[Journal of the Faculty of Science, Hokkaidô University, Japan, Ser. II, Vol. III, No. 8, 1950.]

Heat Transfer from the Surface of the Solid Body in the Turbulent Air Stream

By

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(Received March 2, 1947)*

The problem of cooling was experimentally investigated. The data was analysed from the point of view of the statistical theory of turbulence, and a simple relation between the heat transfer coefficient and the statistical factors of turbulence was obtained.

I. Introduction.

Experimental investigations of the cooling of a solid body in the turbulent air stream have been carried out by many authors. The dependence of the heat transfer coefficient upon the velocity of the main stream, the geometrical form and the surface condition of the cooled body was merely investigated and a quantitative relation between the heat transfer coefficient and the state of turbulent flow was little treated. According to the result of my preliminary experiment, however, the quantitative relation between the heat transfer coefficient and the state of turbulent flow must be indispensable for manifesting the problem of cooling in the turbulent air stream. There are some factors which characterize the turbulent field, and the factors have certain contributions to the heat transfer coefficient. In order to know the mechanism of cooling from the surface of a solid body in the turbulent air stream, it is necessary to grasp a clear conception of the state of turbulent field and to describe it quantitatively from that point of view.

Assuming that the turbulent field is composed from eddies, Taylor(1) succeeded in 1915 in determining a statistical description of

* Presented at the annual Tôkyô meeting of The Physical Society of Japan, May, 1946.

the turbulent field by introducing the following quantities, i.e. the scale of turbulence and the average size of the smallest eddies which are responsible for the energy dissipation by viscosity, and he obtained some interesting result about the decay of turbulence. Thereafter a more generalized theory of turbulence was published by Kármán and Howarth*. These two theories are almost identical in principle. By Kármán and Howarth,[2] the scale of turbulence grows slowly as the observing point moves towards the down stream, while Taylor assumed that the scale of turbulence is some definite fraction of a mesh length of square mesh grid to which the turbulence in the air stream is due. Whether this assumption is appropriate or not, and to what extent it must be modified, must be judged from the experimental results. However, a linear law with respect to the decay of turbulence which is derived as a result of Taylor's assumption, proves to agree satisfactorily with experiments. Accordingly Taylor's theory may be thought to be true in the first approximation. The experiments refered to Taylor's theory have been sufficiently performed, hence in our case the numerical values of the statistical factors of turbulence are easily available. Consequently by using of Taylor's theory the analysis of our experimental results was carried out and the relations between the heat transfer coefficient and the statistical factors of turbulence were sought.

II. Heat Transfer from the Surface of a Cylinder whose Axis is Perpendicular to the Direction of the Main Stream.

A. Case of $d/M = 0.33$.

(i) Method of Experiment.

A brass cylinder was arranged in a wind tunnel of Effel type

* Although Kármán and Howarth's theory is more generalized, the derived formula representing the statistical factors of turbulence contains some constants which have to be determined by experiment. The experimental determinations of these constants have not yet been accomplished. As these numerical values of the constants are necessary for the application of Kármán and Howarth's theory, no attempt was made to use it.

so as to make its axis horizontal and perpendicular to the direction of the main stream. The diameters of the cylinders D were 3.8, 1.9 and 0.95 cm. The cylinder was electrically heated by inserting heating coils into the inner part. A very slender thermojunction (Cu- constantan, B.S.-40) was soldered at the middle point of the axial length of the cylinder and the temperature distribution of the upper half part of the cylinder was measured by rotating the cylinder every 20 degrees of angle. In this case the temperature distributions of the upper and lower parts of the cylinder were nearly symmetrical so that the small difference of the temperature between the both parts was neglected. Temperature of air at a point sufficiently separated from the surface of the cylinder was measured by another thermojunction.

For simplicity we use the following notations:

α_m : the heat transfer coefficient of the cylinder,
θ_s : the surface temperature of the cylinder (in °C),
θ_0 : the temperature of air (in °C),
Q : the amount of heat supplied per unit area of the surface of the cooled body in unit time (in Cal),
$(\theta_s-\theta_0)^m$: the mean value of $(\theta_s-\theta_0)$ measured at each 20° of the circumference of the cylinder,
i : the heating current (in amp),
R : the electrical resistance of the heating coil (in ohm),
D : the outer diameter of the cylinder (in cm),
L : the length of the cylinder (in cm).

