A Method of Determining the Thaw Water Content in Snow Layer

Yosida, Zyungo

北海道大学理学部紀要 = Journal of the Faculty of Science, Hokkaido University. Ser. 2, Physics, 3(4): 91-102

1940-12-20

http://hdl.handle.net/2115/34172

bulletin

3_P91-102.pdf

Hokkaido University Collection of Scholarly and Academic Papers : HUSCAP
A Method of Determining the Thaw Water Content* in Snow Layer

By

Zyungo YOSIDA

1. Introduction

The snow layer which covers the ground through the winter in cold districts shows, as is well known, a very diverse and changeable nature. At the beginning of the formation of the snow layer, its nature is determined by the sort of snow crystals which, manifesting so many varieties in their forms, are to make up the layer. The elements of weather, such as temperature of the atmosphere, wind, sunshine etc., to which the snow layer is exposed afterwards, produce various effects on it and modify its nature conspicuously. In spite of such complex influences affecting the snow layer, the quality which has hitherto been measured for the purpose of specifying its nature, is only the specific gravity. Other properties, such as rigidity and tensile-strength, have also been measured, but only on rare occasions, because the measurement of these quantities requires a very troublesome manipulation.

On the other hand, it is a well known fact that a dry snow layer differs in character from a wet one in so remarkable a manner, that everyone who has been even a little acquainted with snow is aware of this difference. The wetness of the snow layer is evidently due to the presence of thaw water in it. Therefore the thaw water content must act as an important factor in determining the nature of the snow layer. It is desirable that a method of measuring the thaw water content be devised and developed in a convenient form suitable to carry out measurement in the field. By the suggestion of Prof. Nakaya, the present writer tried to realize the measurement last winter (1939–1940) and obtained satisfactory results.


The purpose of this paper is to describe the principle and methods of the measurement as well as to report some results obtained by the experiment.

2. **Principle of Measurement**

The determination of the thaw water content would be an easy matter if it were possible to extract the thaw water by any simple method from a mass of snow taken out of the snow layer. As the thaw water, however, is held in a narrow space between the ice granules composing the snow, or is spread over the surface of these ice granules in the form of a thin film, it is very difficult to separate the thaw water and extract it from the snow. A centrifugal machine may answer the purpose of extraction of the thaw water, but the use of the machine will be accompanied with serious trouble; for it must be very difficult to prevent the melting of the ice part of the snow during the process of extraction.

In view of the fact that the extraction of thaw water from the snow is practically impossible as explained above, other methods of measurement which require no separation of thaw water have been searched for. One method proved promising which takes advantage of the volume change or the latent heat produced at the melting of the ice. The principle of the method is to be explained in the following paragraphs.

It will be convenient to notice beforehand, that the temperature of the snow layer which contains even a small quantity of thaw water is just equal to the melting temperature of ice, viz. 0°C. It has been reported that wet snow layer sometimes shows a temperature below 0°C\(^1\), owing to the fact that the snow layer obtains salt from the atmosphere, and this salt reduces the melting temperature of the ice granules in the layer. But as far as the present writer has measured the temperature of the wet snow layer for himself, the temperature was found to be 0°C in every case within one-hundredth of one degree Centigrade. Therefore, the reasoning in the following will be developed on the belief that the wet snow layer is always exactly at the temperature of 0°C.

---

(i) The method which takes advantage of the volume change at the melting of ice.

Suppose that a mass of snow weighing $M$ gr. contains $m$ gr. of thaw water, and let the sum of volumes of ice-granules and water contained in this mass be denoted by $v_s$ c.c. When the whole of this mass is heated and melted, and then again cooled to the temperature of $0^\circ$C, $v_s$ must have undergone a change $\Delta v_s$ which amounts to $0.0905 (M-m)$ c.c., because 1 gr. of ice decreases in volume by $0.0905$ c.c. when it melts into water of $0^\circ$C. Therefore, if this volume change $\Delta v_s$ could be determined, the thaw water content $m$ would be obtained by the equation.

$$m = M - \frac{\Delta v_s}{0.0905} \quad (1)$$

The actual determination of $\Delta v_s$ may be carried out in the following manner. A mass of snow is put into a glass bottle which is provided with a device to indicate the whole volume enclosed in the bottle without exerting any pressure upon the air contained in it. After the lid of the bottle is closed, it is immersed in a bath of water and ice. In this state, the volume indicating device of the bottle manifests the volume $v_a$ in addition to the volume $v_s$ of the air which has been enclosed in the bottle, the temperature and pressure of the air being $0^\circ$C and one atmospheric pressure respectively. Now the bottle is taken out of the mixture and heated till all the snow in it melts away. The melting having been completed, the bottle is again put into the bath at $0^\circ$C and after a while the volume is read again. The volume change indicated is equal to the change of $v_a$, viz. $\Delta v_s$, because the air in the bottle displays no volume change.

