_i(k_i)$ and $x_i = c_i/q_i$ (average propensity to consume) from the state equation

$$Dk_i = f_i(k_i) - c_i$$

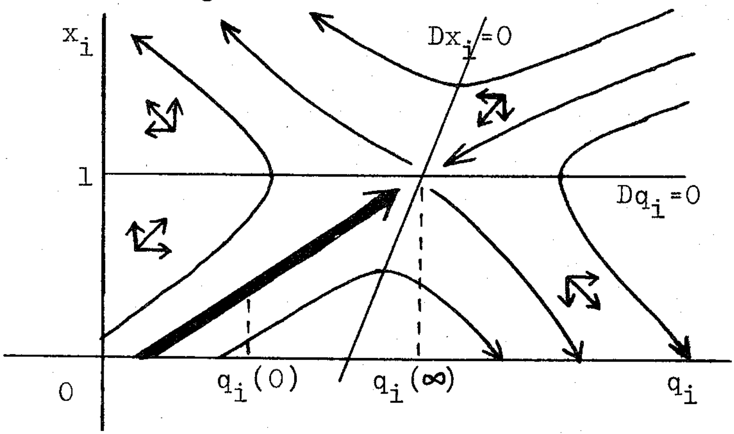
together with

$$Dx_i/x_i = Dc_i/c_i - f_i'(k_i)(1-x_i)$$

where

$$\delta^i(Dc_i/c_i) = f_i'(k_i)$$

under an assumption that the intertemporal preference ordering is homothetic. The solution to the simultaneous differential equations derived above is illustrated in the following phase diagram in which x_i is measured along the vertical axis, while q_i is measured along the horizontal axis.



A unique optimum path is indicated by a heavy arrowed curve. (See Uzawa [17])

5. We have the following simultaneous differential equations by linear approximation of (3) and (10) in the neighborhood of the stationary equilibrium point.

$$\begin{bmatrix} Dk_i \\ Dc_i \end{bmatrix} = \begin{bmatrix} f_i' & -1 \\ -\dfrac{u_i'}{u_i''} f_i'' & (\beta_i - f_i')\left(1 - \dfrac{u_i' u_i'''}{u_i''^2}\right) \end{bmatrix} \begin{bmatrix} k_i - k_i(\infty) \\ c_i - c_i(\infty) \end{bmatrix}$$

The solution to this system can be written as

$$\begin{bmatrix} k_i \\ c_i \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \exp(\theta_1 t) \\ \exp(\theta_2 t) \end{bmatrix} + \begin{bmatrix} k_i(\infty) \\ c_i(\infty) \end{bmatrix}$$

where θ_1 and θ_2 are eigenvalues of the coefficient matrix and a_{ij} is determined by the coefficient matrix and the initial conditions of k_i and c_i.
In the vicinity of the stationary equilibrium point we can approximately regard $\beta_i - f_i'$ as zero, so that the characteristic equation reduces to

$$\theta^2 - f_i'\theta - \frac{u_i'}{u_i''} f_i'' = 0$$

The solution to this is

$$\theta_1, \theta_2 = \frac{1}{2}\left[f_i' \pm \sqrt{f_i'^2 + 4\frac{u_i'}{u_i''} f_i''}\right]$$

Obviously θ_1, θ_2 are real numbers and they take the opposite sign each other, that is $\theta_1\theta_2 < 0$, since

$$\sqrt{f_i'^2 + 4\frac{u_i'}{u_i''}f_i''} > f_i'$$

Thus the stationary equilibrium point $(k_i(\infty),\ c_i(\infty))$ is a saddle point.

6. Phillips [9], Bergstrom [1], [2], Williamson [22], Enthoven [4], Uzawa [19], [20], [21].

APPENDIX I

OUTLINE OF THE DYNAMIC GENERAL EQUILIBRIUM THEORY

In what follows I shall briefly explain the dynamic general equilibrium theory drawing upon my unpublished manuscript (Kato [7]). To begin with, let us fix symbols used in the model.

1. HOUSEHOLD

c_i = real consumption expenditure of the i-th household

a_i = number of stock certificates possessed by the i-th household

b_i = real stock holdings of the i-th household

β = rate of discount of the future utility (const.)

2. FIRM

Q_j = real output of gross products of the j-th firm

K_j = amount of the fixed resources of production (capital) held by the j-th firm

N_j^D = demand for the variable resources of production (labor) of the j-th firm

I_j = real gross investment of the j-th firm

δ = rate of depreciation (const.)

r_j = rate of return of the j-th firm

3. PRICES

p = price of product

w = money wage rate

p^s = nominal par of a stock certificate (const.)

α = yield of the stock (const.)