In a completely stationary state Q is given by the following relation

$$Q = \alpha_m (\theta_s-\theta_0)_m.$$

On the other hand, when i is kept constant Q is given by

$$Q = 0.236\, Ri^2/(\pi DL).$$

Therefore α_m will be calculated by the formula:

$$\alpha_m = \frac{0.236\, Ri^2}{\pi DL}\,\frac{1}{(\theta_s-\theta_0)_m} \qquad \frac{\text{Cal}}{\text{cm}^2, \text{sec}, {}^\circ\text{C}}, \tag{1}$$

where the quantities in the right hand side are all obtainable from experiment.

As shown in Fig. 1, the square mesh grids made from the thick paper 3 mm in thickness were arranged so as to change the distance x from the center of the cylinder. By changing x, the state of the turbulent field (i.e. the values of the statistical factors of turbulence) surrounding the cylinder is altered. For various values of x, the temperature distributions of the surface of the cylinder were measured. The values of the ratio d/M was 0.33 and the values of M were 3.8, 3.0, 2.4, 2.1 and 1.5 cm.

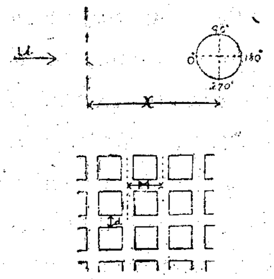


Fig. 1 *Arrangement of apparatus and square-mesh grid.*

d : breadth of thick paper (in cm)

M : mesh length of square mesh as shown in Fig. 1 (in cm)

The decay of turbulence produced by the square mesh grids having different M, but in geometrical similarity, must all obey the same law. In the case of $d/M = 0.33$, the law about the decay of the intensity of turbulence u'/U has been represented by the following formula by Simmons and Salter's experiment[3]:

$$U/u' = -0.7 + 1.32\, x/M, \tag{2}$$

then

$$\lambda/M = 1.95\,(\nu/Mu')^{1/2}, \tag{3}$$

where U : velocity of main stream (in cm/sec),

u' : component of turbulent velocity,

λ : average size of the smallest eddies,

ν : kinematic viscosity.

As the numerical values of u'/U and λ/M for a arbitrarily given x are easily calculated by the above formulae, we can analyse the problem of cooling in the turbulent air stream from the standpoint of statistical theory of turbulence and connect the heat transfer coefficient with the statistical factors of turbulence.

(ii) Experimental Results and Discussion.

(1) *Relation between α_m and x.*

In Fig. 2 α_m is plotted against x for various values of M. Each curve of α_m takes its minimum value at $x=55$ cm, which is almost independent of M, D and U.

(2) *Relation between α_m and u'/U.*

For the given value of U, α_m depends upon the state of turbulent flow, which varies with x. Accordingly, in order to find the effect of the state of the turbulence on the cooling, the relation between α_m and the factors of turbulence at various observing points must be investigated.

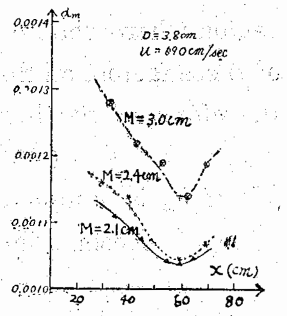


Fig. 2 *Mean heat transfer coefficient α_m as function of x.*

At first u'/U is easily estimated from the formula (2) for a given

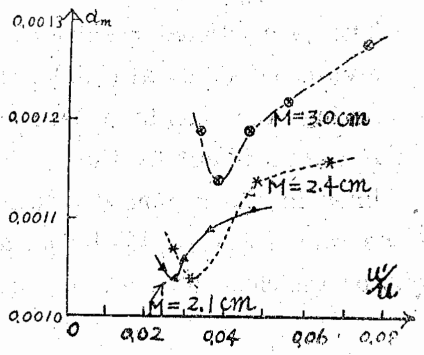


Fig. 3 *Mean heat transfer coefficient α_m as function of intensity of turbulence u'/U.*

