On the Constant Boiling Springs in Izu Peninsula.

By

Takaharu FUKUTOMI.

I. Introduction.

Noboribetu in Hokkaidô, Onikobe in Miyagi prefecture, Beppu and neighbourhood in Ôita prefecture, and the wide-spread area comprising Yugawara, Atami, Atakawa, Katase, Yazu, Mine, Simogamo in Sizuoka prefecture, etc. are the most famous and most active hot spring resorts in Japan. They have intimate relation with volcanic action. There are a number of types of the so-called "Boiling spring" which incessantly ejects great quantities of hot water at boiling temperature and volumes of steam, being the subject of the present report.

The word "Boiling spring" is, generally speaking, used for both the so-called "Geyser" in which ejection of hot water and steam at boiling temperature and repose take place alternately at certain intervals, as well as for the above-mentioned ordinary boiling spring. But, in the following discussion, the writer will regard "Boiling spring" as the ordinary boiling spring only.

There are two sorts of ordinary boiling spring, viz., the natural boiling spring and the boiling spring obtained by boring—the former existing in a group in Onikobe, in Hatimandaira, etc. and the latter in Atami, in Yazu, in Mine, in Simogamo, etc.

As to the ordinary boiling spring, only a few reports[1] have been published, so far as the writer's knowledge is concerned, while as to the geyser the distinct phenomena of its periodic eruption are cautiously treated by the foremost authorities and they have become the subject of their interesting studies. In the previous papers, some

1) T. Fukutomi: Bull. Earthq. Res. Inst. 14 (1936), 259.
T. Fukutomi: Bull. Earthq. Res. Inst. 15 (1937), 113–133.
K. Noguti: Berichte Jap. Gesellsch. Baln. 1 (1941), 17.

Journ. Fac. Sci., Hokkaidô Imp. Univ., Ser. II, Vol. III, No. 6, 1942.

relations between Cl-contents and water temperatures of the boiling springs in Yazu and in Atami in the Izu peninsula were studied by the writer. Also, change of chemical constituents of a boiling spring of Senami in Niigata prefecture for lapse of time was studied by Dr. Noguti and the result was that chemical constituents such as pH, Cl, SO_4, were always constant in extent of two days continuous observation. But, the mechanism of the ordinary boiling spring and the mutual relations between projection velocity, jet-height, underground temperature of the spring, etc. are as yet unknown.

The writer recently had the opportunity to observe some of the physical properties of the boiling springs in Atami, Atakawa, Yazu, Mine and Simogamo in the Izu peninsula, the results of which, together with those already known, enable the establishment of a probable explanation for the mechanism of boiling springs in that locality and the elucidation of the mutual relations between the physical properties of the springs in first approximation.

II. Kind and underground structure of the boiling spring obtained by boring.

Boiling springs and geysers are always distributed in clusters and confined to a narrow area which we call a "Boiling spring basin" as shown in examples of Figs. 1–3.

Cross-hatched areas in the figures represent boiling spring basins, and dots, double circles and crosses indicate the sites of orifices of ordinary hot springs, boiling springs and geysers respectively. It may well be that the underground structure of the boiling spring obtained by boring is simple compared with that of the natural boiling spring for the reason that the outlet of hot water of the former is a simple vertical pipe drilled perpendicular to the ground surface, while that of the latter may be a system of complicated veins. In the Izu district, it is the usual method of boring technic that iron pipes of a diameter of some 5~10 cm in close connection are pushed into the drill-hole until it attains to a moderate depth in order to prevent the intrusion of side soil and to avoid the mixing of cold underground water in shallow strata. In the case where the pipe reaches the bottom of the hole, some side-hole is opened in the lower part of the iron pipe from which hot water can be allowed to stream in.

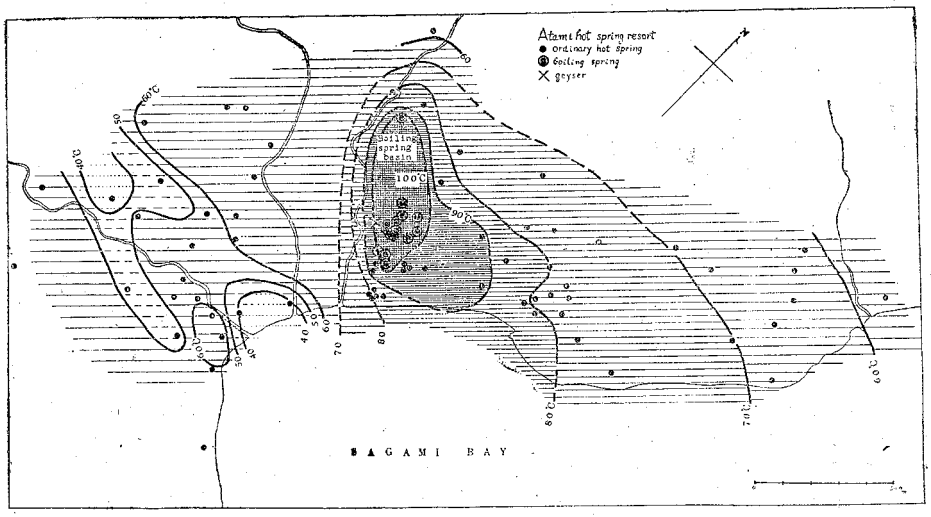

Fig. 1. The boiling spring basin in Atami, Izu peninsula.

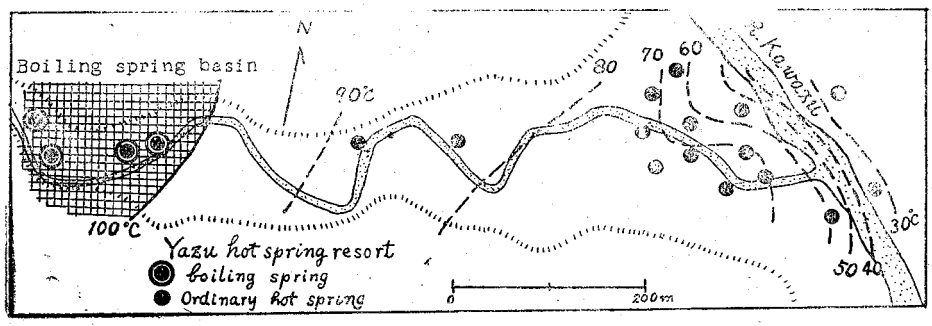

Fig. 2. The boiling spring basin in Yazu, Izu peninsula.

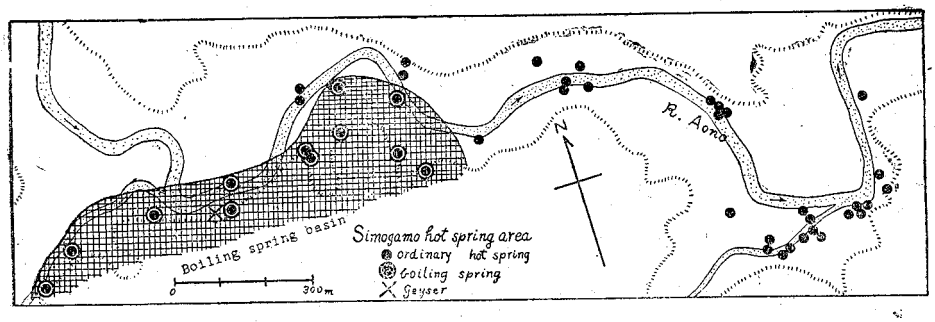

Fig. 3. The boiling spring basin in Simogamo, Izu peninsula.

As to the natural boiling spring, owing to the energy loss by friction or dissipation into the surrounding ground, the orifice temperature, jet-height and projection velocity at the orifice are moderately smaller than those in the boiling spring obtained by boring.

1) Vertical distribution of subterranean temperature in the boiling spring basins, in Izu peninsula.

The subterranean temperature[2] in hot spring regions generally rises with increase of depth until it attains the maximum at a certain depth, whilst below that depth cases where the temperature diminishes somewhat are often seen. In the boiling spring basins, the subterranean temperature shows a similar tendency, and also the underground temperature of hot water from hot spring veins at some 100~250 m depth is always higher than the boiling temperature of water under atmospheric pressure.

Fig. 4 and Table I indicate the vertical distribution of subterranean temperature in the excavation of the boiling springs in the Izu peninsula which were measured successively at the bottom of the bore-hole as the boring proceeded. The writer believes that the successive temperatures reprsent a fairly close approximation to what would have been found if it had been possible to measure temperatures without drilling a hole. Though some disturbance was caused by the circulation of feed-water and by other incidents of drilling, these were probably rather temporary in their effect.

From the results, it may be recognized, so far as the observed depth is concerned, that the subterranean temperature of the boiling spring basins in Izu peninsula is always less than the boiling temperature of water under the pressure of the hydrostatic column of water extending from the orifice to that depth, in other words, that the water in the vertical pipe in hydrostatic state, is always in liquid state, because convection currents can not occur in such a small pipe.

An additional feature of the boiling spring basins in the Izu peninsula is the fact that the subterranean temperature gradients in the

---

2) S. Yoshimura, K. Misawa: Tirigaku Hyoron 7 (1931), 406.
T. Nomitu, K. Yamasita: Memoirs Coll. Sci. Kyoto Imp. Univ. S-A 23 (1940), 97–122.
T. Fukutomi: Disin 12 (1940), 394–403. Jap. Journ. Astro. Geophys. (in Jap. Language) 1 (1941), 64–72.

part where the temperature exhibits more than 100°C are generally small in comparison with the gradients of the above-mentioned (boiling point of water: depth) curve in depths of 0–100 m beneath the ground surface.

