Deformation and Elasticity of Sand Mass

By

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1. The Deformation of a Sand Mass

A sand mass formed as shown in Fig. 1 was loaded on its surface with a weight and its resultant deformation measured by means of a microscope. The results measured are tabulated in Table 1.

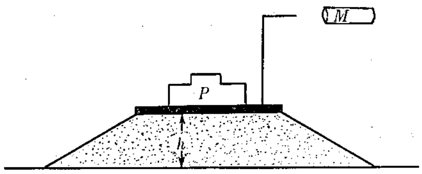

Fig. 1. P = Load, M = Microscope.

Table 1.

h(depth of layer) in cm.	d(contraction)	E	v in m/sec.
	$\times 1.74\times 10^{-3}$ cm.	$\times 10^{6}$	
0.9	1.8	1.78	10.7
1.2	1.5	2.86	13.8
1.9	1.5	4.5	17.3
3.5	1.5	8.3	23 5
4.6	1.3	13.7	30.2
5.0	1.4	13.7	30.2
7.1	1.2	21.0	37.4
10.2	1.0	36.5	49.0
12.7	0.8	55.6	60.6

If a small stress is once applied on the sand mass during a very short time and then the stress is removed, the original formation is recovered without fail. In this case as is evident from the result, it seems that hysteresis needs not be considered and the deformation may be in some meaning considered to be elastic.

It seems that the existence of the hysteresis effect in the deformation of the sand mass is generally to be considered when the stress is always large and the duration of applying stress is also long. However, in the case of stress applied during a short time, the deformation must be considered to be elastic, although the elastic deformation in this case differs from that of the ordinary elastic body.

2. The Partial Elastic Condition

If the thickness of the sand mass is varied, the depression of the sand mass is decreased with the increase of its thickness. The cause of this decrease must be attributed to the following facts.

(i) The direction of the force which produces the stress deviates from the vertical line.

(ii) There is another important factor:—the deformation is concentrated in the domain near where the load is applied.

The stress energy is stored up in the grains of the sand mass in the form of elastic potential and gradually attenuated from the surface on which the load is located. Therefore the strain will decrease with increase of the distance from the surface of the sand mass. The condition to give these deformation is called the partial elastic condition. The deformation to satisfy this condition is called partial elastic deformation.

3. The Model Experiment

Model experiments of the partial elastic deformation were qualitatively carried out by the present author. Many rubber discs whose diameters are all equal to each other are arranged in series in a narrow channel and the end one of them is pushed with a horizontal load. Thus deformation occurs in each disc and strains in discs at the neighbourhood of the load are large as shown in Fig. 2,

Fixed

Fig. 2.

but at the remote part from the load are not remarkable. The mode of this model is not perfectly the same as that of deformation of the sand mass but it is somewhat similar to that. If this model experiment is quantitatively performed, the mechanism on the deformation of the sand mass must be clear. Therefore a few additional model experiments will be described.

4. The Relation between the Partial Elastic Deformation and Propagation Velocity on the Surface of the Sand Mass

In the case when the vibration is propagated on the surface of the powder or granular mass, this partial elastic condition is satisfied. Using the data shown in Table 1, Young's modulus is calculated and shown in the 3rd column of the table. Assuming the apparent density of sand mass as 1.5, the velocity of the elastic wave is immediately calculated and tabulated in the 4th column of the same table. The velocity of the elastic wave thus calculated depends on

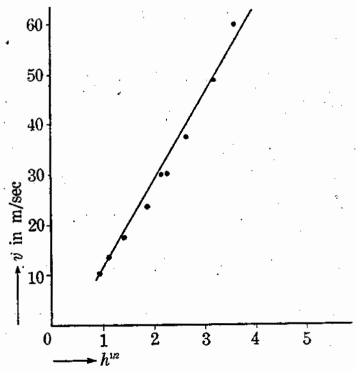

Fig. 3.

the thickness of the sand mass, it is approximately proportional to its square root as shown in Fig. 3. In fact this relation holds in the case where the velocity of the surface wave of the sand mass is measured by an electric method.[1] In order to explain rigorously the deformation of the sand mass, it is permissible to introduce the idea of partial elastic deformation.

1) K. YONETA: On Wave Propagation on the Surface of Sand Mass. Mem. Fac. Eng. Hokkaidô Imp. Univ. Vol. 4, No. 3, p. 265 (1938).

5. Predominance of Surface Wave

If the sand mass is deformed by partial elastic deformation, Young's modulus differs from the surface to the inner portion of the sand mass. Young's moduli for the portion near the surface and for the inner portion of sand mass are given as E_1 and E_2 respectively.

When the stress f is equally given on these portion and each strain is indicated by d_1 and d_2, Young' modulus E_1 and E_2 are given such as

$$E_1 = f/d_1 \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$E_2 = f/d_2 \quad \ldots\ldots\ldots\ldots \quad (2)$$

where $d_1 > d_2$ (from the condition of partial elastic deformation)

$$\text{or} \quad d_1 = nd_2 \quad (n > 1) \quad \ldots\ldots\ldots \quad (3)$$

Substituting (3) and (1) in (2)

$$E_2 = (nf)/(nd_2) = (nf)/d_1 = nE_1 \quad \ldots\ldots \quad (4)$$

W_1 and W_2 are work done per unit volume necessary to produce strain d_1 for portions of Young's modulus E_1 and E_2.
They are given as follows:

$$W_1 = 1/2E_1d_1^2 \quad \ldots\ldots\ldots\ldots \quad (5)$$

$$W_2 = 1/2(nE_1)d_1^2 = nW_1 \quad \ldots\ldots\ldots \quad (6)$$

as is evident from (6) if $n > 1$, W_1 is smaller than W_2. In the case when n is greater than 1 and varied, the ratio of W_2 to W_1 can be readily determined. Thus it is shown that the wave does not easily propagate to the inner part of the sand mass.

In conclusion, the author wishes to express his sincerest thanks to Professor Y. IKEDA for his kind guidance.