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Thermal Conductivity of Frozen Soil

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The thermal diffusivity of frozen soil was measured with the samples which were artificially prepared under the conditions similar to the natural frost formation. Thermal conductivity was computed from the diffusivity thus obtained and the volume specific heat of the soil. Moisture relationship of the thermal diffusivity k and the thermal conductivity K is represented by the following empirical formulae:

$$k = ae^{br}$$

$$K = ce^{dr}$$

In these formulae, r is the moisture ratio of frozen soil and a, b, c and d are constants. These formulae hold only in the range of moisture content below the saturation moisture ratio.

In our climate, when we dig the frozen ground in winter, two modes of segregation of ice are usually observed in the frozen soil; that is, the ice plate of a considerable thickness and the dispersion of innumerable thin layers of ice inbedded in the frozen soil. The ice plate is composed of a bundle of numerous ice filaments and is called "ice filaments layer". Three of these layers are seen in Photo. No. 1 and two in No. 4. Plate I. The type of the dispersion of thin ice layers gives the appearance like a piece of sirloin beef, the ice layers corresponding to the fat layers and the frozen soil to the meat. This type may be tentatively called "sirloin type freezing". One good example being shown in Photo. No. 3 Pl. I. The description and classification of various modes of freezing of soil are given in the paper of Drs. NAKAYA and MAGONO.[1]

The soil samples of these two types of freezing contain usually more water than that the of saturation. In these cases the rate of increase of k with respect to r is smaller than that expressed by the former empirical formula. The variation of the thermal diffusivity in relation to the direction of heat flow is commparatively small.

§ 1. Introduction

For the estimation of the depth of soil freezing in winter for a given condition of climate, it is necessary to know the thermal conductivity of frozen soil. Measurements of the thermal conductivity of frozen soil were carried out by some investigators. But their samples used seem to be far from the natural frozen soils in the process of freezing and consequently in the structure of frozen soil. For instance, the sample was sometimes prepared by the rapid refrigeration of soil in a container used for the conductivity test.

In nature, when the ground is cooled down in the cold weather, the freezing of soil proceeds from the surface virtically to the interior. Various modes of soil frost take place, depending upon the climatic conditions, the soil properties, and the water content of soil.

In our climate, the sirloin type freezing and the ice filaments layer (see Abstract) are very common. It is due to the abundance of water and the property of soil, which is adequate to contain rich water and is characterized by good capillarity. Under these conditions the segregation of ice is much facilitated. The water content of the frozen soil of such types usually exceeds the saturation water content at the normal state. Therefore, the samples of these types of freezing can not be produced in such an ordinary container as that used by the former investigators for the conductivity test.

Furthermore, for the practical application, it is required to measure the conductivity of frozen soil under the condition that the direction of heat flow coincides with that of the natural state.

For these reasons, the author used a special vessel for the preparation of soil samples only, and the measurement of conductivity was carried out with the other apparatus. The soil samples of various modes of freezing could be prepared by the artificial frost-heaving apparatus, and the diffusivity test was carried out by Ångstrom's method.

§ 2. Experimental Procedure

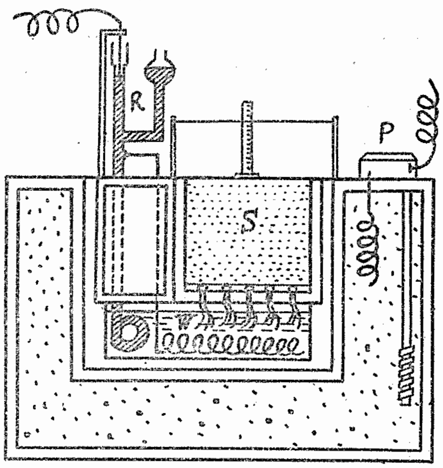
a) *Preparation of the sample*

For the preparation of the sample, special cautions were paid on the method of production, so that the condition being similar to that taking place in nature.

The concerete type freezing; that is, freezing of soil without marked segregation of ice, is obtained when the sample of smaller water content is made to freeze rather rapidly. As the sides and bottom of the wooden box container of sample ($10 \times 10\ cm^2$ in dimension and 4 cm in depth) is thermally insulated with the layer of saw-dust, the moist soil sample in this container is cooled down only from the upper surface. When the container is placed over-night in the cold chamber laboratory, the next morning we can get a sample of the concrete type freezing.