4. AGGREGATIVE VARIABLES

C = aggregate consumption

I = aggregate gross investment

Q = aggregate gross output

N^D = aggregate demand for labor service

N^S = aggregate labor supply (number of households) (const.)

a = total number of stock certificates possessed by the whole household

K = aggregate stock of capital

n = number of firms (const.)

Important assumptions in our system are as follows.

1. There is not uncertenty concerning the price expectation.[7]

2. The perfect competition prevails.
3. There is not money.
4. There is not externality in the productive and consumptive activities.
5. The product is single and labor service is homogeneous.
6. Each variable is continuous with respect to time.
7. Each unit has an infinite time horizon.
8. There is not a stock market.
9. The firm does not issue debts. Namely the stock is an only financial asset.
10. Stocks are issued in the form of par issue.

Our dynamic competitive equilibrium, which is indicated by superscript "*", is defined as follows.

1. HOUSEHOLD

$$\int_0^{\infty} u^i(c_i^*, b_i)\exp(-\beta t)dt$$

$$=\max_{C_i>0} \int_0^{\infty} u^i(c_i, b_i)\exp(-\beta t)dt$$

$$p^* c_i + p^S Da_i = w^* + \alpha p^S a_i, \text{ for each } i$$

2. FIRM

The first step optimization

$$p^*[F^j(K_j, N_j^{D*}) - C_j(I_j)] - w^* N_j^{D*} - p^* I_j$$

$$=\max_{N_j^D>0}[p^*[F^j(K_j, N_j^D) - C_j(I_j)] - w^* N_j^D - p^* I_j]$$

for arbitrary K_j and I_j and for each j

The second step optimization

$$\int_0^\infty [p^*Q_j^* - w^*N_j^{D*} - p^*I_j^*]\exp(-\alpha t)dt$$

$$= \max_{I_j \geq 0} \int_0^\infty [p^*Q - w^*N_j^{D*} - p^*I_j]\exp(-\alpha t)dt$$

$$Q_j = F^j(K_j, N_j^D) - C_j(I_j),\ DK_j = I_j - \delta K_j,\ \text{for each } j$$

3. MARKETS

$C^* + I^* = Q^*$

$N^{D*} = N^S$

4. PRICES

$p^* > 0$

$w^* > 0$

We would briefly explain our system defined above without referring to the mathematical discussion in detail.

1. HOUSEHOLD[8]

In our dynamic model of the household, we are concerned with choosing an optimal consumption plan so as to maximize the utility integral

$$(1)\quad \int_0^\infty u^i(c_i, b_i)\exp(-\beta t)dt$$

where $u_i(\cdot)$ is the utility function of flow c_i and stock b_i which satisfies

$$(2)\quad u^i \geqq 0,\ u^i(0,0)=0,\ u^i(\infty,\infty)=\infty,\ u^i(0,b_i)=0,$$
$$u^i(c_i,0)\geqq 0,\ u^i_1\geqq 0,\ u^i_2\geqq 0,\ u^i_{11}<0,\ u^i_{22}<0,$$
$$u^i_{12}=u^i_{21}<0,\ \begin{vmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{vmatrix}>0$$

among all feasible consumption plans which satisfy the budget constraint

$$(3)\quad p^sDa_i=w+\alpha p^sa_i-pc_i$$

or in real expression

$$(4)\quad Db_i=w/p+(\alpha-Dp/p)b_i-c_i$$

where $b_i = p^sa_i/p$. It is assumed that a household always supplies a unit quantity of labor service and the savings of household is fulfilled by the purchase of stocks alone. Capital gain or loss do not take place, since there is not the stock market. By variational analysis we can obtain the consumption function

$$(5)\quad c_i=c_i(p,w,\alpha,\beta)$$

under an assumption that β is approximately equal to i.

2. FIRM

The optimization behavior of the firm consists of two steps. The first step is determining an optimal amount of employment in such a way that the net cash

flow at each point in time is maximized for arbitrary amount of fixed capital and investment project.
The necessary condition of this short-run optimization is that the marginal productivity of labor is equal to real wage rate. Hence we obtain the tentative labor demand function.

$$(6)\quad N_j^D=\Phi_j(K_j, w/p)$$

The second step of optimization is choosing an optimal investment plan so as to maximize the sum of discounted present values of the expected net cash flow

$$(7)\quad \int_0^\infty [pQ_j-wN_j^D-pI_j]\exp(-\alpha t)dt$$

in which gross output Q_j is represented by

$$(8)\quad Q_j=F^j(K_j, N_j^D)-C_j(I_j)$$

where $F^j(\cdot)$ is the linear homogeneous production function[9] with diminishing marginal productivity and $C_j(\cdot)$ is the adjustment cost function that represents the short-run fixity of capital carrying the property

$$(9)\quad \begin{array}{l} C_j(0)=0,\ \ C_j(I_j)\geqq 0,\ \ C_j(\infty)=\infty, \\ C_j'\geqq 0,\ \ C_j''>0^{10} \end{array}$$

among all feasible investment plans which satisfy

(10) $DK_j = I_j - \delta K_j$

(11) $I_j \geqq 0$

(11) is known as the irreversibility of investment. By variational reasoning we can easily confirm that there exists a unique optimal investment plan and it hinges upon the price of product, money wage rate, yield of equities and rate of depreciation, namely

