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Phase Shifts in D-D Scattering up to 3.5 Mev

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The phase shifts analysis of D-D scattering data by J. M. BLAIR et al. (E_d: 0.87~3.52 Mev) is performed by this author accurately within a few % in deviations. Only S- and P- waves are necessary up to 3.5 Mev.

§ 1. Introduction

Recently, J. M. BLAIR et al.[1] have published D-D elastic scattering data in the higher range of the bombarding energy of 0.87 ~3.52 Mev, while as a reliable data there was hitherto only that in the region of up to 0.83 Mev given by HEYDENBURG and ROBERTS,[2] and the corresponding analysis was also very few, e.g. by KURONUMA, SUEOKA and TOYA.[3] So, this author has performed phase shifts analysis of this new data assuming deuteron to be a Boson of spin 1.

§ 2. Scattering Cross Section

Now, we give the collision cross section per unit solid angle of the laboratory system.

(i) classical Rutherford-Darwin formula for pure Coulombian field considering the similar particle collision,

$$\sigma_{cl.}=\left(\frac{Z_1Z_2e^2}{2\mu v^2}\right)^2\cdot 4\cos\phi\,(\sec^4\phi+\mathrm{cosec}^4\phi)\,, \qquad (1)$$

where μ: reduced mass, ϕ: scattering angle in L.S.

(ii) quantum-mechanical Mott-Massey formula considering the symmetry character of the wave functions and spins,

$$\sigma_{\mathrm{Mott}}=\left(\frac{Z_1Z_2e^2}{2\mu v^2}\right)^2\cdot 4\cos\phi\,\Big(\sec^4\phi+\mathrm{cosec}^4\phi + \frac{2}{3}\sec^2\phi\,\mathrm{cosec}^2\phi\cdot\cos(\eta\, ln\tan^2\phi)\Big)\,, \qquad (2)$$

where $\eta = \dfrac{Z_1 Z_2 e^2}{\hbar v}$.

(iii) scattering formula for similar Fermions is given by e.g. BREIT, CONDON and PRESENT[4] for p-p scattering case, thus we must modify it to use for Bosons.

$$\sigma = 4\cos\phi \cdot P ,$$

$$P = \frac{1}{2s+1}\left\{(s+1)|f(\theta)+f(\pi-\theta)|^2 + s|f(\theta)-f(\pi-\theta)|^2\right\} , \quad (3)$$

where s is the spin quantum number, and for Fermions $(s+1)$ and s are interchanged.

$$f(\theta) = \frac{Z_1 Z_2 e^2}{2\mu v^2} \cdot \frac{1}{\sin^2\frac{\theta}{2}} \cdot e^{-i\eta\, ln \sin^2\frac{\theta}{2} + i2\sigma_0 + i\pi}$$

$$+ \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)(e^{i2K_l}-1)\, e^{i2\sigma_l} P_l(\cos\theta) , \quad (4)$$

where $k = \dfrac{\mu v}{\hbar} = \dfrac{\sqrt{2\mu E}}{\hbar} = \dfrac{1}{\lambda\!\!\!^{-}}$, E : relative $K.E. = E_d/2$

θ : scattering angle in C.M.S., $\theta/2 = \phi$: in L.S.

$\sigma_l = \arg\Gamma(i\eta + L + 1)$: phase constants when pure Coulomb fields holds down to $r = 0$.

K_L : phase shifts depending on the nuclear potential, each for the L-th wave .

Therefore

$$f(\theta) = \frac{Z_1 Z_2 e^2}{2\mu v} \cdot \frac{1}{\sin^2\phi} \cdot e^{-i\eta\, ln \sin^2\phi + i2\sigma_0 + i\pi}$$

$$\times \left\{1 - \sin^2\phi \frac{2}{\eta}\sum_l (2l+1)\sin K_l \cdot e^{i\varphi_l^s} \cdot P_l\right\} ,$$

where

$$\varphi_l^s = K_L + \eta\, ln \sin^2\phi + 2(\sigma_l - \sigma_0) . \quad (5)$$

