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D-D Scattering in Square Well Potential

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Phase shifts of D-D scattering are calculated with square well potentials and compared with the earlier analysis of J. M. BLAIR et al.s' experiment by this author and the velocity dependent forces seem to exist.

§ 1. Introduction

This author had performed the phase shifts analysis of D-D elastic scattering data[1] given by J. M. BLAIR et al.[2] and determined K_0 and K_1 accurately within a few % of mean deviations (max. 5.6% at 0.87 Mev and min. 0.3% at 2.49 Mev). Recently, a new more comprehensive table of Coulomb wave functions are given by I. BLOCH et al.[3], which this author utilized to calculate phase shifts assuming simple square well potentials and compared with the earlier analysis. Agreements between them are pretty good, but a slight deviations in the higher energy region may be considered as an energy dependent nuclear forces.

§ 2. Method of Calculations

The outline of the principle and the method of calculations are given in this author's another paper[4]. So, only the η and ρ values needed here are given in TABLE 1.

TABLE 1.

E (Mev)	η	ρ ($r_0=2.8\cdot10^{-13}$ cm)	ρ ($r_0=7.10^{-13}$ cm)
0.87	0.2395	0.4054	1.0134
1.43	0.1868	0.5197	1.2993
1.96	0.1596	0.6084	1.5210
2.49	0.1415	0.6858	1.7144
3.02	0.1295	0.7552	1.8881
3.52	0.1190	0.8154	2.0384

Unfortunately, however, I. Bloch et al.'s table do not cover the region of $\eta = 0.1585$, or above $E = 2$ Mev in this problem, so this author employed E_q. (7.5) of B. C. P.[5] for smaller radius of $r_0 = 2.8. \times 10^{-13}$ cm case.

$$\rho \frac{F_i'}{F_i} = y \frac{X + (2\, ln\, 2y + f)\, \Phi_0^*}{\Psi_0 + (2 ln\, 2y + f)\, y\Phi_0}\,, \tag{1}$$

where $\quad y = \rho\eta\,, \quad X = (\Psi_0^* + 2y\,\Phi_0)/y$

$$f = -2\, ln\, \eta + q_0/\eta + (C_0^2/\eta) \cot K_0\,.$$

B. C. P. gives the expansion to the 3~4-th order terms, and are sufficient for the smaller radius. For the larger radius of $r_0 = 7.10^{-13}$ cm case, however, the situation is not so good due to the lack of convergence from $y \sim 1$, so this author calculates directly with the following power series expansions to the order of 5~3-th order terms in the line of Yost, and Wheeler, and Breit.[6]

$$\begin{aligned}\Phi_0 = \sum_1^\infty A_j \rho^{j-1} &= 1 + \eta\rho + \left(\frac{\eta^2}{3} - \frac{1}{6}\right)\rho^2 + \left(\frac{\eta^3}{18} - \frac{\eta}{9}\right)\rho^4 \\ &+ \left(\frac{\eta^4}{5\cdot 4\cdot 3^2} - \frac{\eta^2}{36} - \frac{1}{5\cdot 3\cdot 2^2}\right)\rho^3 \\ &+ \left(\frac{\eta^5}{5^2\cdot 3^3\cdot 2^2} - \frac{\eta^3}{5\cdot 3^3\cdot 2} - \frac{17\eta}{10^2\cdot 3^3\cdot 2}\right)\rho^5 + \cdots\,,\end{aligned} \tag{2}$$

$$\Phi_0^* = \sum_1^\infty jA_j\, \rho^{j-1}\,, \tag{2'}$$

$$\begin{aligned}\Psi_0 = \sum_0^\infty a_j \rho^j &= 1 - \left(3\eta^2 + \frac{1}{2}\right)\rho^2 + \left(\frac{\eta}{9} - \frac{14}{9}\eta^3\right)\rho^3 \\ &+ \left(\frac{1}{24} + \frac{43}{108}\eta^2 - \frac{35}{108}\eta^4\right)\rho^4 \\ &+ \left(-\frac{2}{225}\eta + \frac{77}{540}\eta^3 - \frac{151}{2700}\eta^5\right)\rho^5 + \cdots\,,\end{aligned} \tag{3}$$