(12) $I_j = I_j(p, w, \alpha, \delta)$[11]

We can know the time shape of capital accumulation by solving a linear differential equation (10). So that we can write K_j as

(13) $K_j = K_j(p, w, \alpha, \delta)$

Thus we obtain the labor demand function

(14) $N_j^D = N_j^D(p, w, \alpha, \delta)$

substituting (13) into (6). Finally we obtain the product supply function

(15) $Q_j = Q_j(p, w, \alpha, \delta)$

Now it is obvious that the rate of return r_j

(16) $r_j = \dfrac{pQ_j - wN_j^D}{pK_j}$

also depends upon p, w, α and δ.

3. MARKETS

We can obtain the aggregative behavioristic functions on the basis of above results. Namely

$$(17)\quad C=\Sigma_{i=1}^{N^S} c_i(p,w,\alpha,\beta)=C(p,w,\alpha,\beta)$$

$$(18)\quad I=\Sigma_{j=1}^{n} I_j(p,w,\alpha,\delta)=I(p,w,\alpha,\delta)$$

$$(19)\quad Q=\Sigma_j Q_j(p,w,\alpha,\delta)=Q(p,w,\alpha,\delta)$$

$$(20)\quad N^D=\Sigma_j N_j^D(p,w,\alpha,\delta)=N^D(p,w,\alpha,\delta)$$

Thus equilibrium of markets is represented by

$$(21)\quad \begin{aligned} &C(p,w,\alpha,\beta)+I(p,w,\alpha,\delta)=Q(p,w,\alpha,\delta) \\ &N^D(p,w,\alpha,\delta)=N^S \end{aligned}$$

4. PRICES

In the dynamic competitive equilibrium the ratio of prices w/p (so-called "value") is determined, since the Walras' law

$$(22)\quad p[C+I-Q]+w[N^D-N^S]=0$$

holds.

The distributive aspect of our dynamic system is exhibited in TABLE 1.

The reader will presumably be able to understand that our dynamic general equilibrium system reduces to

TABLE 1.

<table>
<tr><td colspan="5">Gross National Product pQ</td></tr>
<tr><td rowspan="4">Wage Cost
wN^D</td><td colspan="4">Gross Profit $pQ-wN^D$</td></tr>
<tr><td colspan="3">Net Profit $pQ-wN^D-p\delta K$</td><td>Depreciation
$p\delta K$</td></tr>
<tr><td colspan="2">Dividend</td><td>Internal Reserve</td><td>Depreciation Allowance</td></tr>
<tr><td>Net Cash Flow
$pQ-wN^D-pI$</td><td>Issue of New Stocks</td><td colspan="2">Gross Savings of Firms</td></tr>
<tr><td colspan="2">Consumption pC</td><td colspan="3">Gross Investment pI</td></tr>
<tr><td colspan="2"></td><td colspan="2">Net Investment pDK</td><td>Replacement Investment</td></tr>
<tr><td colspan="4">Net National Product</td><td rowspan="3"></td></tr>
<tr><td>Wage Income
wN^S</td><td colspan="2">Dividend Income $\alpha p^s a$</td><td rowspan="2"></td></tr>
<tr><td colspan="3">Income of Households</td></tr>
<tr><td colspan="2">Consumption pC</td><td>Savings of Households
$p^s Da$</td><td>Net Savings of Firms</td><td>Depreciation Allowance</td></tr>
<tr><td colspan="2">Consumption</td><td colspan="3">Gross Savings</td></tr>
</table>

the Arrow = Debreu model of static general equilibrium theory replacing variables by vector notation by assuming as if there were not future in the horizon of each unit. We can give the rigorous microeconomic foundation to the Keynesian theory of economic growth by the procedure that has been described heretofore.

NOTES

7. If we assume uncertainty in the true sense of the word, we can not construct any dynamic theory and the actual economy will be thrown into extreme confusion. It appears to me that households and entrepreneurs behave forming some expectation concerning the future in the actual world. So that it is not wise to evade to construct the dynamic theory in view of uncertainty.
8. Our dynamic theory of the household is described in Kato [6] in detail.
9. The assumption of linear homogeneity is not indispensable. There exists a unique optimal solution not only in the case of decreasing returns to scale but in the case of increasing returns to scale so long as increase of costs of adjustment due to expansion of scale dominates the effect of increasing returns. See Treadway [13].
10. The formulation of adjustment cost function of this kind is seen in Gould [5], Treadway [13], Sakai [11]. As representative examples of other type of formulation Lucas [8] and Uzawa [15], [16], [18], [21] are remarkable.