and, $$f(\pi-\theta) = \frac{Z_1 Z_2 e^2}{2\mu v} \cdot \frac{1}{\cos^2\phi} \cdot e^{-i\eta\, ln \cos^2\phi + i2\sigma_0 + i\pi}$$

$$\times \left\{1 - \cos^2\phi \frac{2}{\eta}\sum_l (2l+1)\sin K_l \cdot e^{i\varphi_l^c} \cdot P_l\right\} ,$$

where

$$\varphi_l^c = K_L + \eta\, ln \cos^2\phi + 2(\sigma_l - \sigma_0) .$$

Putting $\frac{s+1}{2s+1} \equiv C_S$, $\frac{s}{2s+1} \equiv C_A$ for Boson, and vice versa for Fermion,

$$P/\left(\frac{\epsilon^2}{2\mu v_2}\right)^2 = \frac{1}{\sin^4\phi} + \frac{1}{\cos^4\phi} + (C_S + C_A)\, 2\frac{\cos\,[\eta\, ln \tan^2\phi]}{S^2 C^2}$$

$$-\frac{4}{\eta}\sum_l (C_S + C_A + (-1)^l (C_S - C_A))\,(2l+1)\left\{\frac{\cos\varphi_l^s}{S^2} + (-1)^l \frac{\cos\varphi_l^c}{C^2}\right\} \sin K^l \cdot P_l$$

$$+\frac{8}{\eta^2}\sum_l (C_S + C_A + (-1)^l (C_S - C_A))\,(2l+1)^2 P_l^2 \cdot \sin^2 K_l$$

$$+\frac{8}{\eta^2}\sum_{l,l'} ((C_S + C_A)(1 + (-1)^{l+l'}) + ((-1)^l + (-1)^{l'})(C_s - C_A))(2l+1)(2l'+1) \times$$

$$\times\; P_l P_l' \sin K_l \sin K_l' \cos(\varphi_l - \varphi_l')\,, \qquad (6)$$

in which in the last term there is no cross term of odd, even l, l', and

$$\varphi_l = K_l + 2(\sigma_s - \sigma_o), \quad \varphi_l^s = \varphi_l + \eta\, ln \sin^2\phi, \quad \varphi_l^c = \varphi_l + \eta\, ln \cos^2\phi$$

$$\sigma_1 - \sigma_0 = \tan^{-1}\eta \quad , \quad \sigma_2 - \sigma_1 = \tan^{-1}\frac{\eta}{2} \quad , \quad \cdots\cdots \sigma_l - \sigma_{l-1} = \tan^{-1}\frac{\eta}{l}\,.$$

Neglecting higher K_L with L larger than 3,

$$P = P_M + (\Delta P)_0 + (\Delta P)_1 + (\Delta P)_2\,. \qquad (7)$$

For a while, divided by a factor $\left(\frac{2\mu v^2}{e^2}\right)^2$ and affix asteric*,

$$P_M^* = \frac{1}{S^4} + \frac{1}{C^4} + (C_S - C_A)\, 2\frac{\cos[\eta\, ln \tan^2\phi]}{S^2 \cdot C^2}\,,$$

$$\Delta P_0^* = -\frac{4}{\eta}\sin K_0 \cdot P_0 \cdot 2C_S\left(\frac{\cos\varphi_0^s}{S^2} + \frac{\cos\varphi_0^c}{C^2}\right) + \frac{8}{\eta^2}\sin^2 K_0 \cdot P_0^2 \cdot 2C_S\,,$$

$$\Delta P_1^* = -\frac{4}{\eta} 3 \sin K_1 \cdot P_1 \cdot 2C_A \cdot \left(\frac{\cos\varphi_1^s}{S^2} - \frac{\cos\varphi_1^c}{C^2}\right)$$

$$+\frac{8}{\eta^2} 3^2 \sin^2 K_1 \cdot P_1^2 \cdot 2C_A\,, \qquad (8)$$

$$\Delta P_2^* = -\frac{4}{\eta} 5 \sin K_2 \cdot P_2 \cdot 2C_S\left(\frac{\cos\varphi_2^s}{S^2} + \frac{\cos\varphi_2^c}{C^2}\right)$$

$$+\frac{8}{\eta^2} 5^2 \sin^2 K_2 \cdot P_2 \cdot 2C_S$$

$$+\frac{8}{\eta^2} 5 \sin K_0 \cdot \sin K_2 \cdot P_0 \cdot P_2 \cos(\varphi_2 - \varphi_0) \cdot 2 \cdot 2C_S\,.$$