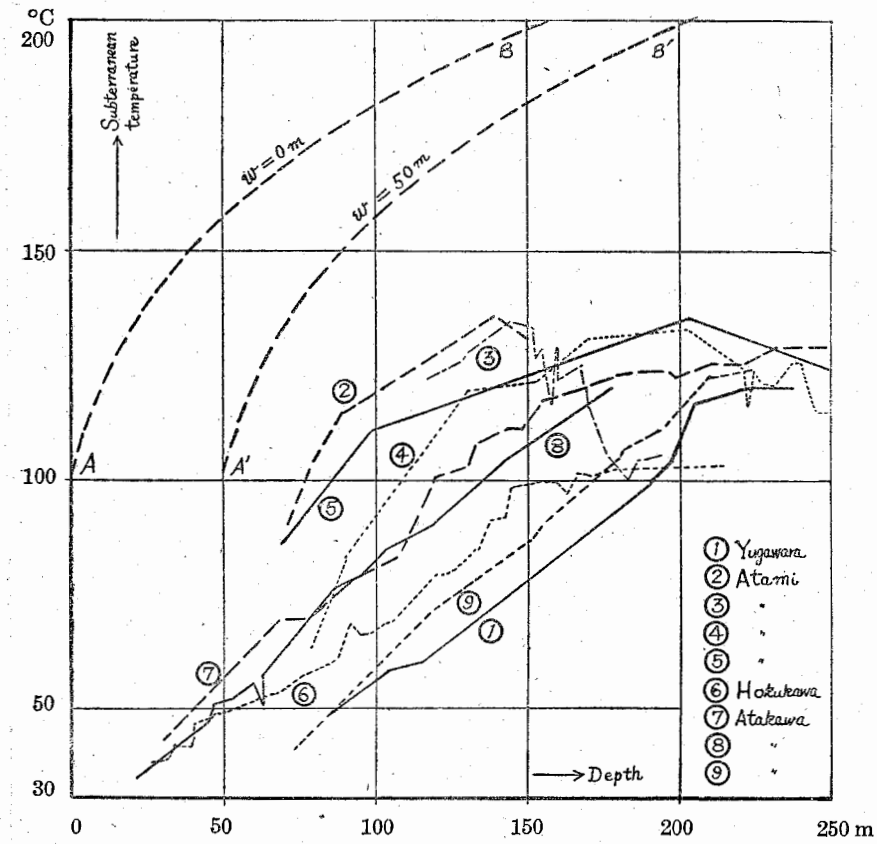

Fig. 4. Vertical distribution of subterranean temperature in the boiling spring basins in Izu peninsula. (See Table I)

There are hot springs in which phenomena of boiling do not occur in spite of their having temperatures at the bottom of the bore-hole higher than boiling point at atmospheric pressure. This may be attributed to the fact that the hot underground water is cooled constantly by the cold surrounding ground until its temperature always falls below boiling, owing to its low speed of ascent. Such springs are not called boiling springs.

Table I. Vertical distribution of subterranean temperature in the excavation of the boiling springs in Izu peninsula.

① No. 84 spring Yugawara (Hirogawara)

Depth	Subterranean temp.
84 m	48°C
97	55
103	58
115	60
131	68
173	90
190	99
198	105
205	117
211	118
220	119
225	120
230	120
237	120

② Ôkura spring Atami

Depth	Subterranean temp.
70 m	90°C
79	103
91	115
137	135
150	130

③ Komatu spring Atami

Depth	Subterranean temp.
115 m	122°C
127	126
129	128
142	134
150	133
152	126
154	128
156	121
157	119
158	116
159	129
160	122
168	125
169	120
174	108
183	100
186	104
195	105

④ Huzisawa spring Atami

Depth	Subterranean temp.
79 m	63°C
91	83
97	90
110	102
129	119
154	121
170	130
192	131
204	132
215	126
218	126
222	124
224	114
225	124
228	121
230	120
231	120
238	125
241	125
245	114
248	124

⑤ Turuya spring Atami

Depth	Subterranean temp.
68 m	86°C
98	110
166	126
204	135
270	120
274	124

⑥ No. 1 spring Hokukawa

Depth	Subterranean temp.
26 m	39°C
31	40
36	42
39	43
41	47
47	49
50	49
56	51
63	53
67	53
72	56
79	59
88	61
91	69
95	66
98	66
103	69
105	69
106	69
109	73
112	75
114	75
117	78
118	79
120	80
123	80
133	85
136	92
142	93
145	99
146	99
152	100
159	100
164	97
166	101
170	101
179	103
187	102
199	103
216	103

(N.B.)
Depth = 224 m
6 inch pipe = 13 m
4 inch pipe = 48 m
Volume output = 117 *l*/min.
Orifice temp. = 99°C

⑦ Tutiya spring Atakawa

Depth	Subterranean temp.
31 m	43°C
68	69
77	69
92	78
108	83
120	101
130	103
133	108
143	111
147	111
156	117
168	119
178	122
186	123
197	123
199	122
212	125
222	125
232	128
251	129

(N.B.)
See Fig. 6.

⑧ No. 3 spring Atakawa

Depth	Subterranean temp.
20 m	35°C
32	41
33	42
35	42
44	47
46	51
52	52
60	56
62	51
62	57
71	65
86	76
97	80
102	84
118	90
142	104
178	120

(N.B.)
Depth = 238
Boiling spring
See Fig. 5.

⑨ No. 1 spring Atakawa

Depth	Subterranean temp.
74 m	41°C
117	71
140	81
151	87
154	90
181	105
182	107
195	111
211	123
226	124

From the facts just mentioned, it is to be concluded that the phenomena of the boiling springs seen in Yugawara, Atami, Hokkawa, Atakawa, Katase, Yazu, Mine, Simogamo, etc. on the eastern side of the Izu peninsula are nothing but those in which the hot spring of liquid state at depth attains, in the course of ascension, to the boiling point of water at shallow depths owing to the decrease of pressure and causes continuous ejection of large quantities of hot water and of steam from the orifice.

2) Bored boiling springs and bored sub-boiling springs.

From their physical nature, boiling springs obtained by boring in the Izu peninsula are divided into two classes, viz., bored boiling springs and bored sub-boiling springs. They may be introduced by examples as below.

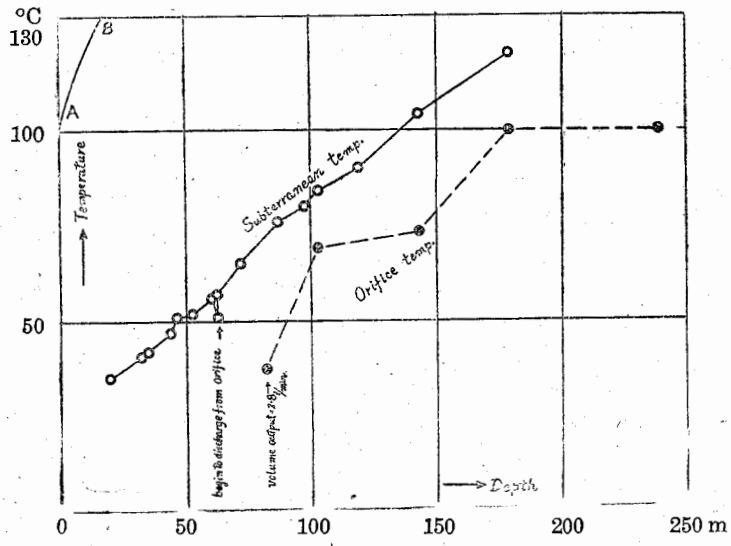

Fig. 5. Vertical distribution of subterranean temperature of No. 3 spring in Atakawa, Izu peninsula, and relation between the orifice temperature and the depth of the excavation.

The first example is No. 3 spring in Atakawa, Izu peninsula. The vertical distribution of subterranean temperatures in the excavation is shown in Fig. 5. The temperature shows as a nearly straight line rising with the increase of depth until at 179 metres it attains to 120°C. The water-head is, at first, below the surface of

ground, but at 62 metres it becomes slightly higher than the ground surface and a slight quantity of water is discharged. At 83 metres the water temperature is 37.5°C and discharge is 2.8 *l/min.* With the increase of depth, orifice temperature and discharge rise as shown by the broken line in Fig. 5. At 179 metres the subterranean temperature and orifice temperature attain to 120°C and 100°C respectively and boiling occurs. At last, at 238 metres orifice temperature of 100°C, volume output of hot water of 194 *l/min* and jet-height of about 15 metres are observed.

We shall call such spring a "Bored boiling spring." One of the characteristics common both to such a boiling spring and to a natural boiling spring is that the hydrostatic head of water is always higher than the level of the orifice.

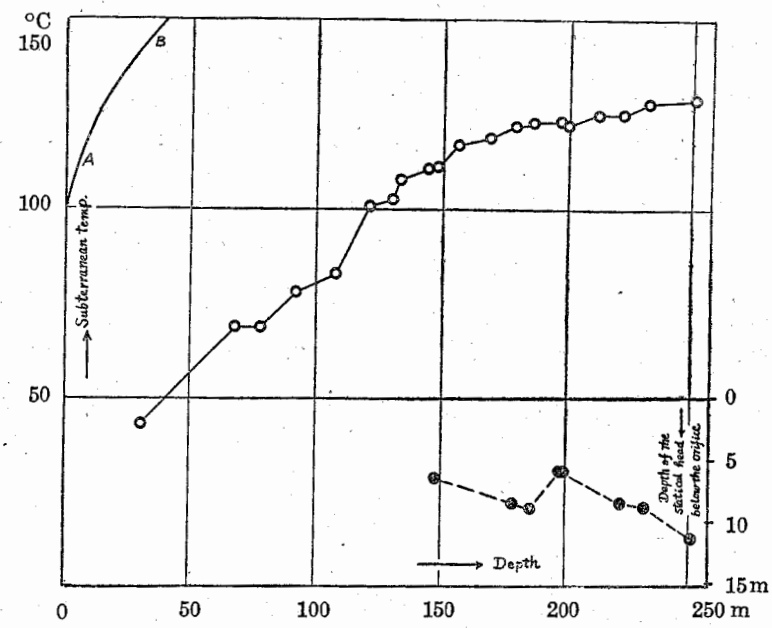

Fig. 6. Vertical distribution of subterranean temperature of Tutiya's spring in Atakawa, Izu peninsula, and relation between the statical head and the depth of the excavation.

The second example is the case of Tutiya's spring in Atakawa. The vertical distribution of subterranean temperature in the excavation was found to be as shown in Fig. 6. The temperature rises in shallow strata and then becomes comparatively homo-thermal. The

final temperature attains 129°C at 250 metres. The hydrostatic head of water in the vertical pipe is always 6~12 metres below the level of the orifice as shown by broken line in Fig. 6, and accordingly the spring can not discharge hot water automatically. By forcing it to discharge water artificially for a certain time interval, ejection of hot water and steam of boiling temperature occurs. When this artificial process is stopped, it continues to eject hot water at the rate of 727 *l/min* and also volumes of steam to the height of about 20 metres above the orifice.

The writer will for convenience call this type of spring a "Bored sub-boiling spring" in order to distinguish it from the preceding. Characteristics of this type of boiling spring are as follows:

(a) The hydrostatic head of water in the vertical well is almost always below the level of the orifice and it does not discharge water automatically.

(b) But, by causing a discharge of water by pumping at a rate over a certain limit for a certain time interval, the continuous ejection of steam and water of boiling temperature in atmospheric pressure are made to occur.

(c) Upon closing the orifice temporarily to stop the continuous ejection of hot water and steam, if the time interval is moderately long, the water-head falls below the level of the orifice and the spring will not erupt again without again causing artificial discharge.

A number of boiling springs in Yazu, Mine, Simogamo, etc. in southern Izu peninsula belong to this type. The difference between two types of the boiling spring just mentioned is caused only by the difference of height of hydrostatic head of water in the vertical tube relative to the level of the corresponding orifice. As to other points, we find no differences in the original characters of these two kinds of springs.

III. A probable explanation of the mechanism of a boiling spring obtained by boring.

First, the mechanism of the bored sub-boiling spring may be discussed. Let us consider the state in which the eruption is stopped, that is to say, when the head of water lies below the level of orifice. Then, water temperature in the vertical tube registers almost the

same as the subterranean temperature of the surrounding ground and rises with the increase of depth as shown in Fig. 4. As already mentioned, we can see that the water in the vertical tube and beneath is always in liquid state.