11. In our model optimal amount of investment is constant.

APPENDIX II

We can classify the contemporary economic theory, whose major concern is confined to the study of market mechanism, as follows.

TABLE 2

Criterion of the Classification	Statics	Dynamics
A system that takes no account of the firm as the unit different from the household		Neoclassical Theory of Economic Growth
A system that takes account of the firm as the unit different from the household	Static General Equilibrium Theory	Dynamic General Equilibrium Theory
		Keynesian Theory of Economic Growth
A system that does not formulate the rational behavior of the unit	Static Inter-industrial Relations Theory	Dynamic Inter-industrial Relations Theory
		von Neumann Growth Theory

The reader will easily be able to understand the position of the neoclassical theory of economic growth in the contemporary economics by this table.

REFERENCES

[1] BERGSTROM, A.R., "A Model of Technical Progress, the Production Function and Cyclical Growth", ECONOMICA, November, 1962.

[2] ――――――――, "Monetary Phenomena and Economic Growth: A Synthesis of Neoclassical and Keynesian Theories", ECONOMIC STUDIES QUARTERLY, December, 1966.

[3] CASS, D. and YAARI, M.E., "Individual Saving, Aggregate Capital Accumulation, and Efficient Growth", in K.SHELL (ed.), "Essays on the Theory of Optimal Economic Growth", THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY PRESS, 1967.

[4] ENTHOVEN, A.C., "A Neo-classical Model of Money, Debt and Economic Growth", in J.G.GURLEY and E.S.SHAW, "Money in a Theory of Finance", THE BROOKINGS INSTITUTION, 1960, pp. 303-359.

[5] GOULD, J.P., "Adjustment Costs in the Theory of Investment of the Firm", REVIEW OF ECONOMIC STUDIES, 1968.

[6] KATO, M., "Dynamic Utility Maximization and the Derivation of Consumption Function", THE ECONOMIC STUDIES (Hokkaido University), 1975.

[7] ――――, "On the Procedure for Dynamization of General Equilibrium Theory in terms of Calculus of Variations", Unpublished Manuscript.

[8] LUCAS, R.E., "Adjustment Costs and the Theory of Supply", JOURNAL OF POLITICAL ECONOMY, August, 1967.

[9] PHILLIPS, A.W., "A Simple Model of Employment, Money and Prices in a Growing Economy", ECONOMICA, November, 1961.

[10] ROSE, H., "Unemployment in a Theory of Growth", INTERNATIONAL ECONOMIC REVIEW, September, 1966.

[11] SAKAI, T., "A Study in the Theory of Investment", Master's Thesis Presented to Hokkaido University, January, 1972.

[12] STEIN, J.L., "Money and Capacity Growth", JOURNAL OF POLITICAL ECONOMY, October, 1966.

[13] TREADWAY, A.B., "On Rational Entrepreneurial Behavior and the Demand for Investment", REVIEW OF ECONOMIC STUDIES, 1969.

[14] UZAWA, H., "On a Neo-classical Model of Economic Growth", ECONOMIC STUDIES QUARTERLY, September, 1966.

[15] ———, "The Penrose Effect and Optimum Growth", ECONOMIC STUDIES QUARTERLY, March, 1968.

[16] ———, "Towards a Neoclassical Theory of Economic Growth", THE JOURNAL OF ECONOMICS (Tokyo University), Vol. 34, No. 4, January, 1969 (in Japanese).

[17] ———, "Reexamining the Theory of Optimum Economic Growth", ECONOMIC STUDIES QUARTERLY, August, 1969 (in Japanese).

[18] ———, "Time Preference and the Penrose Effect in a Two-Class Model of Economic Growth", JOURNAL OF POLITICAL ECONOMY, 1969.

[19] ———, "Towards a Keynesian Model of Monetary Growth", Paper for Conference of the International Economic Association on "The Essence of a Growth Model", April, 1970.

[20] ———, "Dynamic Stability of Processes of Economic Growth", THE JOURNAL OF ECONOMICS, Vol. 36, No. 3, October, 1970 (in Japanese).

[21] ———, "Inflation and Economic Growth—Towards a Keynesian Model of Monetary Growth—", in T.SHIMANO and K.HAMADA (eds.), "Monetary Problems in Japanese Economy", IWANAMI-SHOTEN (Tokyo), 1971, pp. 27-55 (in Japanese).

[22] WILLIAMSON, J., "A Simple Neo-Keynesian Growth Model", REVIEW OF ECONOMIC STUDIES, 1970.