Putting, $\eta\, ln\, S^2$ or $C^2 \equiv \alpha_0$ or β_0 ,

$$2(\sigma_1-\sigma_0)+\alpha_0 \text{ or } \beta_0 \equiv \alpha_1 \text{ or } \beta_1 ,$$

$$2(\sigma_2-\sigma_0)+\alpha_0 \text{ or } \beta_0 \equiv \alpha_2 \text{ or } \beta_2$$

and $C_S = 2/3$, $C_A = 1/3$ for deuteron, then we get

$$P_M^* = \frac{1}{\sin^4\frac{\theta}{2}} + \frac{1}{\cos^4\frac{\theta}{2}} + \frac{2}{3}\frac{1}{S^2\cdot C^2}\cos\left[\eta\, ln \tan^2\frac{\theta}{2}\right] , \text{ (Mott-formula)}$$

$$\begin{aligned}
\Delta P_0^* &= -\frac{1}{\eta}\frac{16}{3}\left(\frac{\cos\alpha_0}{S^2}+\frac{\cos\beta_0}{C^2}\right)\sin K_0 \cos K_0 \\
&\quad + \left(\frac{16}{\eta^2}\frac{2}{3}+\frac{16}{\eta}\frac{1}{3}\left(\frac{\sin\alpha_0}{S^2}+\frac{\sin\beta_0}{C^2}\right)\right)\sin^2 K_0 , \\
\Delta P_1^* &= -\frac{1}{\eta}\frac{8}{3}\left(\frac{\cos\alpha_1}{S^2}-\frac{\cos\beta_1}{C^2}\right)P_1 \sin K_1 \cos K_1 \qquad (9) \\
&\quad + \left(\frac{16}{\eta^2}\frac{3^2}{3}P_1^2+\frac{1}{\eta}\frac{8}{3}\left(\frac{\sin\alpha_1}{S^2}-\frac{\sin\beta_1}{C^2}\right)P_1\right)\sin^2 K_1 , \\
\Delta P_2^* &= -\frac{1}{\eta}\frac{80}{3}\left(\frac{\cos\alpha_2}{S^2}+\frac{\cos\beta_2}{C^2}\right)P_2\cdot\sin K_2\cdot\cos K_2 \\
&\quad + \left(\frac{16}{\eta^2}\frac{5^2}{3}P_2^2+\frac{1}{\eta}\frac{80}{3}\left(\frac{\sin\alpha_2}{S^2}+\frac{\sin\beta_2}{C^2}\right)P^2\right)\sin^2 K_2 \\
&\quad + \frac{1}{\eta^2}\frac{320}{3}P_2\cdot\sin K_0\cdot\sin K_2\cdot\cos(K_2-K_0+2(\sigma_2-\sigma_0)) .
\end{aligned}$$

§ 3. Mott-Massey Cross Section

To perform the phase shifts analysis, we first calculate the so-called Mott-ratio, for which Mott-Massey Cross Section is used.

$$\sigma_{\text{Mott}} = \left(\frac{e^2}{2\mu v^2}\right)^2\cdot P_M^*$$

and $$P_M^* = \operatorname{cosec}^4\phi + \sec^4\phi + 2/3\cdot\Phi\cdot\operatorname{cosec}^2\phi\sec^2\phi ,$$

$$\Phi = \cos\left[\frac{e^2}{\hbar v}\cdot 2\, ln\tan\phi\right] , \quad \phi\,(L.S.) = \frac{\theta}{2}\ \text{(C.M.S.)} .$$

Here, from $\frac{1}{2}M_d v^2 = E_d$ (i) $\eta = \dfrac{e^2}{\hbar v} = \dfrac{(4.770\times10^{-10})^2}{1.042\times10^{-27}}\cdot\dfrac{1}{10^2\sqrt{0.9573E_d}}$

(ii) $\dfrac{e^2}{2\mu v} = \dfrac{4.770\times10^{-10})^2}{2\cdot E_d\cdot 1.5911\times10^{-12}\cdot10^6}$, E_d (Mev)

and their values are given in TABLE I.

