On Terada's Rings of Powder*

By

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Introduction

When some quantity of lycopodium powder is inserted between two plane plates horizontally separated by uniform distances and a bar is made to fall on the plate in such a manner that the axis of the bar is perpendicular to the plate and so that the bar knocks the centre of slice of powder, the very regular concentric figures as shown in Fig. 1 are produced between these two plates.

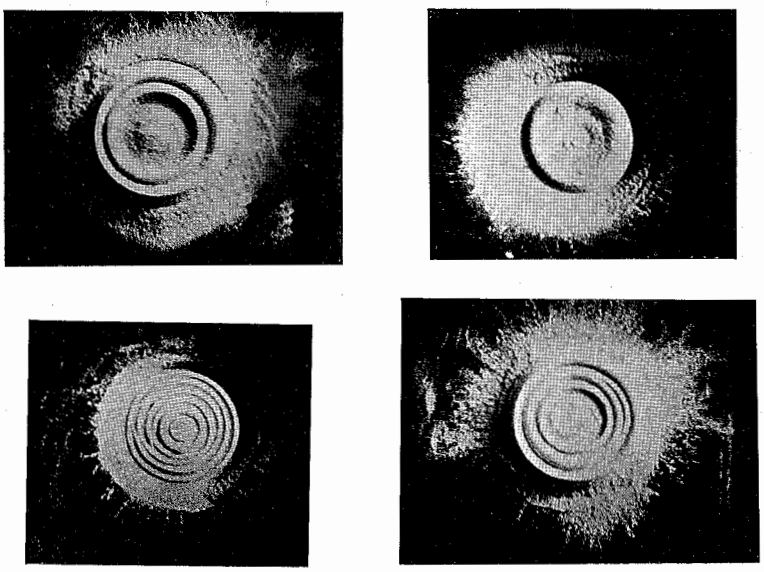

Fig. 1. Glass plate, $D = 20$ mm, $d = 1.1$ mm, $t = 3$ mm.

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* This paper was read by the author at the annual meeting of Physico-Mathematical Society of Japan on 6th April, 1934.

On the occasion of Prof. T. TERADA's visit to Sapporo, he saw this figure and immediately judged that it was be perhaps produced by superposing vortex rings which are concentric to each other and composed of powder and air. This opinion has since been confirmed by the author's observations.

In honour of Prof. TERADA, who keenly prejudged the mode of this figure, it may hereafter be simply called TERADA's Rings of Powder.

The apparatus to produce these figures is schematically shown in Fig. 2.

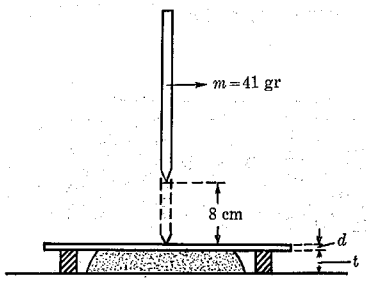


Fig. 2.

(1) Empirical Formula Determining the Number of the Rings.

If these mechanical shocks are still further repeated upon the upper plate, the mode of figures obtained between two plates will be changed with the increase of the number of knocks. A series of this variation is shown in Fig. 3.

The spacing between two adjacent successive rings is increased with the number of shocks and at last becomes constant for greater numbers of N. The first saturated value of the spacing is expressed as s_1. However s is suddenly widened by an increase of N till s reaches a greater value than the saturated one s_1 and s is suddenly widened and rapidly increased with N. This is shown in Fig. 4. $1/s$, the density of rings or number of rings per unit length, is proportional to the thickness of the upper plate unless its sectional dimension and material are changed. Also it depends upon the volume of powder inserted between the two plates. And the data showing these relations are plotted in Fig. 5. From the conditions above stated, the empirical formula determining $1/s$ can be easily found. From the above condition, $1/s$ must be proportional to d_e which indicates the sum of the true thickness d of the upper plate and additional apparent thickness d_0 of plate (here small value). The air in the immediate neighbourhood of the surface of upper plate vibrates simultaneously with the plate and this seems to correspond to the boundary layer in hydrodynamics of a viscous fluid.