By exciting the discharge of water in the vertical tube, the hot water from below rises in compensation and if the speed of ascent is moderately large, the temperature in the tube attains nearly to that in the hot spring veins. The temperature in the hot spring veins, as mentioned above, is beyond the boiling point of water at atmospheric pressure, then when the water temperature attains to the boiling point under the pressure in situ in the course of ascent, it begins to ebullate and eject at moderately high speed to project into the air volumes of hot water and steam of boiling temperature. After boiling occurred, in the course of ascent, the water temperature may decrease along the (boiling point, pressure) curve with the reduction of pressure, and residual heat must be transformed into the latent heat of evaporation for changing the water into vapour. If pressure at the depth of the beginning point of boiling is less than the hydrostatic pressure extending from the hydrostatic head to that depth, the phenomena of boiling may be stable and ejection of hot water and steam may be continued without necessity of causing artificial discharge of water from the tube.

If we close the orifice and force the continuous discharge of hot water and steam to stop, the heat in the tube may be gradually dissipated to the surrounding ground, and then the temperature of the water in the vertical tube may diminish. Then if the orifice is opened again within the time while the temperature of water in vertical tube is yet higher than the boiling temperature, the spring will erupt spontaneously. But, if that is not done, the boiling may no longer occur without artificial discharge of water from the tube being brought about.

Next, the mechanism of the bored boiling spring may be discussed. In this case, as the hydrostatic head is always higher than the level of the orifice, the water in the vertical tube discharges automatically from the orifice without artificial stimulation, and then if the flow speed is above a certain limit, it begins to boil and to eject hot water and steam from the orifice just the same as the bored sub-boiling spring. Also, it is a matter of course that whenever the

orifice is reopened after having been once closed, it may begin to boil spontaneously.

IV. Mathematical study of the phenomena.

In the above, an abstract explanation of the mechanism of the boiling spring obtained by boring has been essayed. In this section, the writer proposes to study the phenomena mathematically in first approximation and also the mutual relation between projection velocity, volume output of hot water in the orifice and jet-height and temperature at the beginning point of boiling, etc.

1) Mutual relation between flow speed v_0 and water temperature θ_1 at the beginning point of boiling, projection velocity v', flow-speed of hot water at the orifice V/A, and jet-height L, etc.

We consider a tube which is perpendicular to the ground surface and of uniform section and through which the hot water in liquid state flows upward from below at speed v_0. Also let it be assumed that the ascending water begins to boil at a certain depth and continuously escapes as hot water and steam from the orifice into the air.

Let V represent the volume output of hot water at the orifice,

y : the rate of reduced water discharge from vapour at the orifice,

v' : the projection velocity of mixed flow of hot water and steam at the orifice,

v_0 : the flow speed at the beginning point of boiling,

ρ : the density of hot water (assumed as constant so far as the present paper concerned),

ρ_v : the density of vapour at the orifice,

A : the sectional area of uniform vertical tube,

c : the specific heat of hot water (assumed as constant so far as the present paper concerned),

c_v : the specific heat of vapour (ditto),

l : the latent heat of evaporation,

θ_2 : the orifice temperature of hot spring (equal to boiling temperature of water in atmospheric pressure),

θ_1 : the temperature of water at the beginning point of boiling,

θ_0 : the air temperature (assumed as equal to the subterranean temperature of shallow stratum of the ground),
S : the effective area of heat loss,
K : the proportional constant,

then the rate of flow at the beginning point of boiling may be written

$$Av_0 = V+y \qquad (1),$$

and the rate of discharge at the orifice may be written

$$Av' = V+\frac{\rho}{\rho_v}y \qquad (2).$$

From the condition of continuity of heat energy, we get approximately

$$Av_0\rho c\theta_1 = V\rho c\theta_2+l\rho y+\rho y c_v\theta_2+KS(\theta_2-\theta_0) \qquad (3)$$

Eliminate V and y from (1), (2) and (3), and we obtain

$$v' = v_0\left[1+\frac{\left(\frac{\rho}{\rho_v}-1\right)}{\frac{l}{c}-\left(1-\frac{c_v}{c}\right)\theta_2}\left\{(\theta_1-\theta_2)-\frac{KS}{Ac\rho v_0}(\theta_2-\theta_0)\right\}\right] \qquad (4)$$

Now put

$$\left.\begin{aligned}
c &= 1.007\\
l &= 539.1 \text{ cal.}\\
\rho &= 0.958 \text{ (value for 100°C)}\\
\rho_v &= 0.606\times10^{-3} \text{ (for saturated water vapour at 100°C, 1 atm. press.)}\\
c_v &= 0.465 \text{ (value for 100°C, 1 atm. press.)}\\
\theta_2 &= 100°\text{C}
\end{aligned}\right\} \qquad (5)$$

into (4) and we get

$$\frac{v'}{v_0} = 1+3.29\left\{(\theta_1-100)-\frac{\lambda'}{Av_0}(100-\theta_0)\right\}, \quad \lambda' \equiv \frac{KS}{\rho c} \qquad (6)$$

From (6), one can calculate the projection velocity of the mixed fluid of hot water and vapour at the orifice v', when v_0, θ_1, λ', θ_0, A are given.

Before going further, the order of λ' which is moderately variable for the shape of the orifices and for thermal properties of the surrounding ground may be calculated. Transform equation (3), substituting $y=0$ and get

$$V = Av_0 = \lambda' \frac{(\theta_2-\theta_0)}{(\theta_1-\theta_2)} \qquad (3)'$$

We obtain $\lambda'=0.66\ l/min$ or 11 cc/sec in the study of Rendaizi thermal spring[3], in the southern part of the Izu peninsula. The value of λ' is also calculated from (3)′ as $\lambda'=74$ cc/sec by the measurements shown in Fig. 5 in which volume output V and the corresponding orifice temperature θ_2 are known, assuming that the underground temperature of hot water θ_1 is equal to the observed temperature at the bottom of the bore-hole, and that θ_0 at Atakawa is 15°C.

Next, we compared the value of the first term in the brackets of equation (6) with the second term for two examples shown in Figs. 5~6, assuming that $\lambda'=43$ cc/sec which is the mean value of the above-mentioned two cases, and got 1:0.23, 1:0.03 respectively corresponding to $\theta_1=105$, 109°C. Therefore, when the temperature of hot spring at the beginning point of boiling θ_1 is moderately higher than 100°C and when 5~20% of error is permitted, we may neglect the second term and obtain

$$\frac{v'}{v_0} = 1 + \frac{\left(\frac{\rho}{\rho_v}-1\right)(\theta_1-\theta_2)}{\frac{l}{c}-\theta_2\left(1-\frac{c_v}{c}\right)} \qquad (7)$$

It will be seen from equation (7) that when the water temperature at the beginning point of boiling is comparatively higher than 100°C, the ratio of the projection velocity at the orifice to the flow speed at the beginning point of boiling v'/v_0 may be moderately large, for example $v'/v_0=50$ for $\theta_1=115$°C. The water temperature at the beginning point of boiling is, in general 105~115°C, then we can recognize that the projection velocity of this type of springs is very much greater than that of ordinary hot springs.

The projection velocity of a boiling spring obtained by boring is so large that the splendid sight of projecting the column of hot

3) T. Fukutomi: Bull. Earthq. Res. Inst. 14 (1936), 267.

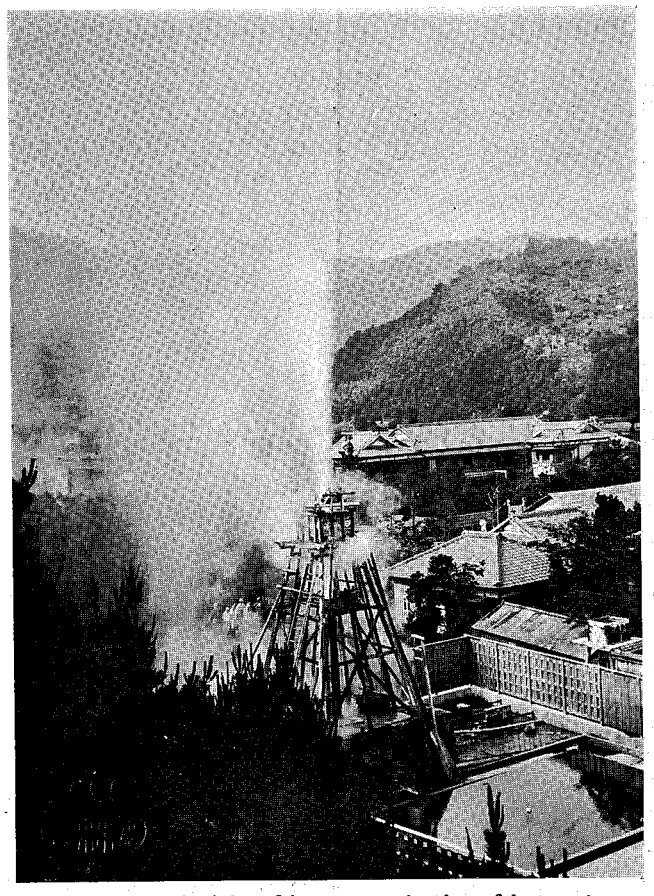

Fig. 7a. A sight of incessant ejection of hot water and steam of boiling temperature of No. 2 spring in Mine, southern Izu peninsula. (Photo Hayasi.)

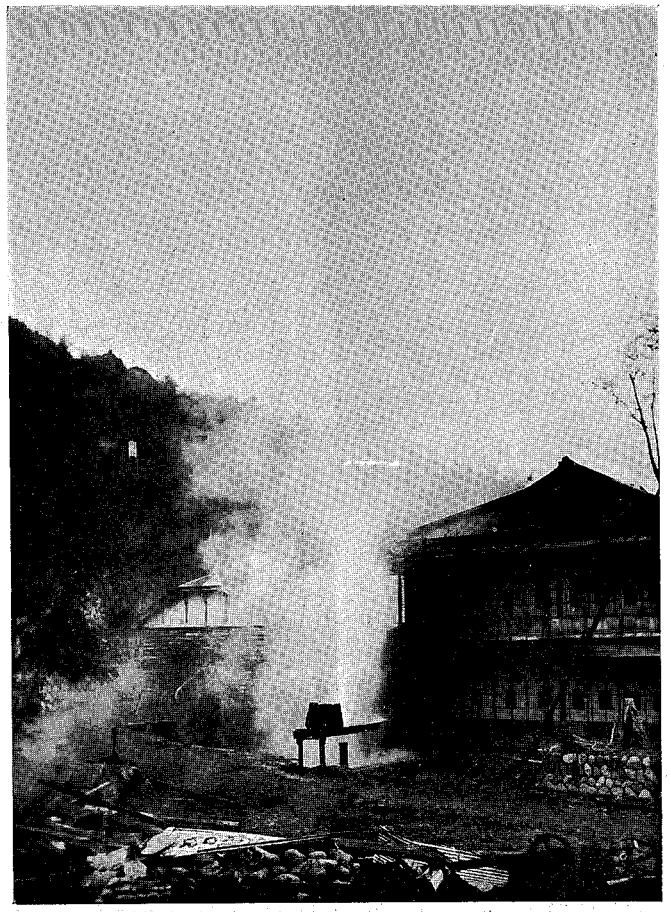

Fig. 7b. A sight of incessant ejection of hot water and steam of boiling temperature of Tamano-yu spring in Mine, southern Izu peninsula. (Dec. 15, 1931. Photo Hayasi.)