TABLE I.

E_d (Mev)	η	$\left(\frac{e^2}{2\mu v^2}\right)^2$ in $10^{-2}b$.
0.87	0.2392	0.6753
1.43	0.1866	0.2500
1.96	0.1594	0.1330
2.49	0.1414	0.0824
3.02	0.1284	0.0560
3.52	0.1189	0.0412
0.72	0.2630	0.9862

The results of Mott-ratio $\sigma_{\text{Obs.}}/\sigma_{\text{Mott}}$ are given in TABLE II.

TABLE II.

ϕ \ E_d	0.87	1.43	1.96	2.49	3.02	3.52
10°	1.457	1.494	1.758	1.926	2.308	2.615
15	1.635	2.193	2.820	3.690	4.835	5.982
20	2.228	3.255	4.942	6.851	9.196	11.606
25	2.808	4.894	7.694	11.390	15.718	19.376
30	3.609	6.680	11.201	15.121	21.896	27.904
35	4.411	8.582	13.735	20.127	—	35.738
40	—	9.910	15.948	23.400	32.143	41.263
45	—	9.939	16.502	24.448	—	—
50°	—	—	—	23.927	31.988	39.368
27°21′	3.170	5.696	9.092	13.636	18.922	23.999

§ 4. *S*-Phase Shifts

Next, we examine the P_0 terms to express the difference of the values in TABLE II. from one.

We determine K_0 at the scattering angle $\phi = 45°$ ($\theta = 90°$) where P-wave contribution vanishes, then the following values are obtained.

TABLE III.

E_d (Mev)	0.87	1.43	1.96	2.49	3.02	3.52
K_0	−19°	−24°58′	−30°54′	−35°58′	−40°10′	−44°30′

And their corresponding deviations i.e. ΔP_0 (Cal.) $-\Delta P_0$ (Exp.) are shown in Fig. 1. with the Mott-ratio including the 0.72 one in the 0.87 Mev case for comparison.

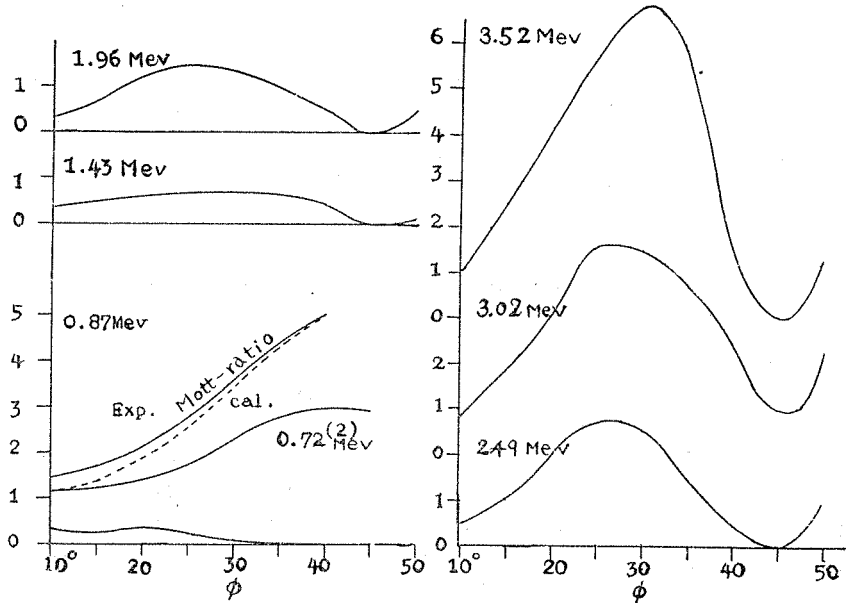

Fig. 1. ΔP_0 (Cal.) $-\Delta P_0$ (Exp.)

§ 5. *P*-wave phase shift

In the third, we examine the P_1 term. We determine K_1 at the scattering angle $\phi = 27°21'$ ($\theta = 54°42'$), where D- wave contribution vanishes, then we have Table IV.