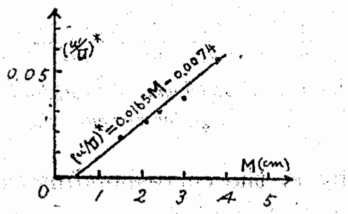


Fig. 4 *Relation between mesh length M and intensity of turbulence which corresponds to the minimum points of curves in Fig. 3, $(u'/U)^*$.*

x. In Fig. 3 α_m is plotted against u'/U for various values of M. Each curve of α_m takes its minimum value at a certain value of u'/U, i.e. $(u'/U)^*$. Its dependence upon M is shown in Fig. 4 and expressed by the following formula:

$$(u'/U)^* = 0.0165\,M - 0.0074. \tag{4}$$

In the region $(u'/U)<(u'/U)^*$, α_m decreases rapidly with increases of u'/U, although the intensity of turbulence must have a great positive influence on the heat transfer. In the region $(u'/U)>(u'/U)^*$, α_m increases with u'/U. From this fact, it may be suggested that there will be some other factors which act on the phenomenon of cooling so as to cancel the effect of the intensity of turbulence. In the region where the intensity of turbulence is rather weak, the effect of these factors on the cooling will become conspicuous by overcoming the effect of the intensity of the turbulence.

(3) *Relation between a_m and λ/M.*

If a_m is assumed to be linearly proportional to u'/U, the ratio $a_m/(u'/U)$ becomes independent of u'/U. When $a_m/(u'/U)$ is plotted against λ/M for various values of M. $a_m/(u'/U)$ increases with λ/M, as can be seen from Fig. 5, and the relation between $a_m/(u'/U)$ and λ/M is shown by the straight lines with inflection at $(\lambda/M)_{\rm I}$. The proportional coefficient increases suddenly at $(\lambda/M)_{\rm I}$. Denoting the points where the λ/M-axis is cut by the straight lines in the region $\lambda/M<(\lambda/M)_{\rm I}$ by a, the corresponding points in the region $\lambda/M>(\lambda/M)_{\rm I}$ by a', and denoting the proportional coefficient or the slope of the two groups

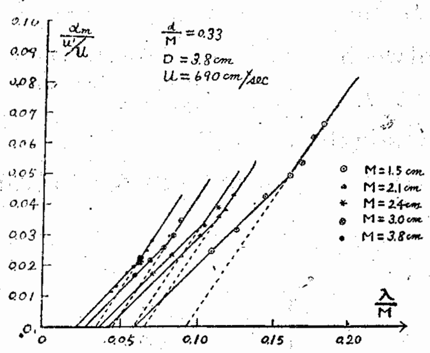


Fig. 5 *Relation between $a_m/(u'/U)$ and λ/M for various values of M,*

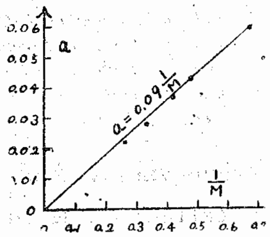


Fig. 6 *Ordinate (a) represents the values of λ/M where λ/M-axis is cut by straight lines which lie in the region smaller than the inflection points in Fig. 5.*

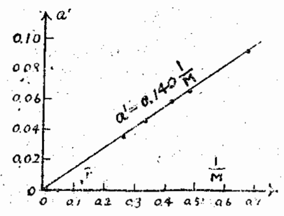


Fig. 7 *Ordinate (a') represents the values of λ/M where λ/M-axis is cut by the straight lines which lie in the region larger than the inflection points in Fig. 5.*

of straight lines by c and c' respectively, the following formulae can be obtained from Fig. 5.

$$a_m/(u'/U) = c\,(\lambda/M - a), \quad c = 0.50 \quad \text{for} \quad \lambda/M<(\lambda/M)_{\rm I}, \qquad (5)$$

$$a_m/(u'/U) = c'\,(\lambda/M - a'), \quad c = 0.77 \quad \text{for} \quad \lambda/M>(\lambda/M)_{\rm I}. \qquad (5')$$

