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Phase Shifts in D-D Scattering up to 3.5 Mev

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(Received August 2, 1951)

The phase shifts analysis of D-D scattering data by J. M. BLAIR et al. \((E_d: 0.87 \sim 3.52 \text{ Mev})\) is performed by this author accurately within a few \(\%\) in deviations. Only \(S\)- and \(P\)-waves are necessary up to \(3.5 \text{ Mev}\).

\section{Introduction}

Recently, J. M. BLAIR et al.\(^1\) have published D-D elastic scattering data in the higher range of the bombarding energy of 0.87 \(-3.52 \text{ Mev}\), while as a reliable data there was hitherto only that in the region of up to 0.83 Mev given by HEYDENBURG and ROBERTS,\(^2\) and the corresponding analysis was also very few, e.g. by KURONUMA, SUEOKA and TOYA.\(^3\) So, this author has performed phase shifts analysis of this new data assuming deuteron to be a Boson of spin 1.

\section{Scattering Cross Section}

Now, we give the collision cross section per unit solid angle of the laboratory system.

(i) classical Rutherford-Darwin formula for pure Coulombian field considering the similar particle collision,

\[ \sigma_{el} = \left( \frac{Z_1 Z_2 e^2}{2\mu \omega} \right)^2 \cdot 4 \cos \phi \left( \sec \phi + \cosec \phi \right), \quad (1) \]

where \(\mu\): reduced mass, \(\phi\): scattering angle in L.S.

(ii) quantum-mechanical Mott-Massey formula considering the symmetry character of the wave functions and spins,

\[ \sigma_{Mott} = \left( \frac{Z_1 Z_2 e^2}{2\mu \omega} \right)^2 \cdot 4 \cos \phi \left( \sec \phi + \cosec \phi \right) \]

\[ + \frac{2}{3} \sec \phi \cosec \phi \cdot \cos \left( \eta \ln \tan \phi \right), \quad (2) \]
where \( \eta = \frac{Z_1 Z_2 e^2}{\hbar v} \).

(iii) scattering formula for similar Fermions is given by e.g. BREIT, CONDON and PRESENT for \( p-p \) scattering case, thus we must modify it to use for Bosons.

\[
\sigma = 4 \cos \phi \cdot P,
\]

\[
P = \frac{1}{2s + 1} \left\{ (s+1)|f(\theta) + f(\pi - \theta)|^2 + 8|f(\theta) - f(\pi - \theta)|^2 \right\},
\]

where \( s \) is the spin quantum number, and for Fermions \((s+1)\) and \( s \) are interchanged.

\[
f(\theta) = \frac{Z_1 Z_2 e^2}{2\mu v^2} \cdot \frac{1}{\sin^2 \theta} \cdot e^{\frac{-i\eta}{2} \ln \sin \frac{\theta}{2} + \frac{i}{2} \sigma_1 + \epsilon_1}
\]

\[
+ \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) (e^{i\eta \delta_1} - 1) e^{i\sigma_1} P_l (\cos \theta),
\]

where \( k = \frac{\mu v}{\hbar} = \frac{\sqrt{2\mu E}}{\hbar} = \frac{1}{\hbar}, \ E: \) relative \( K.E. = E_d/2 \)

\( \theta: \) scattering angle in C.M.S., \( \theta/2 = \phi: \) in L.S.

\( \sigma_1 = \arg \Gamma(i\eta + L + 1): \) phase constants when pure Coulomb fields holds down to \( r = 0 \).

\( K_L: \) phase shifts depending on the nuclear potential, each for the \( L \)-th wave.

Therefore

\[
f(\theta) = \frac{Z_1 Z_2 e^2}{2\mu v^2} \cdot \frac{1}{\sin^2 \theta} \cdot e^{\frac{-i\eta}{2} \ln \sin \frac{\theta}{2} + \frac{i}{2} \sigma_1 + \epsilon_1}
\]

\[
\times \left\{ 1 - \sin^2 \phi \cdot 2 \sum_{l} (2l+1) \sin K_l \cdot e^{i\varphi_l} P_l \right\},
\]

where \( \varphi_l = K_l + \eta \ln \sin \frac{\theta}{2} + 2(\sigma_1 - \sigma_0) \).