Table IV.

E_d(Mev)	0.87	1.43	1.96	2.49	3.02	3.52
K_1	$-3°33'$	$-7°40'$	$-10°30'$	$-14°30'$	$-18°16'$	$-22°30'$

And their final results are given in Table V as the upper figures and the lower bracketted ones are the final deviations from them. In the last row are the mean errors. The effect of D-wave is inside the range of experimental error.

TABLE V.

ϕ \ E_d	0.87	1.43	1.96	2.49	3.02	3.52
10°	1.45	1.49	1.76	1.93	2.30	2.61
	(0.3)	(0.11)	(0.14)	(0.00_8)	(0.07)	(0.00_5)
15	1.63	2.19	2.82	3.69	4.83	5.98
	(0.14)	(0.09)	(0.01)	(−0.09)	(−0.02)	(−0.13)
20	2.23	3.25	4.94	6.85	9.19	11.60
	(0.2)	(−0.01)	(0.09)	(0.03)	(−0.07)	(−0.66)
25	2.81	4.89	7.69	11.30	15.72	19.37
	(0.05)	(−0.03)	(−0.03)	(−0.07)	(0.45)	(−0.5)
30	3.81	6.68	11.20	16.12	21.89	27.90
	(0.01_7)	(0.07)	(0.05)	(−0.21)	(0.5)	(0.32)
35	3.61	8.58	13.73	20.13	—	53.74
	(−0.01)	(0.28)	(9.32)	(−0.21)	—	(0.83)
40	4.41	9.90	15.95	23.40	32.14	41.74
	(0.000)	(0.27)	(-0.00_4)	(−0.03)	(0.37)	(0.63)
45	—	9.94	16.50	24.44	—	—
	—	(0.00_5)	(0.000)	(−0.01)	—	—
50	—	—	—	23.93	31.98	39.37
	—	—	—	(0.49)	(0.22)	(0.000)
Mean |Error|	5.7%	2.6%	1.6%	0.91%	1.59%	1.92%

§ 6. Discussions

The smoothly varying nature of the phase shifts angles are seen in Fig. 2., and are consistent with earlier analysis $K_0 = -11° \sim -13°$, and $K_1 \approx 0$ given by KURONUMA, SUEOKA and TOYA[3] at 0.72 Mev. That both K_0 as well as K_1 are negative means that D-D interaction is repulsive nature in favour to the following facts (i) the ground state of He^4 is a mixture of ${}^1S_0 + {}^3P_0 + {}^5D_0$ [5] and (ii) the nuclear reaction $D+D \rightarrow He$ is very rare if occur, as was found only quite recently[6] with very small probability (its cross section is smaller than 10^{-31}cm^2 at 1 Mev).

It is the most remarkable when compared with $D(d,p)T^3$ and $D(d,n)He^3$ reactions, that D-wave contribution is very small in this case. And this extroadinary difference is attributed to the

large spin-orbit coupling favoring to the latter, which must be explained more basically in the future.

In conclusion, the author must express many thanks to Prof. K. UMEDA for his encouragements.

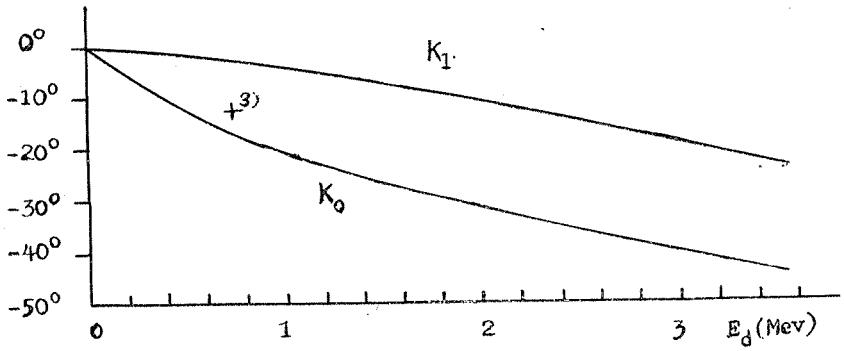

Fig. 2. *D-D*, K_0 & K_1

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