The above explained method seems to have good prospects in its practise, but a serious error may enter into the determination of $\Delta v_s$ as the dead air volume $v_a$ is very changeable with temperature and pressure. In this consideration of the error due to $\Delta v_a$, the writer postponed the actualization of this method to the future.

(2) The method which takes advantage of the latent heat of fusion of ice.

A mass of snow, which weighs $M$ gr. and contains $m$ gr. of thaw water, requires a heat quantity of $79.60 (M-m)$ cal., in order
to melt into water of 0°C; therefore, if this heat quantity is determined as \( Q \) cal., the thaw water content will be found by means of the equation

\[
m = M - \frac{Q}{79.60}
\]

in grams. 79.60 cal. is the latent heat of fusion of ice.

Actual determination of \( Q \) will be most conveniently carried out by the use of an ordinary water calorimeter. Let the heat capacity of calorimeter and its accessories be \( W \) cal/°C, and \( \mu \) gr. of water at the temperature of \( t_i \)°C be contained in the calorimeter. If, after a mass of snow had been put into the calorimeter, its temperature were \( t_f \)°C, \( Q \) could be obtained as

\[
Q = W(t_i - t_f) + \mu(h_i - h_f) - Mh_f
\]

where \( h_i \) and \( h_f \) mean the heat content of water at \( t_i \)°C and \( t_f \)°C, viz. \( \int_0^{t_i} cdt \) and \( \int_0^{t_f} cdt \) respectively, in which \( c \) stands for the specific heat of water. The error in determining the value of \( m \) caused by the adoption of this method can be restricted to a satisfactorily small limit as will be described in the next section.

3. Procedure of Measurement

The calorimeter used for the experiment is shown in Fig. 1. It consists of a cylindrical copper vessel \( C \), about 12 cm. in height, 5 cm. in diameter, and 0.2 mm. thick. The cylindrical brass vessel \( B \) divides into halves the space between the calorimeter and the outermost cylindrical vessel \( A \), and the gum legs \( G_1, G_2 \) attached to the vessel \( B \) by means of screws, serve as heat insulating supports for the calorimeter and the vessel \( B \). All these three vessels are put into a wooden pail \( P \), and the space between the vessel \( A \) and the pail \( P \) is filled with water. As the wooden wall of the pail is a good insulator of heat, the water between \( A \) and \( P \) could be kept at a constant temperature for so long a while that it acted well as a water jacket for the calorimeter. The lid \( D \) to the vessel \( A \) is made of two sheets of brass plate about 1 mm. thick, with a space of 1 mm. between them; and it is so constructed that the lower part of the edge of the lid dips into the water jacket to a depth of about 1 cm.,
A Method of Determining the Thaw Water Content in Snow Layer

The inner sheet of the lid, protected against heat intrusion from the outside by the thin layer of air in the space between the two sheets and connected directly with the water jacket, attains almost the same temperature as that of the water. Thus the calorimeter being surrounded on all sides with walls of the same temperature, the rate of the natural cooling of the calorimeter can be kept fairly well at a constant value. For the convenience the lid D is screwed to the wooden lid D' of the pail, so that both the vessel A and the pail P can be shut at the same time.

The calorimeter is provided with a lid of thin copper plate attached to it by means of a hinge. As the initial temperature $t_i$ of the water in the calorimeter is as high as $30^\circ C$ or so, its vaporization is considerable, and the vapour condenses in a form of many

1) White: Modern Calorimeter (1928), p. 151
droplets attached to the inner surface of the lid. With the intention of reducing the vaporization of water, the calorimeter is designed in a form longer than usual. In the writer's experience, it would be better to make the calorimeter thicker a little bit for the convenience of manipulation throughout the measurement.

The bend stem thermometer $T_1$, used for measuring the temperature of water in the calorimeter, is graduated in tenths of a degree Centigrade over the range $0^\circ$~$50^\circ$C. This thermometer was calibrated in comparison with a sub-standard thermometer certified newly by the Central Meteorological Observatory in Tokyo. $T_2$ is the thermometer used for the determination of the temperature of the water jacket and $S_1, S_2$ are the stirrers for the calorimeter and the water jacket respectively.

The quantity and temperature of water in the calorimeter are chosen as about 150 gr. and $30^\circ$C respectively at the beginning of the measurement. After the rate of natural cooling of the calorimeter has been observed for three or four minutes, a mass of snow obtained from a snow layer is put into the calorimeter. The quantity of snow chosen is such as to bring the final temperature $t_f$ of the calorimeter to $10^\circ$C when it is quite melted. It has been found possible after a few trials to estimate the appropriate quantity of snow with considerable certainty. The reason why the temperature of $10^\circ$C is chosen as an appropriate value for $t_f$ is as follows: if the temperature is below $10^\circ$C, the snow cannot melt easily and the temperature of the calorimeter requires a long time to reach its final value $t_f$.