and,

\[
f(\pi - \theta) = \frac{Z_1 Z_2 e^2}{2\mu v^2} \cdot \frac{1}{\cos^2 \theta} \cdot e^{\frac{-i\eta}{2} \ln \cos \theta} \cdot e^{\frac{i}{2} \sigma_1 + \epsilon_1}
\]

\[
\times \left\{ 1 - \cos^2 \phi \cdot 2 \sum_{l} (2l+1) \sin K_l \cdot e^{i\varphi_l} P_l \right\},
\]

where \( \varphi_l = K_l + \eta \ln \cos \frac{\theta}{2} + 2(\sigma_1 - \sigma_0) \).
Putting \( \frac{s+1}{2s+1} \equiv C_s \), \( \frac{s}{2s+1} \equiv C_A \) for Boson, and vice versa for Fermion,

\[
P_{/\gamma} \left( \frac{e^2}{2\mu a_0^2} \right)^2 = \frac{1}{\sin^4 \phi} + \frac{1}{\cos^4 \phi} + (C_s + C_A) 2 \frac{\cos \left[ \frac{\gamma \ln \tan^2 \phi}{S^2} \right]}{C^2}
\]

\[
- \frac{4}{\gamma} \sum_i (C_s + C_A + (-1)^i (C_s - C_A)) (2l+1) \left[ \frac{\cos \phi_i^*}{S^2} + (-1)^i \frac{\cos \phi_i}{C^2} \right] \sin K_i' \cdot P_i
\]

\[
+ \frac{8}{\gamma} \sum_{i,i'} ((C_s + C_A)(1 + (-1)^i) + ((-1)^i + (-1)^{i'})(C_s - C_A))(2l+1)(2l'+1) \times
\]

\[
\times P_i \cdot P_i' \sin K_i \sin K_i' \cos (\phi_i - \phi_i'),
\]

(6)

in which in the last term there is no cross term of odd, even \( l, l' \), and

\[
\phi_i = K_i + 2 (\sigma_i - \sigma_o), \quad \phi^*_i = \phi_i + \gamma \ln \sin^2 \phi, \quad \phi^*_i = \phi_i + \gamma \ln \cos^2 \phi
\]

\[
\sigma_i - \sigma_o = \tan^{-1} \frac{\gamma}{2}, \quad \sigma_i - \sigma_i = \tan^{-1} \frac{\gamma}{2}, \quad \ldots \ldots \sigma_i - \sigma_{i-1} = \tan^{-1} \frac{\gamma}{2}.
\]

Neglecting higher \( K_L \) with \( L \) larger than 3,

\[
P = P_M + (\Delta P)_o + (\Delta P)_i + (\Delta P)_o^2 .
\]

(7)

For a while, divided by a factor \( \left( \frac{2\mu a_0^2}{e^2} \right)^2 \) and affix asterisk* ,

\[
P_M^* = \frac{1}{S^2} + \frac{1}{C^2} + (C_s - C_A) 2 \frac{\cos \left[ \frac{\gamma \ln \tan^2 \phi}{S^2} \right]}{C^2} \cdot C^2
\]

\[
\Delta P_o^* = - \frac{4}{\gamma} \sin K_o \cdot P_o \cdot 2C_s \left( \frac{\cos \phi_o^*}{S^2} + \frac{\cos \phi_o^*}{C^2} \right) + \frac{8}{\gamma} \sin^2 K_o \cdot P_o^2 \cdot 2C_s
\]

\[
\Delta P_i^* = - \frac{4}{\gamma} 3 \sin K_i \cdot P_i \cdot 2C_A \left( \frac{\cos \phi_i^*}{S^2} - \frac{\cos \phi_i^*}{C^2} \right)
\]

\[
+ \frac{8}{\gamma} 3^2 \sin^2 K_i \cdot P_i^2 \cdot 2C_A
\]

\[
\Delta P_o^* = - \frac{4}{\gamma} 5 \sin K_o \cdot P_o \cdot 2C_s \left( \frac{\cos \phi_o^*}{S^2} + \frac{\cos \phi_o^*}{C^2} \right)
\]

\[
+ \frac{8}{\gamma} 5^2 \sin^2 K_o \cdot P_o \cdot 2C_s
\]

\[
+ \frac{8}{\gamma} 5 \sin K_o \cdot \sin K_o \cdot P_o \cdot P_o \cos (\phi_o - \phi_o) \cdot 2 \cdot 2C_s
\]