From Figs. 6 and 7 the relation between M and a or a' is respectively

$$a = 0.09/M, \tag{6}$$

$$a' = 0.14/M. \tag{6'}$$

Therefore the mean heat transfer coefficient is represented by the formulae:

$$\alpha_m = 0.50\,(u'/U)\,(1/M)\,(\lambda - 0.09) \quad \text{for} \quad \lambda/M < (\lambda/M)_{\mathrm{I}}, \tag{7}$$

$$\alpha'_m = 0.77\,(u'/U)\,(1/M)\,(\lambda - 0.14) \quad \text{for} \quad \lambda/M > (\lambda/M)_{\mathrm{I}}. \tag{7'}$$

The above experimental result is obtained in the case of $D = 3.8$ cm and $U = 690$ cm/sec.

Even if D is changed, the above relations (7) and (7′) between α_m and the statistical factors of turbulence are formally invariant, and c and c' alone increase as D decreases as shown in Figs. 8 and 9. Moreover even if U is varied, the formulae (7) and (7′) hold its

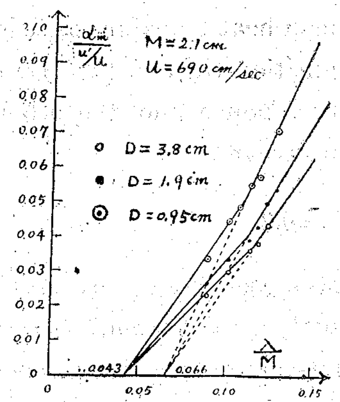


Fig. 8 *Relation between $\alpha_m/(u'/U)$ and λ/M for various values of D.*

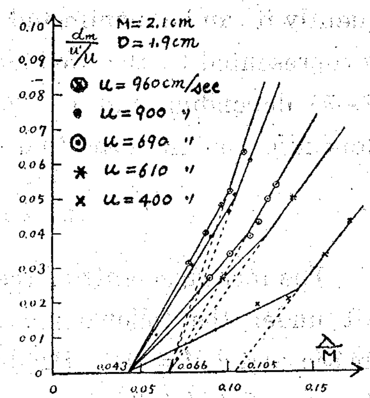


Fig. 10 *Relation between $\alpha_m/(u'/U)$ and λ/M for various values of U.*

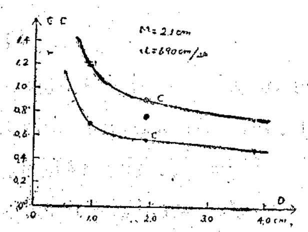


Fig. 9 *Proportional coefficients c and c' as functions of diameter of cylinder D.*

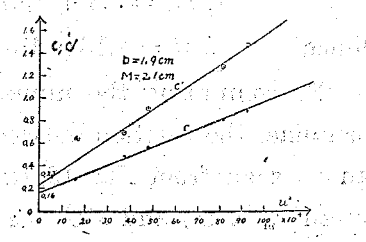


Fig. 11 *Proportional coefficients c and c' as functions of square of velocity of main stream U.*

form unchanged and in this case c and c' increase linearly with the square of U as can be seen from Figs. 10 and 11. In the case of $D = 1.9$ cm, the relation between U^2 and c or c' is shown in Fig. 11 and expressed by

$$c = 0.16 + 0.0085 \times 10^{-4} U^2, \tag{8}$$

$$c' = 0.23 + 0.013 \times 10^{-4} U^2. \tag{8'}$$

However, as can be seen from Fig. 11, when U reduces to about 400 cm/sec the value of c' deviates from the above relation.

Thus the mean heat transfer coefficient can be represented by the following formula;

$$\alpha_m = c\,(u'/U)\,(1/M)\,(\lambda - k). \tag{9}$$

In the formula c and k increase their values at $(\lambda/M)_{\mathrm{I}}$, and c depends upon the geometrical form of the cooled body and the velocity of main stream, while k is independent of them. Consequently it can be manifested that the mean heat transfer coefficient is represented by the product of two quantities, the one $(u'/U)\,(1/M)\,(\lambda - k)$ depending only on the state of turbulence and the other c depending on the condition of the main stream.