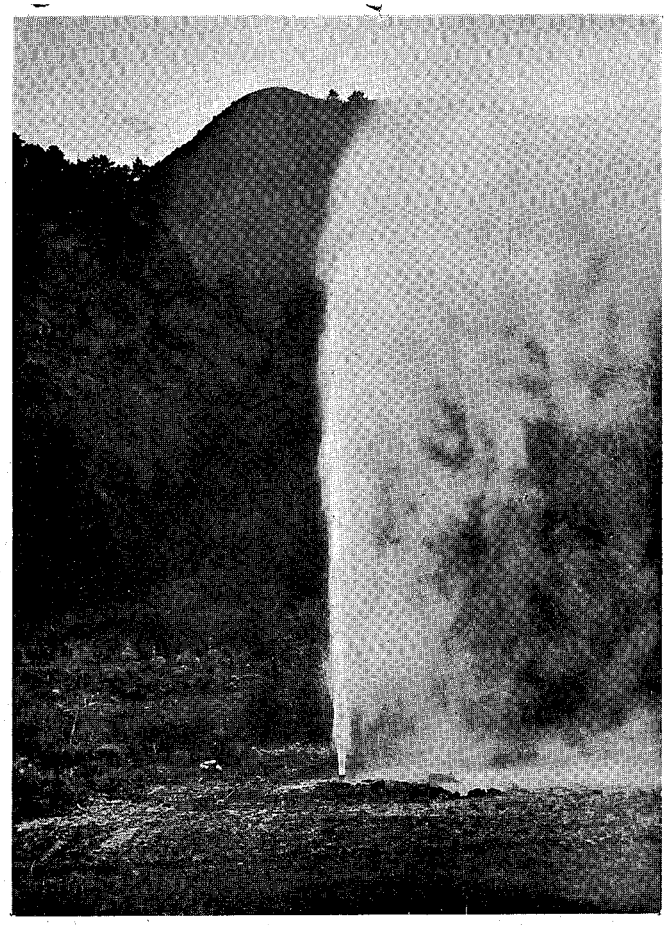

Fig. 7c. A sight of incessant ejection of hot water and steam of boiling temperature of No. 37 spring in Yazu, southern Izu peninsula. (Dec. 26, 1929. Photo Hayasi.)

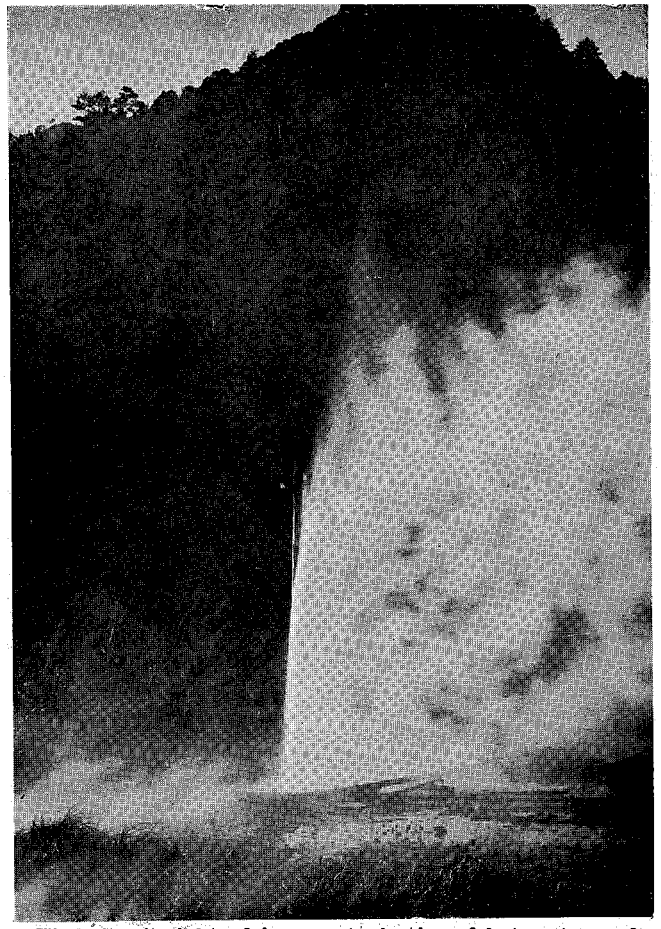

Fig. 7d. A sight of incessant ejection of hot water and steam of boiling temperature of No. 38 spring in Yazu, southern Izu peninsula. (jet height ≒ 11 m). (Mar. 8, 1930, photo.

water and steam to the height of 50~100 m baffles description. We reproduce in Fig. 7 some photos of the sight of this marvelous projections in Yazu and in Mine, southern Izu peninsula.

Let L be the jet-height, g the gravity acceleration at the surface of the earth, then we get approximately

$$L \doteqdot \frac{v'^2}{2g} = \frac{v_0^2}{1960}\left[1+3.29\left\{(\theta_1-100)-\frac{\lambda'}{Av_0}(100-\theta_0)\right\}\right]^2 \quad (8)$$

Eliminating y from (1) and (3), we obtain

$$\frac{(V/A)}{v_0} = \frac{1-\frac{c}{l}\left(\theta_1-\frac{c_v}{c}\theta_2\right)}{1-\frac{c}{l}\left(1-\frac{c_v}{c}\right)\theta_2}+\frac{\lambda'}{Av_0}\frac{(\theta_2-\theta_0)}{\left(\frac{l}{c}+\frac{c_v}{c}\theta_2-\theta_0\right)} \quad (9).$$

Substitute (5) into (9) and we get

$$\frac{(V/A)}{v_0} = 1.112\left\{1-0.00187(\theta_1-46.2)\right\}+\frac{\lambda'}{Av_0}\frac{(100-\theta_0)}{(582.2-\theta_0)} \quad (10).$$

And also, the relation between v' and V/A may be written

$$\frac{v'}{(V/A)} = \frac{1-\frac{c}{l}\left(1-\frac{c_v}{c}\right)\theta_2+\frac{c}{l}\left(\frac{\rho}{\rho_v}-1\right)\left\{(\theta_1-\theta_2)-\frac{\lambda'}{Av_0}(\theta_2-\theta_0)\right\}}{1-\frac{c}{l}\left(\theta_1-\frac{c_v}{c}\theta_2\right)+\frac{\lambda'}{Av_0}\frac{(\theta_2-\theta_0)\left\{1-\frac{c}{l}\left(1-\frac{c_v}{c}\right)\theta_2\right\}}{\left(\frac{l}{c}+\frac{c_v}{c}\theta_2-\theta_0\right)}} \quad (11)$$

Substitute (5) into (11) and we get

$$\frac{v'}{(V/A)} = \frac{482.2+1582\left\{(\theta_1-100)-\frac{\lambda'}{Av_0}(100-\theta_0)\right\}}{582\ 2-\theta_1+\frac{482.2\,\lambda'(100-\theta_0)}{Av_0(582.2-\theta_0)}} \quad (12)$$

Observations of the factors of the boiling spring were carried out with difficulty except the projection height L, orifice temperature θ_2 and rate of discharge of hot water V/A which can be measured comparatively easily, for the reason that the observations without suitable equipment involve no little personal danger.

When jet-height L is known, the value of v' can be calculated from (8). And since v' and V/A are known, θ_1 in first approximation can be estimated, assuming $\lambda'=0$ in equation (12). Next, one may substitute the calculated values of θ_1 and V/A into equation (10), also assuming $\lambda'=0$ in first approximation, and get the value of v_0. Now substitute the value of v_0 into equation (12) assuming $\lambda'=43$ cc/sec, and the value of θ_1 in second approximation can be calculated. After repeating this process with equation (10), the value of v_0 in second approximation can also be calculated. We indicate some examples in which v', v_0, θ_1 can be estimated from the observed values of L and V/A in the following:

Example 1. No. 3 spring in Atakawa (Fig. 5)

$$\left.\begin{array}{l} V/A = 101.6 \text{ cm/sec} \\ L \doteqdot 15 \text{ m} \\ \theta_2 = 100°\text{C} \end{array}\right\}\text{obs.} \qquad \left.\begin{array}{l} v' = 17.2 \text{ m/sec} \\ \theta_1 = 105°\text{C} \\ v_0 = 102.6 \text{ cm/sec} \end{array}\right\}\text{calc.}$$

Example 2. Tutiya's spring in Atakawa (Fig. 6)

$$\left.\begin{array}{l} V/A = 69.0 \text{ cm/sec} \\ L \doteqdot 20 \text{ m} \\ \theta_2 = 100°\text{C} \end{array}\right\}\text{obs.} \qquad \left.\begin{array}{l} v' = 19.8 \text{ m/sec} \\ \theta_1 = 108.7°\text{C} \\ \quad (108.3°\text{C in first appro.}) \\ v_0 = 70.1 \text{ cm/sec} \\ \quad (70.2 \text{ cm/sec} \quad ,, \quad) \end{array}\right\}\text{calc.}$$

Example 3. Toyozato spring in Simogamo

$$\left.\begin{array}{l} V/A = 95.3 \text{ cm/sec} \\ L \doteqdot 15 \text{ m} \\ \theta_2 = 100°\text{C} \end{array}\right\}\text{obs.} \qquad \left.\begin{array}{l} v' = 17.2 \text{ m/sec} \\ \theta_1 = 105.5°\text{C} \\ v_0 = 96.3 \text{ cm/sec} \end{array}\right\}\text{calc.}$$

2) Another method of estimating θ_1.

As to the normal springs of Atami, Rendaizi and Yazu etc., a positive linear relation $x=A'\theta_1+B'$ (13) exists between the Cl-contents x and orifice temperatures θ_1 as shown by an example in Fig. 8, where A', B' are the constants proper to the locality. As to springs, the Cl-contents of which is above a certain limit, the orifice temperatures are always constant at about 100°C displaying the phenomena of a boiling spring. This may be attributed to the loss of

excess heat of water of high temperature at some depth as latent heat of evaporation due to smaller pressure at the earth's surface. Then, it is not so absurd to assume that the same linear relation must be satisfied between Cl-contents and the temperatures before the phenomenon of boiling occurs.

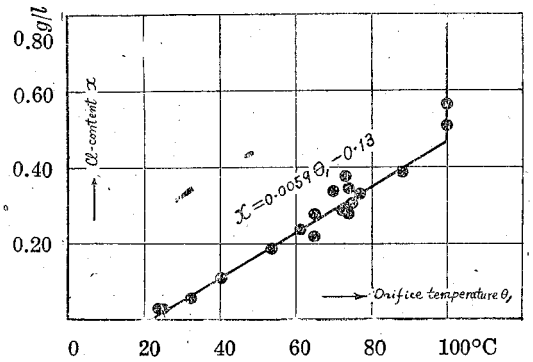

Fig. 8. Relation between Cl-contents and corresponding orifice temperatures of Yazu hot springs.