The sirloin type freezing and the ice filaments layer are produced in the artificial frost-heaving apparatus of open system. The general view of the apparatus is shown in Fig. 1. This is similar to the one which was originally designed by Dr. NAKAYA.[2] The water is supplied through hanging clothes from the cistern placed below, and the freezing proceeds from the upper surface. As the segregation of ice takes place in the soil sample, the frost-heaving is observed.

Controlling the temperatures of the air and the water in the cistern, sample of various types of segregation of ice are produced. Some examples of these samples are shown in Photos. No. 1, No. 3 and No. 4, Pl. I. As a test specimen, a block of about $10 \times 10 \times 4\,\mathrm{cm}^3$ in size is cut from the sample thus produced.



S: Soil sample
W: Water
R: Regulator for water temperature
P: Regulator for container temperature

Fig. 1.

Two hair thermojunctions are inserted, along the central line of the specimen, at different distances from the surface. For this purpose, two fine holes were drilled from a side. Photos. No. 2, No. 3 and No. 4, Pl. I., are the samples of the concrete type, the sirloin type and the ice filaments layer respectively. The holes for insertion of the thermojunctions are seen as two black dots in these photographs.

b) *Measurements of the thermal diffusivity.*

Thermal diffusivity is measured by Ångstrom's method, using a newly designed apparatus which had been described in the previous paper.[3] The specimen and the heater which should be used to give the sinusoidal temperature change at the surface of the specimen are set in a thermostat placed in the cold chamber laboratory. The working temperature of the thermostat is about $-20°C$. Leading wires for the heater and the thermojunctions are drawn out of the cold chamber, and the operation of the measurements are carried out in the ordinary room. The apparatus and the procedure for the measurements are the same as in the case of former experiment.

Adjusting the power supply to the heater and the temperature of the thermostat, the mean temperature between two thermojunctions in the specimen was maintained at about $-5°C$ in this case. Thermal diffusivity was computed from the phase difference of the sinusoidal temperature waves observed at two points of thermojunctions and the vertical distance between two junctions.

The amplitude of the temperature wave at the upper thermojunction was kept within a range of about 2°C. Sinusoidal form of the temperature waves always showed sufficient accuracy, as shown in Fig. 2.

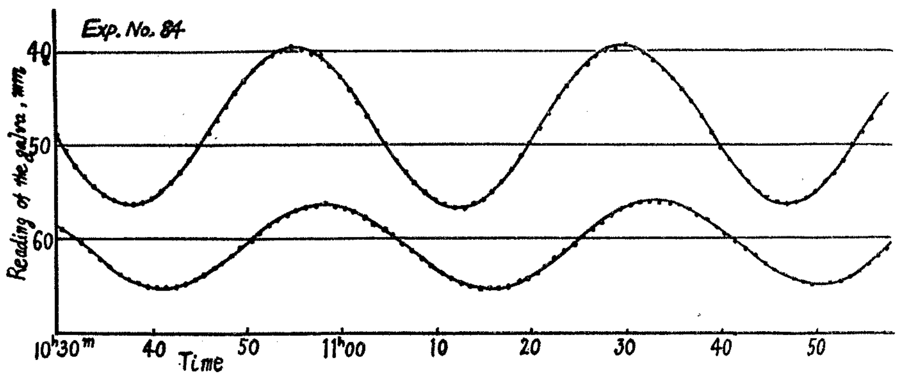


Fig. 2.

c) *Measurement of the specific heat of soil at low temperature.*

In order to compute the thermal conductivity from the diffusivity obtained above, it is necessary to know the volume specific heat of frozen soil. It can be calculated from the specific heat of dry soil and that of ice. The specific heat of dry soil at low temperature $(-5°C)$ was measured by the method of mixture, using an ordinary calorimeter, In this experiment the alcohol cooled below $-30°C$ was used for the working liquid in the calorimeter.

d) Miscellaneous.

Specific gravity of the soil material was measured by a pycnometer. Mechanical analysis of the soil was carried out by the method of hydrostatic sedimentation. Saturation moisture ratio was determined by the ordinary gravimetric method.

The saturation was obtained by absorbing water from below by the capillary action of soil.

§ 3. Results of the Experiments

Three volcanogenous soils were tested, which were taken at Memuro in Hokkaido. These three soil samples are designated as A, B and C for the convenience sake. Results of the mechanical analysis and the other physical constants are tabulated in TABLE I.