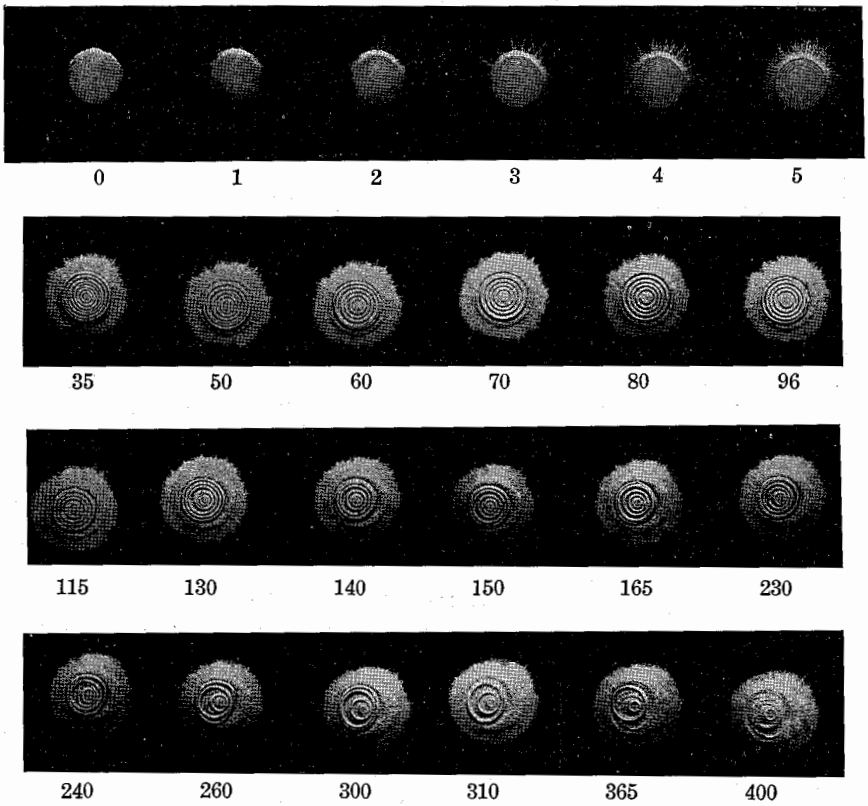


Fig. 3. Glass plate, D = 33 mm, d = 1.1 mm, t = 3 mm.
Arabic numerals indicate the number of shocks.

Writing the dimensional relation among these quantities, the following equation is obtained

$$1/s[o] = kd_e[L]\left\{\text{volume } [L^3]\right\}^{-1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Equating the volume of powder inserted between two plates to R^2t as a thin cylindrical form where R is the radius of the figure and t its thickness, (1) can finally be reduced to the following form.

$$1/s = kd_e R^{-3/2}t^{-1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Plotting $1/s$ to $R^{-3/2}$ or $t^{-1/3}$ respectively in x, y directions of rectangular coordinates, it will be seen that the curves showing the

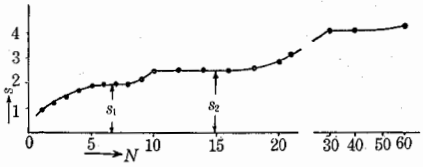


Fig. 4.

$R = 17$ mm, $t = 3.07$ mm, $s = 2.0$ mm.

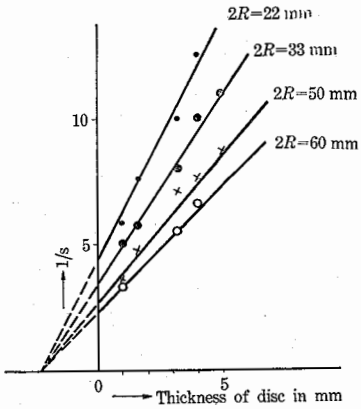


Fig. 5. Brass disc, Diameter of disc = 10 cm, $t = 1.05 \sim 1.25$ mm.

relation between these two quantities are both straight lines, through the origin of the coordinate. From the experimental results, these relations above stated are shown to be perfectly satisfied. Some of them are represented in the dotted lines of Fig. 6. The data showing these relation are tabulated in Table 1, 2, 3 and 4.