Let X g/l be the amount of a chemical constituent in the spring before boiling, m g/l the amount of the same material in water at the orifice, and neglect the effect of heat dissipation from the pipe into surrounding ground or from the orifice to air, then we get

$$(V+y)\rho c\theta_1 = V\rho c\theta_2 + ly\rho + \rho y c_v \theta_2 \qquad (3)''$$

$$x(V+y) = mV \qquad (14)$$

Eliminating y from these equations, we obtain

$$m = \frac{x}{1-\left(\frac{c}{l}\right)\frac{(\theta_1-\theta_2)}{1-\frac{(c-c_v)}{l}\theta_2}} \qquad (15)$$

Now put (13) into (15), then we get (16) or (16)′

$$\theta_1 = \frac{m\left(1+\frac{c_v}{l}\theta_2\right)-B'\left\{1-\frac{(c-c_v)}{l}\theta_2\right\}}{m\frac{c}{l}+A'\left\{1-\frac{(c-c_v)}{l}\theta_2\right\}} \qquad (16)$$

$$\theta_1 = \frac{1.0862\,\mathrm{m}-0.8994\,B'}{0.001868\,\mathrm{m}+0.8994\,A'} \qquad (16)'$$

Then, if A', B' and m are known, θ_1 can be calculated from (16)$'$.

Let us consider an example in which v', v_0, θ_1, L are estimated from the observed value of m, V/A etc.

Example 4. No. 39 spring in Yazu.

$A'=0.0059$, $B'=-0.13$ for Yazu springs (calculated from Fig. 8)

$$\left.\begin{array}{l} V/A = 90.2 \text{ cm/sec} \\ \theta_2 = 100°\text{C} \\ m = 0.508 \text{ g}/l \\ L \doteqdot 15\sim20 \text{ m} \end{array}\right\}\text{obs.} \qquad \left.\begin{array}{l} \theta_1 = 107°\text{C} \\ v_0 = 91.5 \text{ cm/sec} \\ v' = 18.4 \text{ m/sec} \\ L = 17.0 \text{ m} \end{array}\right\}\text{calc.}$$

It will be seen from the results that the calculated value of L is fairly in agreement with that observed.

Regarding this section the writer had already discussed the matter in a previous paper,[4] in which he did not take into consideration the term $\varrho y c_v \theta_2$ in equation (3)$''$. Therefore he again proposed to discuss this problem.

3) Fundamental idea with respect to the ascent speed v_0 at the beginning point of boiling.

Since many hot springs in Japan are of the compressed ground water type, examples of which are the hot springs of Beppu,[5] Itô[6], etc., it is not so absurd to assume that many boiling springs also belong to the same type. Indeed, it is known that the boiling spring in Yazu, Simogamo, etc. may belong to this type. As to the ordinary hot springs of the compressed ground water type, the relation between flow speed v_0 and height of statical head above the level of the orifice $\mathfrak{H}$ is shown experimentally by the linear equation[7]

$$v_0 = \kappa \mathfrak{H} \tag{17}$$

where $\varkappa$ is the constant proper to the individual hot springs. According to the Darcy-Thiem theory of the stratified ground water

4) T. Fukutomi: Bull. Earthq. Res. Inst. 14 (1936), 259. Disin 8 (1936), 457–468.

5) T. Momitu, K. Seno, K. Yamashita: Memoirs Coll. Sci. Kyoto Imp. Univ. S-A. 23 (1940), 75–95.

6) T. Fukutomi: Bull. Earthq. Res. Inst. 15 (1937), 506–535. Disin 12 (1940), 404–418.

7) T. Nomitu, K. Seno, K. Yamashita: loc. cit. 6).
T. Fukutomi: Disin 12 (1940), 404–418.

under pressure condition, the flow speed v_0 in the vertical well is given by the following equation[8]:

$$v_0 = \frac{2\pi km}{A \log_e \frac{R}{r}}(H-l) \tag{18}$$

where k is the coefficient of permeability, m the thickness of the water-bearing stratum between the impermeable layers, A the sectional area of well, r and R the radius of the well and of the effecting

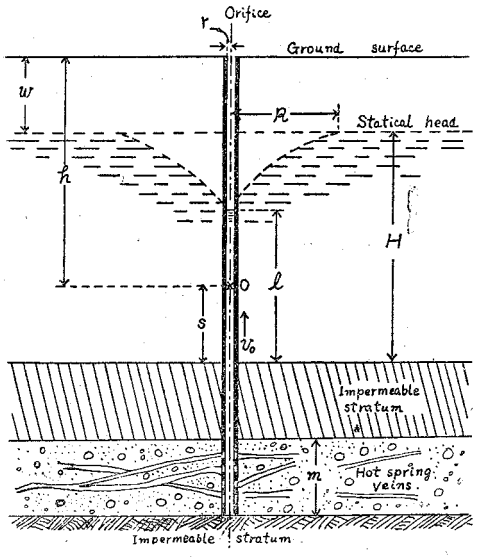

Fig. 9. The elevation sketch map of a stratified hot spring under compressed condition.

area respectively, H and l the height above the basal rock of the statical pressure head and of the orifice. By putting $\mathfrak{H} \equiv (H-l)$, $\varkappa = \frac{2\pi km}{A \log_e \frac{R}{r}}$ in equation (18), one obtains (17).

The relation which is specified by (17) may be, in general, satisfied for the hot springs of all the compressed ground water type, not only for those of the stratified type above mentioned, if the flow

8) H. Kimizima: "Underground water" (in Japanese), 265.

speed in the hot spring vein is proportional to the pressure gradient in it.

But, let us continue the discussion regarding the case of stratified ground water under pressure condition as shown in Fig. 9, in order to understand easily its physical meaning.

Let 0 be a point in water of the vertical well, P_0 the pressure at 0, s the height of 0 above the basal rock, π the atmospheric pressure, and assume that v_0 is not very large, then we get approximately

$$(P_0-\pi)+s\rho g = l\rho g \tag{19}$$

Put (19) into (18) and get

$$v_0 = \frac{\kappa}{\rho g}\left\{\rho g(H-s)-(P_0-\pi)\right\} \tag{20}$$

Now assume that the spring is a boiling spring obtained by boring and put 0 as the beginning point of boiling. Also let w be the height difference between the orifice and the statical pressure head (the sign of w is positive or negative corresponding to whether the spring is a bored sub-boiling one or a bored boiling spring respectively), and let h be the depth of the beginning point of boiling below the orifice, then (20) may be written

$$v_0 = \kappa\left\{(h-w)-\frac{(P_0-\pi)}{\rho g}\right\} \tag{21}$$

It may be said, in general, that the ascending flow speed at the beginning point of boiling v_0 is approximately proportional to the pressure difference between the pressure of the hydrostatic column of water extending from the orifice to that depth and the pressure in situ at the point subtracting the atmospheric pressure.

4) Approximate estimation of the depth of the beginning point of boiling below the orifice h.

Let v' and v_0 represent the projection speed at the orifice and the flow speed at the beginning point of boiling in the vertical well of the boiling spring respectively, $\bar{\rho}$ the mean density of mixed phase of hot water and steam at any point between the orifice and the beginning point of boiling, and p (kg/cm^2) the pressure at the point,

then from Bernoulli's law of hydrodynamics the following relation may be written

$$\frac{1}{2g}(v'^2-v_0^2)+h=\int_{\pi}^{P_0}\frac{1}{\bar{\rho}}\,dp \tag{22}$$

As it is not so absurd to assume that the density ϱ of hot water is independent of the change of pressure, we transform equation (22) displaying the caution at this point and get

$$h=\frac{1}{\rho}\int_{\pi}^{P_0}\frac{\rho}{\bar{\rho}}dp-\frac{v_0^2}{2g}\left\{\left(\frac{v'}{v_0}\right)^2-1\right\} \tag{23}$$

We neglect the effect of heat disipation of hot water in the vertical well to surrounding ground, and substitute the value of (v'/v_0) of (7) and of v_0 of (21) into (23), then we get

$$h=\frac{2JN\kappa^2-1+\sqrt{1+4J\kappa^2\left(\frac{I}{\rho}-N\right)}}{2J\kappa^2} \tag{24}$$

where

$$\left.\begin{aligned} I&\equiv\int_{\pi}^{P_0}\frac{\rho}{\bar{\rho}}\,dp\\ J&\equiv\frac{1}{2g}\left\{\left(\frac{v'}{v_0}\right)^2-1\right\}=\frac{1}{2g}\left[\left\{1+\frac{\left(\frac{\rho}{\rho_v}-1\right)(\theta_1-\theta_2)}{\frac{l}{c}-\theta_2\left(1-\frac{c_v}{c}\right)}\right\}^2-1\right]\\ N&\equiv w+\frac{P_0-\pi}{\rho}\,. \end{aligned}\right\} \tag{25}$$

J and N may be calculated, if the values of θ_1, w and P_0 are known. And P_0 is calculated from (27), if θ_1 is known.

In temperatures of 100~200°C, the relation[9] between the boiling point θ of water and the pressure p(kg/cm²) may be written approximately

$$\log_{10}p=A''-\frac{B''}{d+\theta} \tag{26}$$

9) S. Nukiyama: Heat Engine (in Japanese) (Iwanami Kôza), 4 page.

where $A''=5.6485$, $B''=2101.1$, $d=273.2°C$. And then change the form and get

$$P_0 = e^{\frac{1}{M}\left(A''-\frac{B''}{d+\theta_1}\right)} \tag{27}$$

where $M=0.4343$.

Next, we may calculate $I \equiv \int_{\pi}^{P_0} \frac{\rho}{\bar{\rho}} dp$. Let v'' and θ be the ascending speed and water temperature respectively at any point between the orifice and the beginning point of boiling, then we get

$$\frac{\rho}{\bar{\rho}} = \frac{v''}{v_0} \tag{28}$$

Substituting v'', $\varrho_v \frac{p(d+\theta_2)}{\pi(d+\theta)}$, θ instead of v', ϱ_v, θ_2 of equation (7) respectively, we get

$$\frac{v''}{v_0} = 1 + \frac{\left\{\frac{\rho}{\rho_v}\cdot\frac{\pi(d+\theta)}{p(d+\theta_2)}-1\right\}(\theta_1-\theta)}{\frac{l}{c}-\theta\left(1-\frac{c_v}{c}\right)} \tag{29}$$

As the water temperature θ at any point between the orifice and the beginning point of boiling is considered to vary along the (boiling point of water, pressure) curve from θ_1 to θ_2, it may be given by the following equation which is another expression of (26).

$$\theta = \frac{(B''-dA'')+dM\log_e p}{A''-M\log_e p} \tag{30}$$

From (28), (29) and (30) we get

$$\Delta \equiv \frac{I}{\rho} - \frac{(P_0-\pi)}{\rho} = \frac{1}{\rho}\int_{\pi}^{P_0}\left\{\mu\frac{\pi}{p}\frac{D}{(A''-M\log_e p)}-1\right\}\frac{(\eta-\zeta\log_e p)}{(\gamma-\xi\log_e p)}dp \tag{31}$$

$$\left.\begin{aligned}
\text{where}\quad \mu &\equiv \frac{\rho}{\rho_v} = 1583 \\
D &\equiv \frac{B''}{d+\theta_2} = 5.63 \\
\gamma &\equiv A''\left(\frac{l}{c}\right)-(B''-dA'')\left(1-\frac{c_v}{c}\right) = 2727.4 \\
\xi &\equiv M\left\{\frac{l}{c}+d\left(1-\frac{c_v}{c}\right)\right\} = 296.6 \\
\eta &\equiv A''(\theta_1+d)-B'' = 5.6485\theta_1-558 \\
\zeta &\equiv M(\theta_1+d) = 0.4343\theta_1+118.6
\end{aligned}\right\} \tag{32}$$

Put the value of (32) into (31) and calculate the value of Δ which is indicated in Table II.