TABLE I. *General physical properties of sample soils*

Soil symbol	Soil nature	Depth from ground surface cm	Mechanical analysis % Sand >0.05	Silt 0.05–0.005 mm	Clay 0.005>	Specific gravity g/cc.	Specific heat of dry soil at −5°C cal/g°C	Saturation moisture ratio at close packing
A	Black cultivated soil	0–10	61.7	18.5	19.8	2.43	0.17	0.6
B	Brown subsoil	25–30	72.8	20.6	6.6	2.56	0.19	0.7
C	Yellow brown subsoil	50–60	52.8	30.2	17.0	2.59	0.19	0.6

a) Results in the region below the saturation water content.

Results of the measurements of the thermal diffusivity of frozon soils k in relation to their moisture ratio r are tabulated in TABLE II. In this Table, the moisture ratio r means the ratio of the mass of ice to that of dry soil, and the dry density M is the mass of the dry soil in unit volume of the sample.

These results are shown graphically in Fig. 3. The white dots represent the thermal diffusivity k in TABLE II. This is the thermal diffusivity in the frozen state. The black dots represent the thermal diffusivity of the same soil at room temperature measured for comparison. This will be written as k'. Full lines are drawn by smoothing these white dots. In the region of the water con-

Table II *Thermal diffusivity of frozen soils at* $-5°C$

Soil symbol	Exp. no.	r	$k\times10^3$ C.G.S.	M g/cc.	Soil symbol	Exp. no.	r	$k\times10^3$ C.G.S.	M g/cc.
A	51	0.50	5.69	1,09	B	76	0.65	5.67	0.83
	52	0.38	2.68	0.94		77	0.68	6.21	—
	53	0.29	3.92	0.90		78	0.65	5.27	0.91
	54	0.56	5.17	1.00		80	0.16	1.37	0.94
	55	0.55	5.31	1.05		82	0.53	3.56	0.84
	56	0.23	3.48	1.04		83	0.73	5.37	0.83
	57	0.54	6.38	0.96		84	0.06	0.95	0.99
	58	0.00	2.28	1.10		86	0 01	1.12	1.05
	61	0.36	3.78	0.98	C	101	0.72	4.13	0.90
	62	0.13	2.46	1.17		102	0.30	2.17	—
B	71	0.25	2.38	—		103	0.58	4.25	1.02
	72	0.34	2.32	0.79		104	0.10	1.65	0.90
	73	0.44	2.66	0.86		105	0.18	2.14	0.88
	74	0.54	2.95	0.81		106	0.56	4.16	1.05
	75	0.58	4.36	0.86		108	0.42	3.30	1.04

tent below saturation, these values of k are expressed by an empirical formula of the form

$$k = ae^{br}$$

where a and b are constants. The numerical values of these constants for the dotted lines drawn in Fig. 3 are tabulated in the left half of Table III. In Fig. 3, we see that the dotted line coincides with the full line below saturation.

Table III.

Soil symbol	Thermal diffusivity		Thermal conductivity	
	$a\times10^3$ C. G. S.	b	$c\times10^4$ C. G. S.	d
A	2.25	1.62	4.81	2.84
B	1.00	2.23	2.12	3.29
C	1.57	1.56	26.8	3.46