(8)
Putting, \( \eta \ln S^2 \) or \( C^2 = \alpha_0 \) or \( \beta_0 \),
\[
2(\sigma_1 - \sigma_0) + \alpha_0 \text{ or } \beta_0 \equiv \alpha_1 \text{ or } \beta_1, \\
2(\sigma_2 - \sigma_0) + \alpha_0 \text{ or } \beta_0 \equiv \alpha_2 \text{ or } \beta_2
\]
and \( C_S = \frac{2}{3}, \ C_A = 1/3 \) for deuteron, then we get
\[
P_M = \frac{1}{\sin^2 \frac{\theta}{2}} + \frac{1}{\cos^2 \frac{\theta}{2}} + \frac{2}{3} \frac{1}{S^2 \cdot C^2} \cos \left[ \eta \ln \tan^2 \frac{\theta}{2} \right], \ (\text{Mott-formula})
\]
\[
\Delta P_n = -\frac{1}{\eta} \frac{16}{3} \left( \frac{\cos \alpha_0}{S^2} + \frac{\cos \beta_0}{C^2} \right) \sin K_0 \cos K_0
\]
\[
+ \left( \frac{16}{\eta} \frac{2}{3} + \frac{16}{\eta} \frac{1}{3} \left( \frac{\sin \alpha_0}{S^2} + \frac{\sin \beta_0}{C^2} \right) \right) \sin^2 K_0,
\]
\[
\Delta P_i = -\frac{1}{\eta} \frac{8}{3} \left( \frac{\cos \alpha_1}{S^2} - \frac{\cos \beta_1}{C^2} \right) P_i \sin K_i \cos K_i
\]
\[
+ \left( \frac{16}{\eta} \frac{3}{3} P_i^2 + \frac{1}{\eta} \frac{8}{3} \left( \frac{\sin \alpha_1}{S^2} - \frac{\sin \beta_1}{C^2} \right) P_i \right) \sin^2 K_i,
\]
\[
\Delta P_z = -\frac{1}{\eta} \frac{80}{3} \left( \frac{\cos \alpha_2}{S^2} + \frac{\cos \beta_2}{C^2} \right) P_z \sin K_z \cos K_z
\]
\[
+ \left( \frac{16}{\eta} \frac{5}{3} P_z^2 + \frac{1}{\eta} \frac{80}{3} \left( \frac{\sin \alpha_2}{S^2} + \frac{\sin \beta_2}{C^2} \right) P_z \right) \sin^2 K_z
\]
\[
+ \frac{1}{\eta} \frac{320}{3} P_z \cdot \sin K_0 \cdot \sin K_z \cdot \cos (K_z - K_0 + 2(\sigma_2 - \sigma_0)).
\]

§ 3. Mott-Massey Cross Section

To perform the phase shifts analysis, we first calculate the so-called Mott-ratio, for which Mott-Massey Cross Section is used.
\[
\sigma_{Mott} = \left( \frac{e^2}{2\mu v^2} \right)^2 \cdot P_M^2
\]
and
\[
P_M = \csc^2 \phi + \sec^2 \phi + \frac{2}{3} \cdot \varphi \cdot \csc^2 \phi \cdot \sec^2 \phi,
\]
\[
\phi = \cos \left[ \frac{e^2}{h \nu} \cdot 2 \ln \tan \phi \right], \ \varphi (L. S.) = \frac{\theta}{2} \ (\text{C.M.S.).}
\]
Here, from \( \frac{1}{2} M_\phi \cdot v^2 = E_d \) (i) \( \gamma = \frac{e^2}{h \nu} = \frac{(4.770 \times 10^{-10})^2}{1.042 \times 10^{-2}} \cdot \frac{1}{10^2 \cdot 0.9573E_d} \)
\[
(\text{ii}) \frac{e^2}{2\mu v} = \frac{4.770 \times 10^{-10}}{1.5911 \times 10^{-12} \cdot 10^8}, \ E_d \ (\text{Mev})
\]
and their values are given in Table I.
Phase Shifts in D-D Scattering up to 3.5 Mev

TABLE I.

<table>
<thead>
<tr>
<th>$E_d$ (Mev)</th>
<th>$\gamma$</th>
<th>$\left(\frac{e^2}{2\mu v^2}\right)^2$ in $10^{-2}b$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.87</td>
<td>0.2392</td>
<td>0.6753</td>
</tr>
<tr>
<td>1.43</td>
<td>0.1866</td>
<td>0.2500</td>
</tr>
<tr>
<td>1.96</td>
<td>0.1594</td>
<td>0.1330</td>
</tr>
<tr>
<td>2.49</td>
<td>0.1414</td>
<td>0.0884</td>
</tr>
<tr>
<td>3.02</td>
<td>0.1284</td>
<td>0.0560</td>
</tr>
<tr>
<td>3.52</td>
<td>0.1189</td>
<td>0.0412</td>
</tr>
<tr>
<td>0.72</td>
<td>0.2630</td>
<td>0.9882</td>
</tr>
</tbody>
</table>

The results of Mott-ratio $\sigma_{\text{Obs.}}/\sigma_{\text{Mott.}}$ are given in Table II.

TABLE II.