B. Case of $d/M = 0.2$.

The measurements of the heat transfer coefficient were carried out under the following condition, namely $D = 1.9$ cm, $U = 900$ cm/sec and $d/M = 0.2$. Dryden's[(4)] measurements have been done in the case of $d/M = 0.2$. According to his results, the decay of turbulence obeys the following formula:

$$U/u' = 8.8 + 1.072\, x/M, \tag{10}$$

Hence

$$\lambda/M = 2.16\,(\nu/Mu')^{1/2}. \tag{11}$$

By computing the numerical values of u'/U and λ/M from the formulae, the relation between $\alpha_m/(u'/U)$ and λ/M was obtained. As can be seen from Fig. 12, in the case of $d/M = 0.2$, the relation between $\alpha_m/(u'/U)$ and λ/M is represented by the straight lines which have no inflection as former case and is expressed by the formula:

$$\alpha_m = 1.30\,(u'/U)\,(1/M)\,(\lambda - 0.14). \tag{12}$$

The value of the proportional coefficient c' ($=1.30$) in this formula agrees with the value, which was obtained in the case of $D=1.9$ cm, $U=900$ cm/sec and $d/M=0.33$. Moreover the constant k' has the value 0.14, which is the same value in the case of $d/M=0.33$ (Fig. 13).

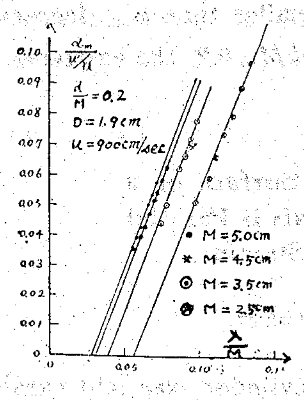


Fig. 12 *Relation between $a_m/(u'/U)$ and λ/M for various values of M, $d/M=0.2$.*

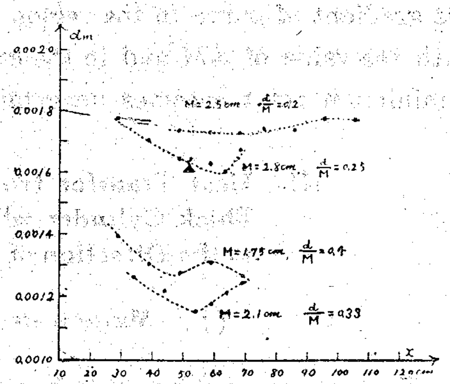


Fig. 14 *Relation between a_m and x for various values of M and d/M.*

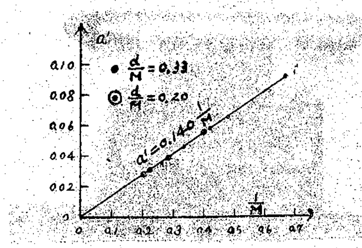


Fig. 13 *Ordinate (a') represents the values of λ/M where λ/M-axis is cut by the straight lines in Fig 12.*

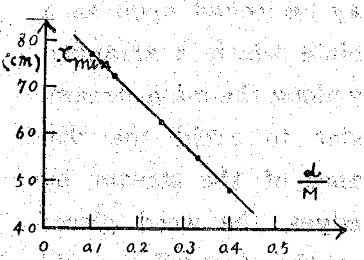


Fig. 15 *Relation between d/M and x which corresponds to the minimum points of curves in Fig. 14, x_{min}.*

Thus it can be concluded that the relations between the mean heat transfer coefficient and the statistical factors of turbulence in the two cases where $d/M=0.2$ and $d/M=0.33$ are expressed by the same formula (9) at least in the region past the inflection point. Thus the formula (9) seems to hold in the various turbulent fields produced by the square mesh grid having various values of d/M. a_m in the four cases $d/M=0.2$, 0.25, 0.33, and 0.40 were plotted against

x in Fig. 14. As shown in this figure α_m takes its minimum value in each case at a certain value of x, i.e. x_{min}, which corresponds nearly to the inflection point of the straight line representing the relation between $\alpha_m/(u'/U)$ and λ/M. The value of x_{min} is a function of d/M and decreases with increasing d/M, as shown in Fig. 15. Moreover the gradient of curve in the region of x smaller than x_{min} decreases with the value of d/M and in the case of $d/M=0.2$, the existence of a minimum point becomes uncertain.