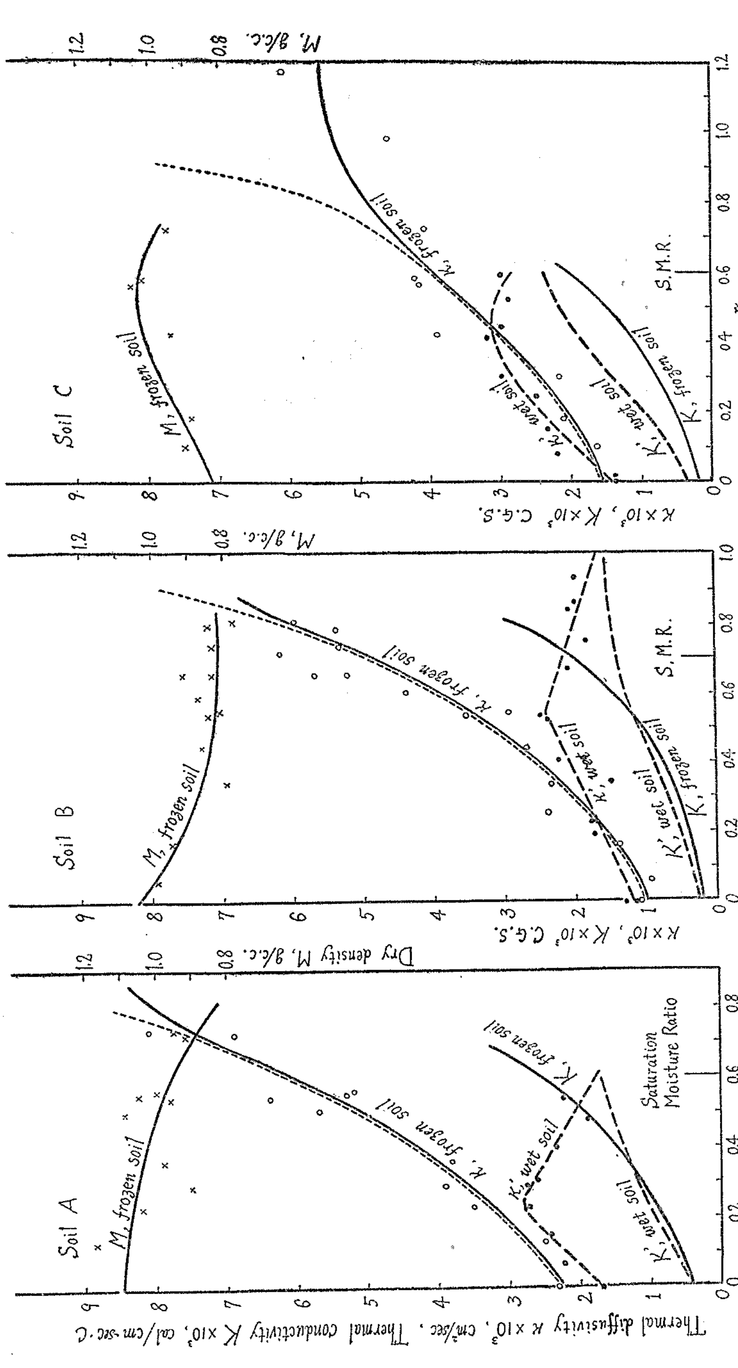

Soil A
Soil B
Soil C
M, frozen soil
κ, frozen soil
K, frozen soil
K', wet soil
κ', wet soil
Saturation Moisture Ratio
S.M.R.
Moisture ratio r
Dry density M, g/c.c.
M, g/c.c.
κ×10³, K×10³ C.G.S.
Thermal diffusivity κ×10³, cm²/sec, Thermal conductivity K×10³, cal/cm·sec·°C

Fig. 3.

The broken lines in Fig. 3 are obtained by smoothing the values of black dots. They show the relationship between the thermal diffusivity of the same soil at room temperature k' and the moisture ratio r. The curve for the soil B is reproduced from the previous paper,[3] with some correction.

Thermal conductivity K is given by the equation $K = c\rho k$ where $c\rho$ is the volume specific heat of frozen soil sample. Neglecting the specific heat of air void, it is represented as the following equation:

$$c\rho = sM + irM\,.$$

where M is the dry density, r the moisture ratio, s the specific heat of dry soil and i the specific heat of ice. The values of s for three samples of soils measured at $-5°C$ are tabulated in TABLE I, and i is 0.485 cal/g°C at $-5°C$.

When the samples are produced in the state of close packing, M varies with r. The values of M are picked up from TABLE II and protted against r in Fig. 3 with cross marks. Because of the difficulty for getting constant degree of packing, the marks are rather scattered. The M-r curves are drawn by smoothing these cross marks.

Values of M are read on M–r curve for every 0.1 of r, and the values of $c\rho$ are calculated for every 0.1 of r using the values of M thus obtained. They are tabulated in the left half of TABLE IV. Similary the values of k are read from k–r curves in Fig. 3, and K are computed by the equation $K = c\rho k$ for every 0.1 of r. Results of these calculations are tabulated in the right half of TABLE IV, together with the values of k.

TABLE IV.

Soil symbol	r	M g/cc.	$c\rho$ cal/cc.	$k \times 10^3$ C. G. S.	$K \times 10^3$ C. G. S.
	0.0	1.09	0.185	2.25	0.42
	0.1	1.08	0.235	2.60	0.61
A	0.2	1.07	0.285	3.10	0.83
	0.3	1.05	0.331	3.65	1.21
	0.4	1.03	0.374	4.30	1.61
	0.5	1.00	0.411	5.05	2.08