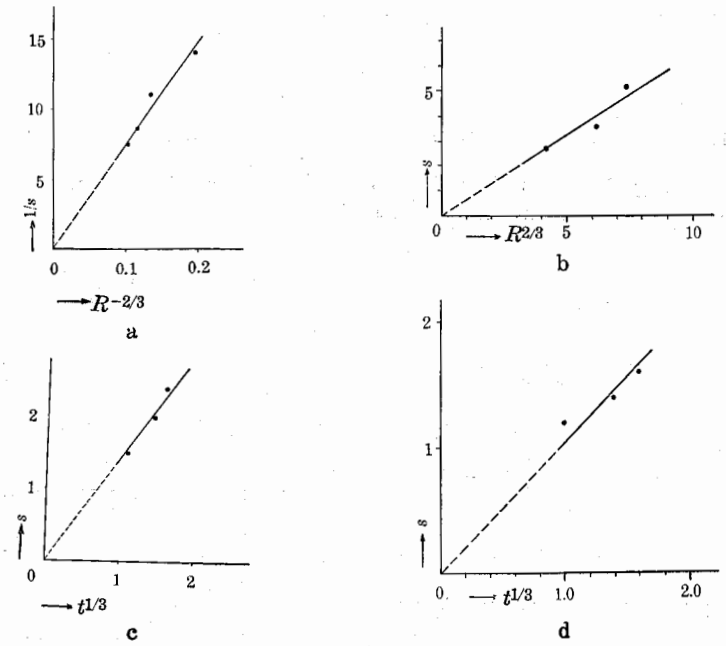


Fig. 6.

Table 1.

Brass disc, Diameter = 10 cm, $d = 5$ mm, $t = 1.05 \sim 1.25$ mm.

R in mm	$1/s$	$R^{-2/3}$
11.0	14.0	0.196
16.5	11.0	0.135
25.0	8.6	0.118
30.0	7.5	0.104

Table 2.

Glass plate, $t = 0.33$ mm, V = Volume of Powder, D_1 = Diameter of outermost ring, D_2 = Diameter of innermost ring.

V in cc	R in mm	D_2 in mm	D_2 in mm	N	$s = (D_1\text{-}D_2)/N$	$R^{2/3}$
1	8.75	17 18	4 5	5~6 5	2.6	4.24
2	13.90	28.5 27.2	7 6	6 6~7	3.5	6.22
4	20.25	41 40	2.5	7 8	5.1	7.37

Table 3.

Glass plate, $d = 1.05$ mm, $R = 17$ mm.

t in mm	s in mm	$t^{1/3}$
4.12	2.4	1.6
3.07	2.0	1.4
1.35	1.5	1.1

Table 4.

Glass disc, Diameter = 10 cm, $d = 3.1$ mm, $R = 25$ mm.

t in mm	s in mm	$t^{1/3}$
1.0	1.2	1.0
2.8	1.4	1.4
4.2	1.6	1.6

(2) Motion of Individual Particles in the Figure.

(i) Radial Sediment.

In order to investigate from what part of the slice of powder the radial sediment may flow, a glass plate is placed in the position

near the periphery of the slice as shown in Fig. 7. When a knock is given on the upper plate, the radial sediment appears almost the same as before in the form. The form and size of this radial sediment is not affected by inserting the plate in such a place. From

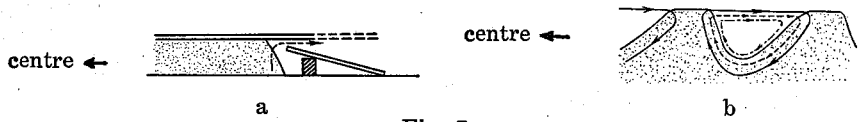


Fig. 7.

the fact that the powder is not caused to flow in the space below the inserted plate, it is evident that the powder of radial sediment is conveyed by the air stream that flows along the back surface of the upper glass as shown in Fig. 7.

(ii) Concentric Rings.

(a) Outermost Ring.

When a very small quantity of red powder coloured bluish by eosin is placed on the outward foot of the outermost ring and the knock is repeated, the profile through the centre of the rings will become as shown in Fig. 8. The black part in the figure shows the red powder.