Fig. 2 is an example of the cooling curve of the calorimeter. As is apparent from the figure, the temperature reaches its final value within an interval of 15~20 sec. By the virtue of the shortness of the interval, the correction to be made to the observed values due to the natural cooling during this interval is found to be only a few hundredths of a degree Centigrade, assuming
the validity of Newton's law of cooling. The smallness of the correction is an important factor in diminishing the error of this measurement.

In order to determine the value of the final temperature \( t_f \), it is convenient that this temperature be almost equal to that of the water jacket. For this reason water in the pail was kept at the temperature nearly 10°C, by adding from time to time an appropriate quantity of hot water. In cases when the temperature of the atmosphere was of 2 or 3°C, the decrease in the temperature of water during the interval of one measurement was only \( 0.2°C \) owing to the excellency of the heat insulation of the wall of the pail.

In order to take snow from the snow layer and carry it to the calorimeter, it was found convenient to use a measuring cylinder of 3 cm. diameter. Holding it at the bottom with a gloved hand, there was no danger of melting the snow in the cylinder.

Inasmuch as the temperature of calorimeter can be determined within one one-hundredth of a degree Centigrade, the heat content of water \( h=\int_0^t \! c \, dt \) must not be replaced by \( t=1 \times (t-0) \), because \( h \) and \( t \) differ in the second decimal place. The values of \( h \) adopted in this measurement are taken from the table recently proposed by W. A. Roth\(^1\), a part of which is reproduced in Table 1.

<table>
<thead>
<tr>
<th>( t , (^\circ \text{C}) )</th>
<th>( h , (\text{cal/gr}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>5.025</td>
</tr>
<tr>
<td>10</td>
<td>10.040</td>
</tr>
<tr>
<td>15</td>
<td>15.050</td>
</tr>
<tr>
<td>20</td>
<td>20.048</td>
</tr>
<tr>
<td>25</td>
<td>25.040</td>
</tr>
<tr>
<td>30</td>
<td>30.025</td>
</tr>
<tr>
<td>35</td>
<td>35.005</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( M , (\text{gr}) )</th>
<th>79.60 ( M - Q , (\text{cal}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.8</td>
<td>25</td>
</tr>
<tr>
<td>11.9</td>
<td>30</td>
</tr>
<tr>
<td>17.7</td>
<td>27</td>
</tr>
<tr>
<td>19.1</td>
<td>37</td>
</tr>
</tbody>
</table>

\(^1\) ZS. Phys. Chem. Abt. A. 183, 38, (1938).
heat quantity necessary to melt this block of ice, viz. 79.60 \( M \) cal. The difference between the experimentally determined value of \( Q \) and 79.60 \( M \) will then give the error in the measurement. To keep the temperature of the block of ice exactly at 0°C, the block was floated for a long time on water, then picked out, wiped with a piece of cloth and put into the calorimeter. The results obtained by four experiments are shown in Table 2. The difference 79.60 \( M - Q \) is always positive and the values are scattered near the mean value 30, although \( M \) is varied over a considerably wide range. This result led to the consideration that this 30 cal. is a systematic error in the determination of \( Q \) due to the heat supplied to the calorimeter by some indetectable but constant ways during the experiment, and so equation was revised to

\[
Q = W(t_i - t_f) + \mu(t_i - t_f) - MH_f + 30.
\]

The results in the next section were all calculated by this revised equation.

\( Q \) is usually of the order of 1500, while its accidental error is about 5 as learned from the table above. So the relative error in \( Q \) is estimated as 0.3%. The relative error in \( m \), whose determination is the aim of this measurement, is, however, not so small as this value. As is clearly understood by equation (2), the error in \( m \) increases or decreases accordingly as \( m \) becomes small or large in comparison with \( M \). When \( m/M \) is 5%, the error in \( m \) is 6%; when \( m/M \) is 1%, the error is 30%. Although the error in \( m \) is rather large as shown, yet it can be determined correct to two significant figures when \( m/M \) is large and to one significant figure when \( m/M \) is small.

When it is more adequate to use \( m/M \) for indicating the thaw water content than to use \( m \) itself, the name “thaw water content” should be applied to \( m/M \) after this. It is evident that the relative error of \( m/M \) is equal to that of \( m \), because \( M \) can be determined very accurately compared with \( m \).

As it is more adequate to use \( m/M \) for indicating the thaw water content than to use \( m \) itself, the name “thaw water content” should be applied to \( m/M \) after this. It is evident that the relative error of \( m/M \) is equal to that of \( m \), because \( M \) can be determined very accurately compared with \( m \).