Soil symbol	r	M g/cc.	$c\rho$ cal/cc.	$k\times10^3$ C. G. S.	$K\times10^3$ C. G. S.
A	0.6	0.96	0.444	6.10	2.71
	0.7	0.90	0.467	7.30	3.40
B	0.0	1.04	0.198	1.00	0.20
	0.1	0.96	0.228	1.25	0.29
	0.2	0.91	0.261	1.60	0.42
	0.3	0.87	0.291	2.05	0.60
	0.4	0.85	0.325	2.60	0.85
	0.5	0.84	0.362	3.20	1.16
	0.6	0.83	0.398	4.00	1.59
	0.7	0.825	0.435	5.15	2.24
	0.8	0.82	0.464	6.20	2.88
C	0.0	0.80	0.152	1.60	0.25
	0.1	0.87	0.207	1.80	0.37
	0.2	0.93	0.267	2.10	0.55
	0.3	0.98	0.328	2.40	0.80
	0.4	1.02	0.391	2.95	1.15
	0.5	1.03	0.444	3.55	1.58
	0.6	1.02	0.489	4.00	1.96

From TABLE IV, we get the curves of K as a function of r. They are shown in Fig. 3. K' curves shown in Fig. 3 with broken lines correspond to k' curves, namely they represent the thermal conductivity of the same soil at room temperature. K's are computed in the same manner described in the previous paper.

These K–r curves for frozen soils are expressed by an empirical formula of the similar form as the case of k–r curves.

$$K = ce^{dr},$$

where c and d are constants. The numerical values of these constants for the three kinds of soils are tabulated in the right half of TABLE III. This form of the empirical formula also holds only in the region below the saturation moisture ratio.

Dr. KERSTEN proposes an empirical formula of the form, $K = A + Br$ for the frozen soils.[4] His formula seems to be the first approximation of our formula, and for the smaller values of r these two equations come to the same expression. The exponential relation will be the general form between K and r for frozen soils.

b) Results in the region above the saturation water content.

Because of the segregation of ice, the samples of sirloin type freezing or ice filaments layer type usually contain excessive water than the saturation water content. Measurements of k are also carried out for these samples; that is, those containing excessive water than saturation.

The results are tabulated in TABLE V. From these results we see that the difiusivity in the region above saturation water content also increases with the moisture ratio, but the rate of increase is smaller than the case below saturation.

These results are also shown in Fig. 3. It is clearly seen that k curves in this region have smaller values than the exponential curves, which hold well for the region below saturation. For far larger values of r, the thermal difiusivity of these frozen soils will approach to the value of ice; that is, 0.011 C.G.S.

TABLE V

Soil symbol	Exp. no.	Type of freezing	r	$k\times10^3$ C. G. S.	M g/cc.
A	59	sirloin type	0.72	6.88	0.91
	63	"	0.73	8.12	0.85
	64	"	2.13	9.40	0.38
	68	ice filaments layer	2.77	8.30	0.26
B	79	sirloin type	0.79	5.39	0.84
	85	"	0.80	6.02	0.77
C	91	sirloin type	1.33	7.47	0.82
	93	"	1.84	6.73	0.41
	95	"	2.38	6.04	0.31
	97	"	1.17	6.09	0.63
	107	"	1.08	4.62	0.66

c) Variation of the thermal diffusivity due to the direction of heat flow.

When the ground is cooled gradually from the surface and sufficient water is supplied from the soil below, ice layers are segregated from the soil. In nature, these ice layers lie in a horizontal position; namely, perpendicular to the direction of heat flow.

Those frozen soils with segregated ice layers may have different thermal properties for different directions. If the difference might exist, it would appear markedly in the case of the sirloin type freezing. All the measurements described above are in the case, when the direction of heat flow is parallel to the direction of freezing. This case is expressed by the symbol $\|$. The thermal diffusivity is measured also for the same specimens in the case when the directions are perpendicular with each other. The value is marked with $\perp$.

The drilling of the holes for the thermojunctions were made carefully, so that the measurement could be done in the neighbourhood in both cases. The section views of the sample used in this experiment are shown in Photos. No. 5a and b, Pl. I. No. 5a is the parallel case $\|$, and No. 5b is the case when the specimen is rotated by right angle around the horizontal axis; that is $\perp$. In both cases, the direction of heat flow for measuring the thermal diffusivity is from top to bottom in the photograph.

The results obtained for various values of moisture ratio are tabulated in TABLE VI. In five tests, three cases are $\| > \perp$ and two cases $\| < \perp$. No simple relation is observed with moisture ratio. At present we are lead to the conclusion that the difference of the thermal diffusivity for different directions of heat flow is small and complicated so that no simple relation is observed. It is assured that the effect of the direction of heat flow can be neglected for the practical application.

TABLE VI.