Table II. Relation between Δ and θ_1.

θ_1 (°C)	100	102	104	106	108	110	115	120	130	140
Δ (m)	0	2.6	9.6	22.1	39.6	62.0	141	252	566	987

Assuming $\varkappa=0.5$ 1/sec (a typical example of the boiling spring obtained by boring), the writer calculated the value of the depth of the beginning point of boiling below the orifice h corresponding with θ_1 from equation (24) and represented in Fig. 10. Since $w=-2\sim+20$ m and $\theta_1=100°\sim120°$C for the boiling spring obtained by boring, $h=0\sim35$ m were obtained from the figure.

Then it is to be concluded that the phenomena of boiling seen in the case of the boiling spring obtained by boring occur in very shallow strata of less than 30 or 40 metres depth from the surface of the ground.

5) Approximate calculation of ascent speed v_0 at the beginning point of boiling.

Now, put the value of h of (24) into (21) and the following equation is obtained which is the approximate solution of neglecting the effect of dissipation of heat energy to surrounding ground from the vertical pipe near the orifice.

$$v_0 = \frac{1}{2J\kappa}\left\{\sqrt{4J\kappa^2(\Delta-w)+1}-1\right\} \tag{33}$$

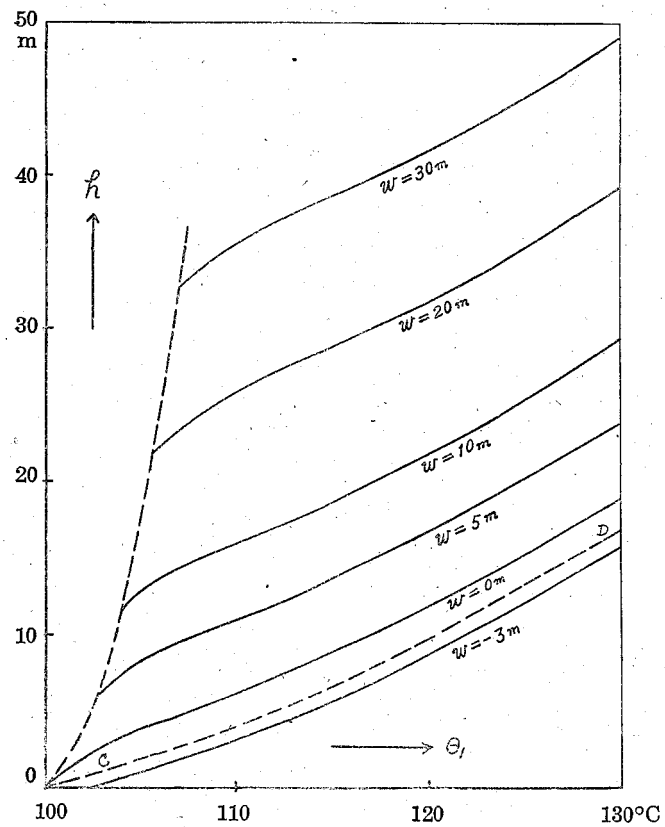

Fig. 10. Relation between the depth of the beginning point of boiling below the orifice h, the water temperature at the point θ_1 and the depth of the statical head below the orifice w when $\kappa = 0.5$ 1/sec.

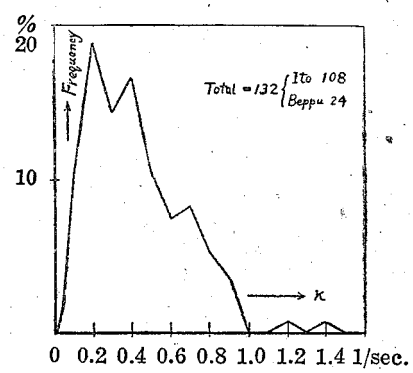

Fig. 11. The frequency distribution of value of κ of the boiling springs obtained by boring.

Thus, the ascent speed v_0 at the beginning point of boiling is found to be the function of θ_1, w and $\varkappa$.

The frequency distribution of the value of $\varkappa$ of the bored springs which is estimated for 108 orifices at Itô[10] and 24 orifices at Beppu[11] was found to be as shown in Fig. 11. It will be seen that 98% of the total number of springs show values of less than 0.9 1/sec. The value of $\varkappa$ for only two natural springs was also estimated at Beppu, at the small values of 0.0027 and 0.0003 1/sec respectively. Though

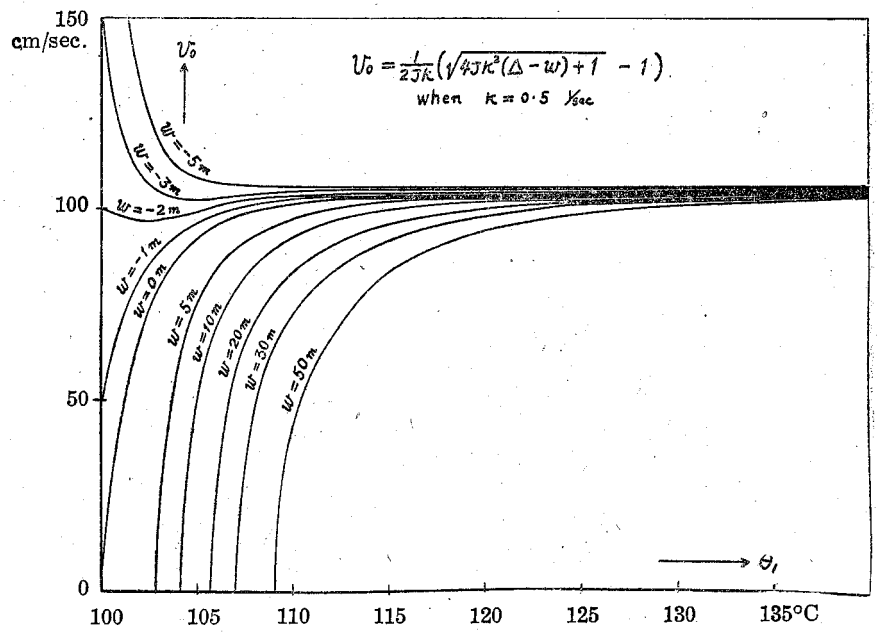

Fig. 12. Relation between the flow speed at the beginning point of boiling v_0, the water temperature at the point θ_1, and the depth of the statical head below the orifice w (assume $\kappa = 0.5$ 1/sec.).

it is not so certain on account of unsufficient data, it may be said that the value of $\varkappa$ is very small in the case of the natural spring in comparison with the case of the boring spring.

The relation between v_0, θ_1 and w when the value of $\varkappa$=0.50 1/sec is shown in Fig. 12, corresponding to the boiling spring obtained by

10) T. Fukutomi: loc. cit. 6).
11) T. Nomitu, K. Seno, K. Yamashita: loc. cit. 5).

boring. Relation between v_0, θ_1 and $\varkappa$ when $w=0$ is also shown in Fig. 13.

It will be seen from the figure that in the case of the boiling spring obtained by boring v_0 tends to approach a constant value of about 105 cm/sec with increasing value of θ_1.

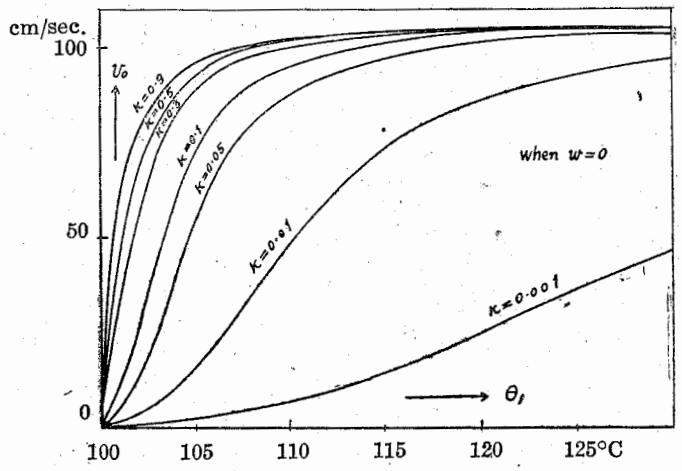

Fig. 13. Relation between v_0, θ_1 and κ when $w = 0$.

When the value of v_0, $\varkappa$, θ_1 of a boiling spring is known, the value of w can be calculated from equation (33), and also it can be estimated approximately whether the spring is a boiling spring or a sub-boiling spring by noting if the sign of the calculated value of w is positive or negative. Practical examples are given in the following.

1. Example 1 in section 1).

bored boiling spring	obs.	$w = -2 \sim -3$ m (assume $\kappa = 0.5$) $h \doteqdot 1.5$ m a boiling spring	calc.

2. Example 2 in section 1).

$w = 11.4$ m a bored sub-boiling spring	obs.	$w \doteqdot 20$ m (assume $\kappa = 0.5$) $h \doteqdot 25$ m a sub-boiling spring	calc.

3. Example 3 in section 1).

a bored sub-boiling spring	obs.	$w \doteqdot 0$ $h \doteqdot 4$ m ambiguous	calc.

4. Example 4 in section 2).

a bored sub-boiling spring	obs.	$w \doteqdot 4$ m (assume $\kappa = 0.5$) $h \doteqdot 9$ m a sub-boiling spring	calc.

6) Approximate calculation of projection velocity at orifice v' and jet-height L.

Put the value of v_0 of (33) into (4) and (8), and we get approximately

$$v' = \frac{\sqrt{2gJ+1}}{2J\kappa}\left\{\sqrt{4J\kappa^2(J-w)+1}-1\right\} \tag{34}$$

$$L = \frac{(2gJ+1)}{8gJ^2\kappa^2}\left\{\sqrt{4J\kappa^2(J-w)+1}-1\right\}^2 \tag{35}$$

Fig. 14 shows the values of v' and L changing with values of $\varkappa$ and θ_1 respectively. As the value of $\varkappa$ is 0.1~0.7 l/sec, and the value of w is generally +20~−2 metres for the boiling spring obtained by boring, the projection velocity at the orifice v' and jet-height L increase remarkably with the rise of temperature θ_1. For example, v'=50 m/sec and L=140 m for θ_1=115°C are obtained.