In conclusion of this section, some points respecting the practice of the measurement should be mentioned. It needs at least three persons to execute this experiment; two for the calorimetry and one for collecting snow. After trials the writer and assistants were able to carry out the measurement within ten minutes. The instruments necessary for this measurement, except the calorimeter and
its accessories, are a balance sensitive to 0.1 gr., a stop watch, a measuring cylinder and several beakers. It is necessary, too, to have a stove to get hot water when the measurement is to be done outdoors.

4. Results of the Measurement

The measurement was done in the campus of the Hokkaido Imperial University, Sapporo. A hole was dug vertically in the snow layer, and snow samples were taken from the wall of the hole at several points of various heights. Figs. 3a, 3b, 3c show the results of the measurement conducted on the dates indicated there. The first column of each figure is a sketch of the cross section of the snow layer, the second shows the distribution of thaw water content along the vertical height and the last that of specific gravity. The arrows mark the points where the snow samples were taken out.

After a long continuation of cold days in winter, warm and sunny days came in the beginning of March, then, everyone noticed that the snow layer began to decrease. March 4th (Fig. 3a) was a sunny day, and the snow on the roofs of houses melted plentifully.
According to the report of Hirata\textsuperscript{1}) the snow layer melts in cold winter days little by little at the lower part of the layer by heat supply from the ground, and when it becomes warmer, the temperature of the atmosphere fluctuating near 0°C, the snow layer begins to melt from its surface. As is apparently seen in Fig. 3a, the thaw water content is large near the surface of the snow layer and this fact shows that it was melting at the top in accordance with Hirata's report. It was cloudy in March 13th (Fig. 3b). The height of the snow layer had been diminished from 70 cm. to 50 cm. during the ten days after March 4th. The thaw water content became considerable also at the deep part of the snow layer. In the night before March 14th (Fig. 3c) rain fell and the thaw water content was observed to have increased in comparison with that of the previous day over the whole height of the layer.

The following experiment was done with the intention to know to what amount the snow layers could hold thaw water. A part of snow above a definite height from the ground was removed from the layer, and the remaining part was soaked with water of 0°C, a mixture of tap water and a large quantity of snow. The thaw water content obtained from snow in such a condition will roughly give the maximum value of thaw water the snow can contain. The broken line in the middle column of Fig. 3c shows the distribution of such a maximum thaw water content. The specific gravity of the snow soaked with water is shown by the broken line in the last column. As is evident from the figure, the upper part of the snow layer was almost saturated with thaw water while the lower part was far from being saturated.

Although the maximum value of thaw water content should

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig3c}
\caption{Fig. 3c (March 14th)}
\end{figure}

\textsuperscript{1}) Ooyō-Buturi (Applied Physics) 7, 309. (1938); in Japanese.
vary according as the nature of the snow is changed, it can be said that in general this value is not greatly different from 20%. Nevertheless, the uppermost part of the snow layer shown in Fig. 3b contained so much thaw water that the content amounted to 40%. After examination of the thin sheet of grey snow shown in the figure by a line beneath the uppermost part of the layer, it was found that this sheet was almost agglomerated to form a thin layer of icy crust, and the thaw water in the snow above this sheet could not permeate through it. Such an agglomeration of ice granules to an icy crust in grey snow band, found in the section of the snow layer in several places, was not an uncommon phenomenon. The cause of defects found in several parts of the columns of Fig. 3 representing the distribution of specific gravity is that it could not be determined because of the agglomeration of the snow in their corresponding points.

At first sight, the white part of the snow layer seems dry, and grey one, wet. But, one will see in the figure that it is only a superficial impression. A white snow band is sometimes very wet while the grey snow band adjacent to it contains only a small quantity of thaw water.

The experiment was undertaken, as mentioned at the end of section 1, by the suggestion of Prof. Nakaya and conducted under his constant direction. The author should like to express here his deep gratitude to him. Thanks are also due to Mr. H. Watanabe and Miss H. Nakayama, students of the University, for their assistance in executing the investigation.

Summary

A method of determining the thaw water content in snow layer by means of calorimetry is described. When a mass of snow weighing $M$ gr. contains $m$ gr. of thaw water, the heat quantity required to melt the mass should be equal to $79.60(M-m)$ cal., where 79.60 cal. is the latent heat of fusion of ice. If this heat quantity is determined by calorimetry the thaw water content $m/M$ can be obtained. Calorimetry was carried out by means of an ordinary water calorimeter. Errors were found to be restricted within a
small limit, testing this method of measurement by the application of it to a block of ice at 0°C, which can be considered as a special form of snow containing no thaw water.

The results of the measurement on the snow layer in the campus of the Hokkaido Imperial University, Sapporo, are shown in diagrams, which represent the distribution of the thaw water content along the height of the cross section of the snow layer.