Soil symbol	Exp. no.	r	$k \times 10^3$ C. G. S.	
			$\|$	$\perp$
A	64 & 65	2.14	9.4	8.1
C	91 & 92	1.33	7.5	6.4
	93 & 94	1.84	6.7	8.2
	95 & 96	2.38	6.0	5.6
	97 & 98	1.17	6.5	6.5

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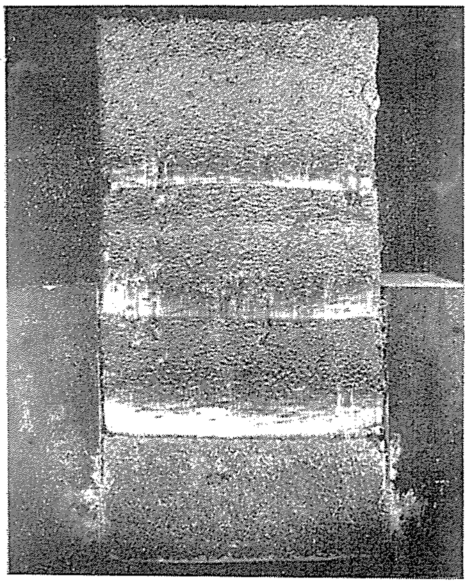
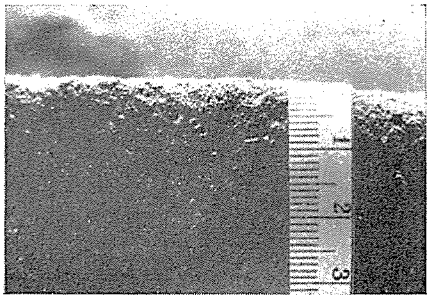
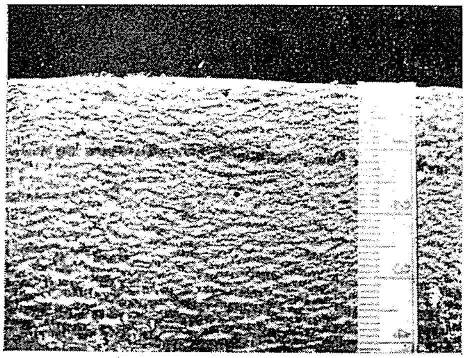


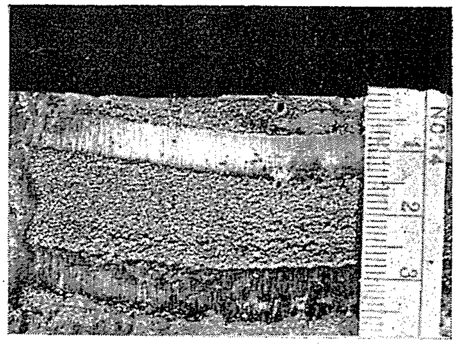
No. 1 A sample with remarkable frost heaving by the segregation of ice.
Soil. C.



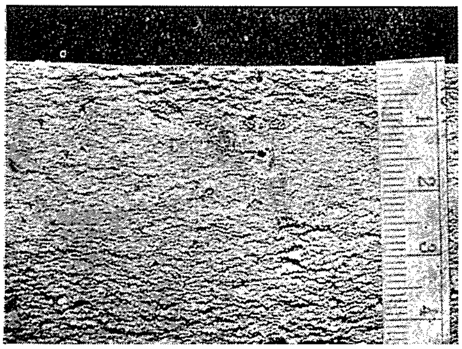
No. 2 Concrete type freezing.
Soil B Exp. No. 73
r = 0.44 M = 0.86



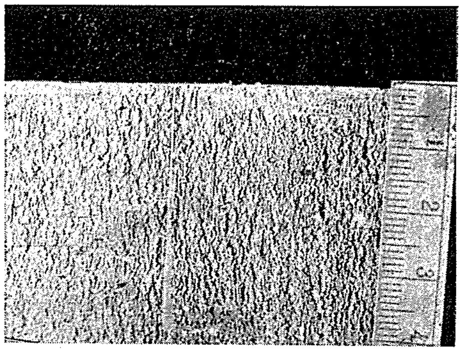
No. 3 Sirloin type freezing
Soil C. Exp. No. 95
r = 2.38 M = 0.31



No. 4 Ice filaments layer
Soil A. Exp. No. 68
r = 2.77 M = 0.23



Heat flow ↓



No. 5a Exp. No. 97 || **Soil C.** No. 5b Exp. No. 98 ⊥

r=1.17 M=0.63