In the case of the natural boiling spring the effect of heat dissipation into the surrounding ground in shallow strata was moderately large and θ_1 is low in comparison with case of the bored spring, then the value of v' or L may be less than those shown in (34) or (35) and Fig. 14. But, it can be estimated approximately from these equations and figure that values of v' or L of the natural boiling springs are at least less than 7 m/sec or less than 2 m respectively owing to the small value of $\varkappa$.

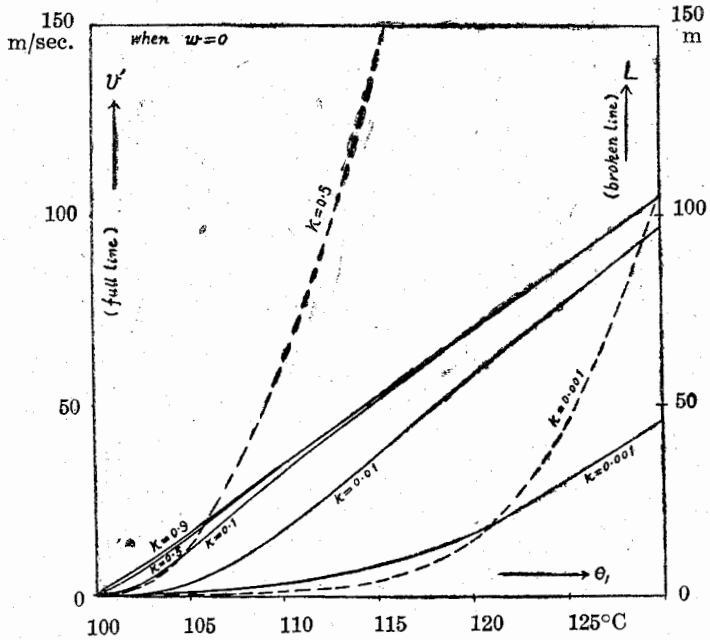

Fig. 14. Relation between the projection velocity v', the jet-height L and κ, θ_1 when $w = 0$.

7) Approximate calculation of the flow speed of hot water at orifice V/A.

Put (33) into (9) and get the flow speed of hot water at orifice V/A as follows:

$$V/A = \frac{\left\{\sqrt{4J\kappa^2(\Delta-w)+1}-1\right\}\left\{1-\frac{c}{l}\left(\theta_1-\frac{c_v}{c}\theta_2\right)\right\}}{2J\kappa\left\{1-\frac{c}{l}\left(1-\frac{c_v}{c}\right)\theta_2\right\}} \tag{36}$$

Relation between V/A, θ_1 and $\varkappa$ when $w=0$ is shown in Fig. 15. It will be seen from the figure that V/A tends to approach a constant value of about 100 cm/sec with increasing value of θ_1 just the same as the case of v_0. Generally in the case of a boiling spring obtained by boring, in which $\varkappa=0.3\sim0.9$ and $w=-3\sim15$ m, V/A indicates the value of about 50~100 cm/sec for $\theta_1>105°$C.

Volume outputs of hot water at orifices V of the boiling springs obtained by boring in Simogamo, Mine, Yazu, Atakawa and Atami measured by engineers of boring and owners of the springs are shown in Table III.

We calculated V/A from the values of V and A, and also indicate the results in the table. Values of V/A just mentioned, are considered to contain moderate errors due to the inaccurate methods and equipment for measurement of V, therefore the mean value of V/A for every locality was calculated and compared with the theoretical value. It will be seen from the table and the figure that the observed values approximately agree with the theoretical values. We believe that fair coincidence may be obtained, if calculations be carried out in future using the results of more refined measurements.

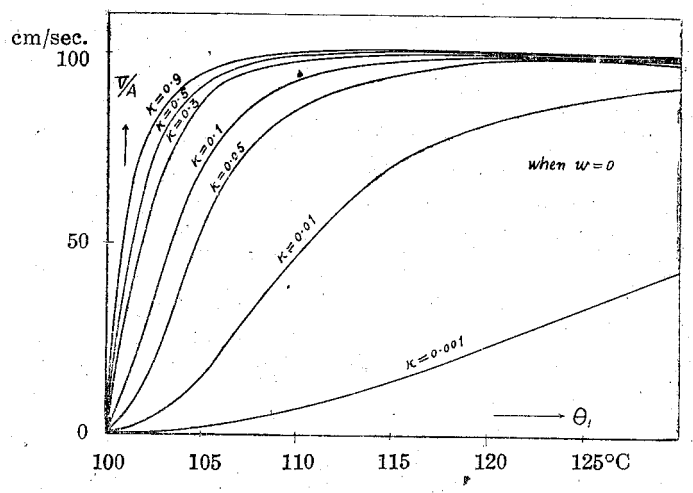

Fig. 15. Relation between the flow speed of hot water at the orifice V/A, κ and θ_1 when $w = 0$.

V. Criterion for determining the boiling or the sub-boiling spring and classification of hot springs by them.

1) Conditions for the beginning of boiling of the bored boiling spring.

The characteristics of the bored boiling spring are that the level of the statical head is always higher than that of the orifice and that the phenomena of boiling are always going on.

Table III. Observed values of the flow speed of hot water at orifice of the boiling spring V/A.
(Cases of the boiling springs obtained by boring)

Hot spring	Locality	Depth of excavation	Dia. of orifice	Sectional area A	Volume output of hot water V	V/A	mean value	N.B.
			cm	cm²	l/min.	cm/sec.		
Toyozato spring	Simogamo hot spring, Izu.	39m	6.35	31.8	182	95	cm/sec. 107	Mean flow speed of 11 ordinary hot springs = 26.7 cm/sec.
Sirasaka spring		91	,,	,,	27	14		
Kotobuki spring		114	,,	,,	273	143		
Takara No. 1 spr.		56	,,	,,	182	95		
Takara No. 2 spr.		56	,,	,,	182	95		
Gaku spring		51	,,	,,	182	95		
Tamano-yu		35	,,	,,	182	95		
Tôryû spring		33	,,	,,	273	143		
Takasima No. 1		52	,,	,,	22	11		
Takasima No. 2		73	,,	,,	546	286		
Kiku spring	Mine hot spring, Izu.	90	9.0	63.5	154	41	cm/sec. 102	Mean flow speed of 4 ordinary hot spr. =16.4 cm/sec.
Tamano-yu		48	10.16	81.0	910	187		
No. 2 spring		18	15.23	181.3	840(?)	77		
No. 36 spring	Yazu hot spring, Izu.	46	6	28.2	177	104	cm/sec. 85	Mean flow speed of 14 ordinary hot spr. =29.1 cm/sec.
No. 37 spring		38	6	,,	90	53		
No. 38 spring		46	8	50.2	272	90		
No. 39 spring		37	8	,,	272	90		
Tutiya's spring	Atakawa hot spr., Izu.	251	15	176	727	69	cm/sec. 85	
No. 3 spring		238	6.35	31.8	194	102		
No. 7 spring	Atami hot spring, Izu.	75	7.6	45.2	200	74	cm/sec. 49	Pump was used for ordinary hot springs
No. 16 spring		34	,,	,,	289	107		
No. 18 spring		73	,,	,,	319	118		
No. 29 spring		29	,,	,,	12	4		
No. 30 spring		17	15 2	181.2	63	6		
No. 34 spring		73	7.6	45.2	105	39		
No. 35 spring		67	,,	,,	112	41		
No. 40 spring		60	,,	,,	87	32		
No. 57 spring		218	,,	,,	70	25		
No. 85 spring		122	,,	,,	189	42		

(Case of a bored-spring nearly similar to natural boiling spring)

Hot spring	Locality	Depth of excavation	Dia. of orifice	Sectional area A	Volume output of hot water V	V/A	N.B.
		c.a.	cm	cm²	l/min.		
Ômoto spring	Simogamo hot spring, Izu.	1 m	6.35	31.8	18.2	9.5 cm/sec.	As the depth of excavation is very shallow, it may be considered as a natural boiling spring.

Now, consider a hot spring which has at depth the underground temperature Θ higher than boiling temperature at orifice θ_2 and attains just to the boiling point θ_2 at the orifice on account of heat dissipation to surrounding ground in the course of ascent at flow speed v_0'. If it had been possible to increase its flow speed from v_0' to v_0 in this case without changing any other conditions, then the phenomena of boiling would have been found at a lower point in the vertical tube below the level of the orifice.

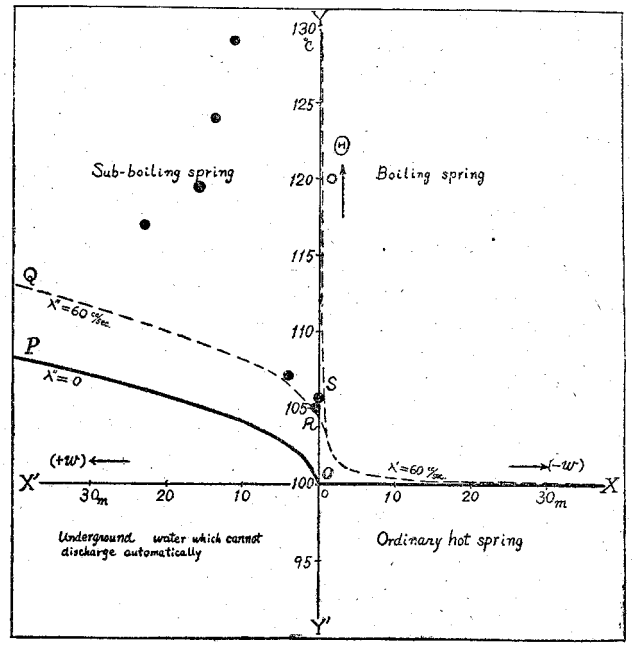

Fig. 16. The critical curves determining the boiling spring and the sub-boiling spring. Dots and circle indicate the observed bored sub-boiling springs and the observed bored boiling spring respectively.

The critical velocity v_0' was obtained in first approximation by substituting v_0', Θ instead of v_0, θ_1 respectively into equation (3)'. Therefore, the height of the statical head above the orifice $\mathfrak{H}'$ or $(-w)'$ which gives just the flow speed v_0' is obtained by (17) as following:

$$\mathfrak{H}' = (-w)' = \frac{\lambda'}{\kappa A}\frac{(\theta_2-\theta_0)}{(\Theta-\theta_2)} \tag{37}$$

We calculate the corresponding value of $(-w)'$ for Θ, for instance, putting $\varkappa=0.5$ l/sec, $A=31.64$ cm^2, $\lambda'=60$ cc/sec, $\theta_2=100°$C, $\theta_0=15°$C and plot the points in Fig. 16 with $(-w)'$ as abscissae and corresponding Θ as ordinates in order to ascertain more exactly the relation between these two quantities, and get a rectangular hyperbola marked YSX in the diagram. The position of the hyperbola varies more or less with the change of values of λ', A, $\varkappa$, and in special case where heat dissipation to surrounding ground is neglected, that is $\lambda'=0$, the curve tends to approach the segments of the straight lines YO and OX.