III. Heat Transfer from the Surface of a Thick Cylinder whose Axis is Parallel to the Direction of Main Stream.

(i) Method of Experiment.

The uppermost generating line of the cylinder was laid parallel to the direction of the main stream. When the diameter of the cylinder is sufficiently large, it may be looked upon as a flat plate which is arranged to lay along the main stream. In oder to avoid the disturbance of the stream by the edges, the wood pieces having the streamline profil were attached to the both ends of the cylinder as seen from Plate I.

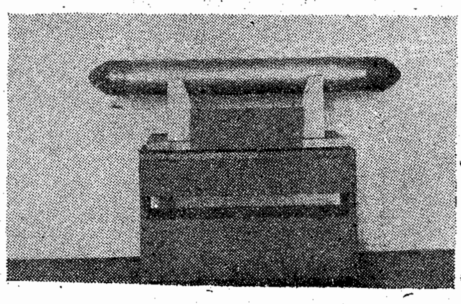


Plate 1

The thermojunctions were soldered at five points along the uppermost generating line of the cylinder which laid horizontally. In this measurement the temperature difference between the positions of various angles was neglected and the temperature change along the axis only was considered.

The distances between the front edge and each measuring point l are 14.6, 21.8, 29, 36.1 and 43.3 cm respectively. The values of M are 5, 4.5, 3.5 and 2,5 cm, and the value of d/M equals 0.2.

The local heat transfer coefficient α_l is given by the following formula

$$\alpha_l = \frac{0.236\, Ri^2}{S} \frac{1}{(\theta_s - \theta_0)} \quad \left(\frac{\text{Cal}}{\text{cm}^2, \text{sec}, {}^\circ\text{C}}\right) \tag{13}$$

where S is the surface area of the cylinder in cm^2. During this run, U was 740 cm/sec.

(ii) Experimental Results and Discussion.

(1) *Relation between α_l and l.*

In the case of $M = 4.5$ cm, the curves of α_l vers l namely the distribution of α_l along the uppermost generating line of the cylinder, are shown in Fig. 16 for various values of x, the distance between grid and the front edge of the cylinder. As seen from this figure, α_l decreases with increasing l.

(2) *Relation between α_l and $x + l$.*

In Fig. 17 α_l is plotted against $x + l$, the distance of each measuring point from the grid, for various values of l. When l is comparatively small, there exists a certain region on each curve where

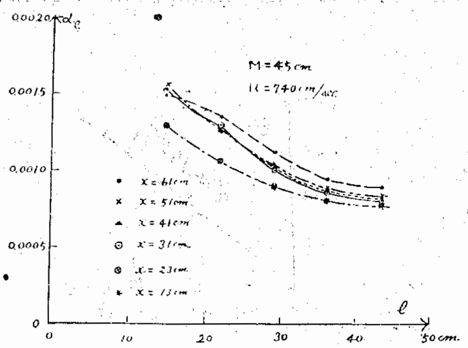


Fig. 16 *Local heat transfer coefficient α_l as function of l.*

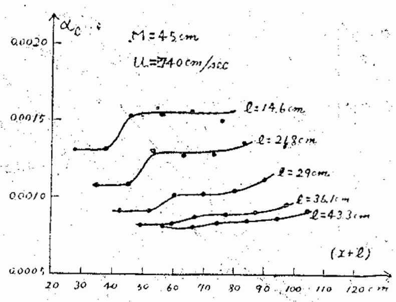
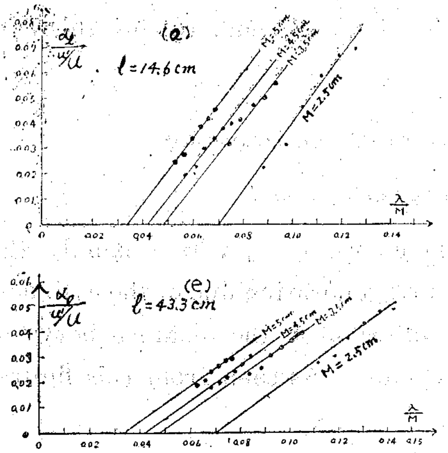