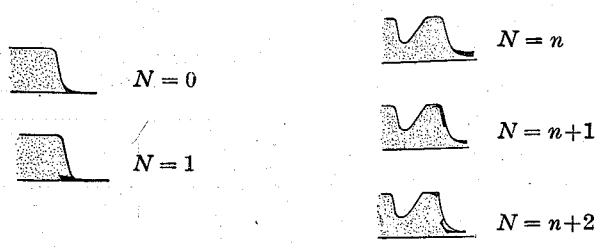


Fig. 8.

From (1) and (2) (a), it is evident that air stream is produced along the path as shown in full line of Fig. 7 near the outside of the ring and that powder is transported by this air stream.

(b) Inner Rings.

Many vortex rings are concentrically produced in the centre of the figure in the same sense as to the direction of the rotation and

at equal distances apart. When the upper plate is struck with a bar, the production of these vortex rings is started from the periphery of the figure and developed towards the centre of the figure with increase of shock numbers.

The direction of the rotation of these vortex rings is shown easily by laying coloured powder on the part of the figure to be tested before knocking the plate. Now if a small quantity of coloured powder is placed on any point in the valley between two adjacent successive heaps in the case when the number of knocks is n and TERADA's ring is produced and one knock is given, this red powder is chiefly elongated to the left after the knock.

Next if further a knock is again repeated, the band of red powder is enlarged till it reaches the left heap. The powder movement in this case is shown in Fig. 9.

Next if a very small quantity of coloured powder is placed on any point of the heap in the case when the TERADA's rings are produced and a knock is once more repeated, the red powder on the heap is elongated till it reaches the outer heap. The powder movement in this case is shown in Fig. 7b.

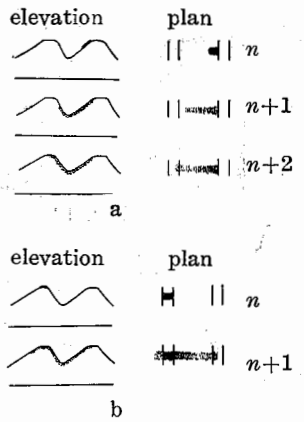


Fig. 9.

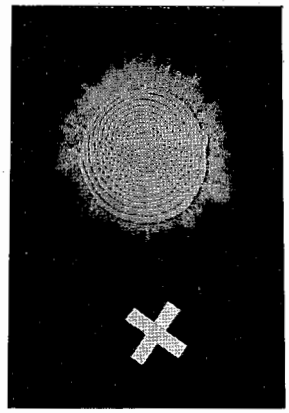


Fig. 10. Glass plate, $D = 40$ mm, $d = 1.1$ mm, $t = 1$ mm.

(3) The Case When the Knock is Given Eccentrically from the Centre of Slice of Powder.

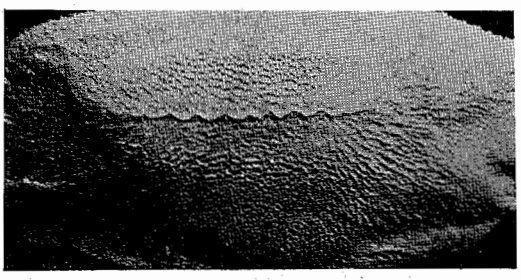
If the knock is given on the plate eccentrically to the centre of the circle of the slice of the powder mass, the figure is different to

that obtained by one which is given on the centre of the slice. One of these examples is shown in Fig. 9 in which the cross denotes the position of the knock.

(4) The Long Wave of the Powder Mass.

If knocks are given on the upper plate in the case when the form of powder mass is not circular but considerably long compared to its width, a beautiful sinusoidal wavy heap is produced along the central long axis of the powder mass. One of these figures is shown in Fig. 11. These wavy forms are sometimes found on the coast line of a sandy beach or on a desert. The cause of the production of these wavy forms seems to be the interference of the waves produced by two winds which proceed in opposite directions from each other.

One example from a sandy beach is reproduced in Fig. 12.



Flg. 11.

Fig. 12.

In conclusion, the author wishes to express his cordial thanks to Professor Y. IKEDA for his kind guidance and also to the late Professor T. TERADA who has offered many valuable suggestions.