When one knows the height of the statical head above the orifice $(-w)$ and the underground temperature of a boiling spring Θ, he can ascertain whether the spring is a bored boiling spring or not by the criterion of the position of the point which is specified by $(-w)$ and Θ in the diagram whether it exists on right hand side or in left hand side of the above-mentioned curve. We may, therefore, call the curve the "critical curve of determining the bored boiling spring."

2) Condition for stoppage of boiling of the bored sub-boiling spring.

In the case of the bored sub-boiling spring, when the phenomenon of boiling is occurring by application of artificial method already mentioned, let us stop the artificial process and consider the conditions under which the phenomena of boiling will just proceed.

Let Θ be the underground temperature of water in the hot spring vein, θ'_0 the subterranean temperature of the surrounding ground at the beginning point of boiling, v_0'' and θ_1' the artificial flow speed and corresponding water temperature at the beginning point of boiling respectively, and assume, for purposes of approximation, that the relation between θ_1' and Θ is the same relation as (3)', then we get

$$\theta_1' = \frac{\Theta + \dfrac{\lambda''}{Av_0''}\theta_0'}{1+\dfrac{\lambda''}{Av_0''}} \tag{38}$$

In the equation, θ_1 becomes Θ for $v_0''\to\infty$, and becomes θ_0' for $v_0''\to 0$.

After stoppage of the artificial process, flow speed changed from v_0'' to the value of v_0 that satisfy equation (33) for θ_1'. As the flow speed changes, θ_1 subsequently changes according to (38). Thus, the values of v_0 and θ_1' change alternately and at last they tend respectively to the values v_0 and θ_1 which satisfy simultaneously equations (38) and (33). Therefore, the relation between θ_1, Θ and w, in this ultimate case, may be written in the form of an implicit function as follows.

$$\frac{\lambda''(\theta_1-\theta_0')}{A(\Theta-\theta_1)}=\frac{1}{2\kappa J(\theta_1)}\left[\sqrt{4\kappa^2 J(\theta_1)\{\Delta(\theta_1)-w\}+1}-1\right] \qquad (39)$$

If we assume that Θ is constant in equation (39), and let the corresponding maximum value of w for the change of θ_1 to the extent of $100\leqq\theta_1\leqq\Theta$ as $w_{\max}$, then the boiling may proceed when $w<w_{\max}$ and stop when $w>w_{\max}$.

For instance, $(w_{\max}, \Theta)$ curve was calculated for $\lambda''=60$ cc/sec, $A=31.64$ cm^2, $\varkappa=0.5$ 1/sec, $\theta_0'=15°$C and shown as curve QR in Fig. 16. Here, we may call the QR curve the "critical curve of determining the bored sub-boiling spring." When $\lambda''=0$, the QR curve coincides with PO curve which represents the relation between Δ and θ_1.

3) Classification of hot springs by two critical curves of the boiling spring just mentioned.

If the critical curves YSX and QR determining the boiling spring and the sub-boiling spring respectively be drawn on a diagram with the height of the statical head above the level of the orifice $(-w)$ as absissae and the water temperature in the hot spring veins Θ as ordinates, Fig. 16 is obtained as already mentioned. Let a point be also plotted on this diagram for the practical value of $(-w)$ and Θ of a spring, then it can be determined whether the spring may belong to the class of boiling springs, sub-boiling springs, ordinary springs or underground water which can not flow automatically as shown by the relative position of the point for the critical curves. If the point lies on the concave side of YSX curve, it may be a boiling spring, but if it lies in the wedge-shaped area between QRX and YSX curves, it may be a sub-boiling spring. If the point lies in the wedge-shaped area XRY′, it may be an ordinary hot spring or an ordinary cold spring. If the point lies in the residual

space QRY′, the spring may classify as underground water which can not discharge automatically.

The critical curves XSY and QR may more or less change their positions with the change of effect of the heat dissipation to the surrounding ground from the vertical pipe is as already mentioned above. Furthermore, we plotted the points in Fig. 16 for the observed values of $(-w)$ and Θ of the bored boiling springs and the bored sub-boiling springs in Izu pennisula (see Table IV), and found that the points are all in the places expected from the theory.

Table IV. Relation between the underground temperature of boiling spring Θ and the depth of the statical head below the orifice w.

Hot spring	Locality	Depth of excavation	Θ	w	Dia. of orifice	Kind of boiling spring	N.B.
					cm		
Sirasaka spr.	Simogamo, Izu	91m	105°C	0.3m	6.35	sub-boiling	Θ : calc.
Toyozato spr.	,, ,,	39	105.5	nearly 0	,,	,,	Θ, w : calc.
No. 39 spring	Yazu, Izu	37	107	4∼5	8	,,	Θ, w : calc.
No. 3 spring	Atakawa, Izu	238	120	−2∼−3	6.35	boiling	w : calc.
Tutiya's spr.	,, ,,	251	129	11	15	sub-boiling	
Komatu's spr.	Atami, Izu	197	117	23	7.6	,,	
Turuya spr.	,, ,,	297	124	14	,,	,,	
No. 84 spring	Yugawara (Hirogawara), Izu	237	120	16	—	,,	

4) Proposition of a new hypothesis of the mechanism of a geyser.

As a matter of interest, the above-described diagram suggests a possible idea as to the mechanism of a geyser. The main idea is described as follows:

The geyser is a liquid hot spring of constant high temperature in the hot spring veins in the depth, just like the bored boiling springs in the Izu peninsula as described above. The character that differs from the bored boiling spring is that the statical head of this kind of geyser gradually decreases with the process of continuous ejection of hot water and steam, whereas that in the boiling spring is almost constant. The physical meaning of this phenomenon may be attributed to the smallness of the dimension of something such as the hot water bearing stratum.

Now, if the critical curves already mentioned be drawn on a diagram with the height of the statical head above the level of the orifice $(-w)$ as absissae and the water temperature at the beginning point of boiling θ_1 as ordinates, line segments AO, OB, and curve CD are obtained as shown in Fig. 17 corresponding to curve YSX, curve QR in Fig. 16 respectively. (We assume that the diagram be drawn with the orifice temperature or the water temperature near water head in the vertical tube as ordinates respectively for the ordinary hot springs or for the underground water which can not discharge automatically.) At any rate, from the state of the boiling

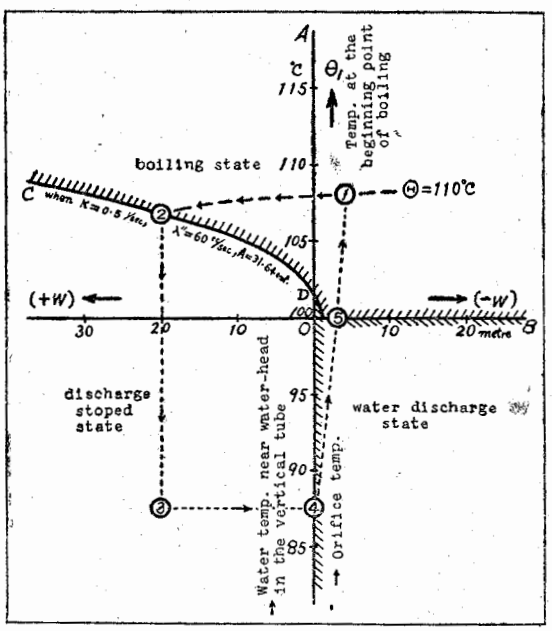

Fig. 17. Three states in the cycle of geyser.

spring which is represented at first by the point ① in Fig. 17, a geyser changes gradually along the curve ① ② which is specified by equation (39) to the state of a sub-boiling spring, and at point ② it stops boiling. The curve ① ② in Fig. 17 indicates the relation between θ_1 and w in (39) when $\Theta=110°C$, $\theta_0'=15°C$, $\varkappa=0.5$ 1/sec, $\lambda''=60$ cc/sec, $A=31.64$ cm^2. After the cessation of boiling, the water temperature in the vertical tube diminishes gradually until it attains to the subterranean temperature of the surrounding ground due to

heat dissipation, and the condition varies from ② to ③. With gradual recovery of the statical head, it moves from ③ to ④. At ④ it begins to discharge from the orifice the comparatively cold water in the vertical tube, and the orifice temperature gradually rises due to the ascending hot water from below; then conditions move from ④ to ⑤. At ⑤ the geyser begins to boil and at last it returns to state ① and repeats the course in cycle.

Gensu geyser in Simogamo, Izu peninsula, may belong to this type.

Although it is far from the writer's intention to assert that all geysers are of this type, he does believe at least that geysers which exist in the boiling spring basins having characteristics similar to those described above may be of this type.

In this type of geyser, the discharge of comparatively cold water from the orifice may be expected before the boiling occurs. Indeed we know that in almost all the geysers such facts are recognizable.

The writer will reserve the problem for further detailed investigations in the future.

VI. Conclusion.

The main points are summarized as follows:

(1) The boiling springs are divided into two kinds, viz., natural boiling springs and boiling springs obtained by boring. The latter are also divided into two classes, viz., bored boiling springs and bored sub-boiling springs.

(2) The vertical distribution of subterranean temperature in the boiling spring basins in the Izu peninsula was studied. The result from this is that the subterranean temperature of the boiling spring basins is always less than the boiling temperature of water under the pressure of the hydrostatic column of water extending from the orifice to that depth. In other words, the water in the vertical pipe, in hydrostatic state, is always in liquid state, so far as the observed depth is concerned.

From the facts just mentioned, it is to be concluded that the phenomena of the boiling springs seen in the boiling spring basins in Izu peninsula, are nothing but those in which the hot spring of liquid state at depth attains, in the course of ascension, to the boiling point of water at 0~40 m depth owing to the decrease of pressure

and causes continuous ejection of large quantities of hot water and of steam from the orifice.

(3) The phenomena of the boiling spring were theoretically studied in first approximation, and the relation between the rate of discharge of hot water at orifice V/A, the projection speed v', the jet-height L, the water temperature θ_1 and the flow speed v_0 at the beginning point of boiling and the depth of the statical head below the orifice w were discussed. Furthermore, the observed values were compared with the theoretical values and it was recognized that these two values were approximately agreeable.

(4) Probable explanations of the mechanisms of the bored boiling spring and bored sub-boiling spring were offered.

(5) Criteria for classifying springs as bored boiling springs and bored sub-boiling springs were discussed. By means of those criteria, hot springs were divided into four classes, namely, boiling springs, sub-boiling springs, ordinary hot springs, and underground water which can not discharge automatically.

(6) A new hypothesis of the mechanism of a geyser was also offered.

A part of the cost of this research was defrayed from the Scientific Research Expenditure of the Department of Education. The writer wishes to express his sincere thanks to the authorities concerned.