Fig. 17 *Local heat transfer coefficient α_l as function of $x+l$.*

α_l rapidly increases with $x+l$. As l increases, that region is shifted into the larger $x+l$ and the rate of increase of α_l in this region decreases. When l becomes about 43.3 cm, the region almost disappears.

(3) *Relation between a_l and the statistical factors of turbulence.*

For each measuring point (the distance from the grid $= x + l$) the statistical factors of turbulence (i.e. u'/U and λ/M) were computed



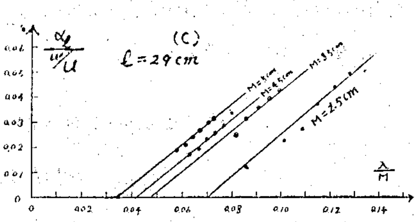


Fig. 18 *Relation between a_l / (u'/U) and λ/M for various values of M when l is fixed.*

and $a_l/(u'/U)$ was plotted against λ/M, for various values of l and M in Figs. 18 and 19.

As shown in these figures, the relation between $a_l/(u'/U)$ and λ/M is represented by the straight lines and the slope of these lines is a function of l and if l is fixed, the slope is quite independent of M.

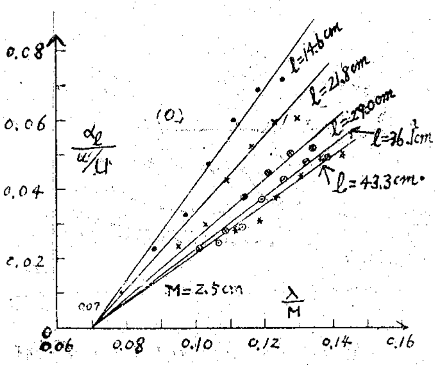


Fig. 19 *Relation between $a_l/(u'/U)$ and λ/M for various values of l when M is fixed.*

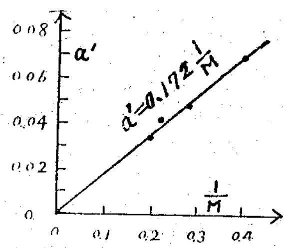


Fig. 20 *Ordinate (a') represents the values of λ/M where λ/M-axis is cut by the straight lines in Fig. 18.*

Denoting the points where the λ/M-axis is cut by these straight lines in Fig. 18 by a', a' is plotted against M in Fig. 20. The relation between a' and M is independent of l. Moreover denoting the proportional coefficient of $a_l/(u'/U)$ for λ/M by c', each straight line in Fig. 18 is represented by the following formula:

$$a_l/(u'/U) = c'(\lambda/M - a'). \tag{14}$$

From Fig. 20,

$$a' = 0.172/M. \tag{15}$$

Hence

$$a_l = c'(u'/U)(1/M)(\lambda - 0.172), \tag{16}$$

where c' is a function of l.

c' is plotted against l in Fig 21. As can be seen from the figure,

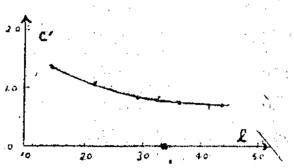


Fig. 21 *Proportional coefficient c' as function of l.*

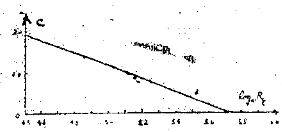


Fig. 22 *Proportional coefficient c' as function of $\log_{10}R_l$.*

the influence of the state of turbulence on a_l becomes smaller with increases of l.