<table>
<thead>
<tr>
<th>$E_d$</th>
<th>0.87</th>
<th>1.43</th>
<th>1.96</th>
<th>2.49</th>
<th>3.02</th>
<th>3.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1.457</td>
<td>1.494</td>
<td>1.758</td>
<td>1.926</td>
<td>2.308</td>
<td>2.615</td>
</tr>
<tr>
<td>20</td>
<td>1.635</td>
<td>2.193</td>
<td>2.820</td>
<td>3.690</td>
<td>4.385</td>
<td>5.982</td>
</tr>
<tr>
<td>30</td>
<td>2.808</td>
<td>4.884</td>
<td>7.094</td>
<td>11.390</td>
<td>15.713</td>
<td>19.376</td>
</tr>
<tr>
<td>35</td>
<td>3.609</td>
<td>6.680</td>
<td>11.201</td>
<td>15.121</td>
<td>21.896</td>
<td>27.904</td>
</tr>
<tr>
<td>40</td>
<td>4.411</td>
<td>8.582</td>
<td>13.735</td>
<td>20.127</td>
<td>25.738</td>
<td></td>
</tr>
<tr>
<td>50°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>55°</td>
<td>2.170</td>
<td>5.696</td>
<td>9.992</td>
<td>13.635</td>
<td>18.922</td>
<td>23.999</td>
</tr>
</tbody>
</table>

§ 4. S-Phase Shifts

Next, we examine the $P_0$ terms to express the difference of the values in Table II. from one.

We determine $K_0$ at the scattering angle $\phi = 45^\circ (\theta = 90^\circ)$ where P-wave contribution vanishes, then the following values are obtained.

TABLE III.

<table>
<thead>
<tr>
<th>$E_d$(Mev)</th>
<th>0.87</th>
<th>1.43</th>
<th>1.96</th>
<th>2.49</th>
<th>3.02</th>
<th>3.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0$</td>
<td>-19°</td>
<td>-24°58'</td>
<td>-30°54'</td>
<td>-35°58'</td>
<td>-40°10'</td>
<td>-44°30'</td>
</tr>
</tbody>
</table>
And their corresponding deviations i.e. $\Delta P_0 (\text{Cal.}) - \Delta P_0 (\text{Exp.})$ are shown in Fig. 1. with the Mott-ratio including the 0.72 one in the 0.87 Mev case for comparison.

![Graph showing $\Delta P_0 (\text{Cal.}) - \Delta P_0 (\text{Exp.})$ for different energies.]

**§ 5. $P$-wave phase shift**

In the third, we examine the $P_i$ term. We determine $K_i$ at the scattering angle $\phi = 27°21' (\theta = 54°42')$, where $D$-wave contribution vanishes, then we have Table IV.

<table>
<thead>
<tr>
<th>$E_r (\text{MeV})$</th>
<th>0.87</th>
<th>1.43</th>
<th>1.96</th>
<th>2.49</th>
<th>3.02</th>
<th>3.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_i$</td>
<td>$-3°33'$</td>
<td>$-7°40'$</td>
<td>$-10°30'$</td>
<td>$-14°30'$</td>
<td>$-18°16'$</td>
<td>$-22°30'$</td>
</tr>
</tbody>
</table>

And their final results are given in Table V as the upper figures and the lower bracketted ones are the final deviations from them. In the last row are the mean errors. The effect of $D$-wave is inside the range of experimental error.
The smoothly varying nature of the phase shifts angles are seen in Fig. 2., and are consistent with earlier analysis \( K_0 = -11^\circ \sim -13^\circ \), and \( K_1 \approx 0 \) given by Kuronuma, Sueoka and Toya\(^5\) at 0.72 Mev. That both \( K_0 \) as well as \( K_1 \) are negative means that \( D-D \) interaction is repulsive nature in favour to the following facts

(i) the ground state \( He^1 \) is a mixture \( ^1S_0 + ^3P_0 + ^5D_0 \) and (ii) the nuclear reaction \( D+D \rightarrow He \) is very rare if occur, as was found only quite recently\(^6\) with very small probability (its cross section is smaller than \( 10^{-33} \text{ cm}^2 \) at 1 Mev).

It is the most remarkable when compared with \( D(d, p)T^3 \) and \( D(d, n)He^3 \) reactions, that \( D \)-wave contribution is very small in this case. And this extraordinary difference is attributed to the
large spin-orbit coupling favoring to the latter, which must be explained more basically in the future.

In conclusion, the author must express many thanks to Prof. K. UMEDA for his encouragements.

![Graph showing $\theta$, $K_0$, and $K_1$](image)

**Fig. 2.** $D-D, K_0$ & $K_1$

**References**

5) L. ROSENFIELD, *Nuclear Forces*, (1948). *Chapter 17, §2*