From this fact it may be considered that in the cooling by forced convection, the heat transfer coefficient is controlled by the state of flow in the boundary layer, while the state of flow is controlled by not only the state of turbulence contained in the main stream, but also the history of the flow along the solid wall.

In the case of flat plate or the flow along the axis of the cylinder, the influence of the history of the flow along the wall on the heat transfer coefficient becomes more important than the case of the flow perpendicular to the axis. In the other word the influence of the history increases with l and predominates over the effect of turbulence in the main stream.

The relation between c' and $\log_{10}R_l$ is shown in Fig. 22 or TABLE I and is expressed by the following formula:

$$c' = 8.97 - 1.56 \log_{10} R_l \tag{17}$$

where R_l is Reynolds number with regard to l. Accordingly α can be expressed as follows:

$$\alpha_l = (8.97 - 1.56 \log_{10} R_l)\,(u'/U)\,(1/M)\,(\lambda - 0.172). \qquad (18)$$

That is to say, the relation between the heat transfer coefficient and the statistical factors of turbulence is represented by the formula of the same type (9) no matter whether the axis of the cylinder may be parallel or perpendicular to the direction of main stream.

TABLE I. *Proportional coefficient and Reynolds number.*

l (cm)	$R_l = Ul/\nu$	$\log_{10} R_l$	c
14.6	$7.2 \cdot 10^4$	4.857	1.35
21.8	$10.7 \cdot 10^4$	5.031	1.10
29.0	$14.3 \cdot 10^4$	5.155	0.84
36.1	$17.8 \cdot 10^4$	5.250	0.75
43.3	$21.3 \cdot 10^4$	5.329	0.72

IV. Conclusion.

In the study on the relation between the heat transfer coefficient and the statistical factors of turbulence, it is manifested that the heat transfer coefficient is represented by the following simple formula (9):

$$\alpha = c\,(u'/U)\,(1/M)\,(\lambda - k).$$

Thus the heat transfer coefficient can be represented by the product of two quantities, the one is $(u'/U)\,(1/M)\,(\lambda - k)$ which depends upon the state of turbulence alone and the other is c which depends upon the state of main stream alone. The above described formula holds no matter whether the axis of the cylinder may be arranged so as to be perpendicular or parallel to the direction of main stream. The physical meaning of c, however, differs slightly from each other, namely in the former case c is a function of the diameter of the cylinder and the velocity of main stream, while in the latter case c is a function of Reynolds number, provided that the distance is taken as the one from the front edge of the cylinder.

In the treatment of the experimental results the values of the statistical factors of turbulence used above are those which are available in the isotropic turbulent field. The values of the statistical factors of turbulence which influence the heat transfer coefficient most for practical purposes, however, are those which are available only in the neighbourhood of a solid wall. When the turbulence is produced by the square mesh grid, the turbulent field sufficiently far from the grid may be looked upon as the isotropic turbulent field, and the effect of shadow of the grid no longer appears.

If the solid body is placed in the isotropic turbulent field, however, the tangential component of the turbulent velocity u' in the neighbourhood of the surface of solid becomes larger than the normal component v'. Consequently the condition of the isotropic turbulence is not satisfied in this region. As long as the condition of the isotropic turbulence is satisfied, however, the following formulae hold:

$$U/u' = \text{const} + (5/A^2)(x/M),$$

$$\lambda/M = A(\nu/Mu')^{1/2}.$$

Accordingly the values of u'/U and λ/M used in this paper differ from the values in the neighbourhood of the solid wall. It may be thought that the values of u'/U and λ/M in the neighbourhood of the solid wall are certain fraction of the values in the isotropic turbulent field (i.e. the values used in this paper). Accordingly if the values in the neighbourhood of solid wall are used, a different proportional coefficient should be used, and notwithstanding the formula representing the relation between the heat transfer coefficient and the statistical factors of turbulence may be kept valid.

From this experiment a clue may be given to the investigation of the mechanism of the heat transfer phenomena from the surface of solid body in the turbulent air stream.

In conclusion, the author wishes to express her sincere thanks to Professor Y. Ikeda for his kind guidance.

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