$$\Psi_0^* = \sum_1^\infty j a_j\, \rho^j\,, \tag{3'}$$

$$\Theta_0 = \Psi_0 + \rho\,(2\eta\, ln\, 2\rho + q_0)\,\Phi_0\,, \tag{4}$$

$$\begin{aligned} q_0 = 2\eta &\left[\gamma - \frac{1}{1+\eta^2} + (S_3 - 1)\eta^2 - (S_5 - 1)\,\eta^4 + (S_7 - 1)\,\eta^6 - \cdots\right], \\ &\gamma = 0.5772\cdots, \\ &S_3 = 1.2021\,, \quad S_5 = 1.0369\,, \quad S_7 = 1.0083_5\,,\end{aligned} \tag{5}$$

$$F_0 = C_0 \rho \Phi_0 , \qquad (6), \qquad C_0 = \left[\frac{2\pi\eta}{e^{2\pi\eta} - 1} \right]^{\frac{1}{2}}. \qquad (7)$$

Thus obtained values needed are as follows. Here, 1.96 (interp.) are given from interpolation of I. BLOCH et al.'s table for comparison and this table shows that the accuracy they call is certainly within a few %.

TABLE II. (*for* $r_0 = 7.10^{-13}$cm)

E (Mev)	1.96 (interp.)	1.96	2.49	3.05	3.52
Φ_0^* / Φ_0	0.4182	0.4005	0.1272	− 0.1769	− 0.4916
$\Phi_0\ \Theta_0$	0.2189	0.2007	0.0583	− 0.0438	− 0.1124
F_0	0.9922	0.9919	1.0278	1.0243	0.9953

Further, for reference's sake in the region not tabulated in their table, the Coulomb wave functions derived from (1) are also given in TABLE III. (though this author did not use these values, but the relation (1) directly).

TABLE III. (*for* $r_0 = 2.8.10^{-13}$cm)

E (Mev)	1.96 (interp.)	2.49	3.02	3.52
Φ_0^* / Φ_0	0.9730	0.9331	0.9247	0.8531
$\Phi_0 \Theta_0$	0.7831	0.7464	0.7030	0.6650
F_0	0.4800	0.5493	0.5891	0.6609

Similar expansions for $l=1$ case are as follows,

$$\Phi_1 = \sum_2^\infty A'_j \rho^{j-2} = 1 + \eta\rho + \left(\frac{\eta^2}{5} - \frac{1}{10}\right)\rho^2 + \left(\frac{\eta^3}{5\cdot 3^2} - \frac{\eta}{15}\right)\rho^3$$
$$+ \left(\frac{\eta^4}{7\cdot 5\cdot 3^2\cdot 2} - \frac{\eta^2}{7\cdot 4\cdot 3} + \frac{1}{280}\right)\rho^4$$
$$- \left(\frac{\eta^5}{8\cdot 7\cdot 5^2\cdot 3^2} - \frac{29\eta^3}{8\cdot 7\cdot 5^2\cdot 3^2\cdot 2} - \frac{31\eta}{8\cdot 7\cdot 5^2\cdot 3\cdot 2^2}\right)\rho^5 + \cdots, \qquad (8)$$

$$\Phi_1^* = \sum_2^\infty j A'_j \rho^{j-2} . \qquad (8')$$

But, this author tried with values extrapolated graphically from I. BLOCH's table for K_1 case for convenience's sake.

§ 3. Results and Discussions

Comparison between the observed and the theoretical phase shifts are shown in Fig. 1 (K_0) and Fig. 2 (K_1). For K_0, the next sets of values are probable,

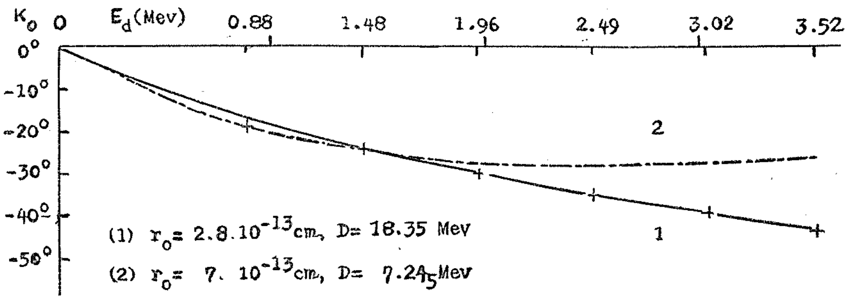

Fig. 1 *D-D* Scattering K_0

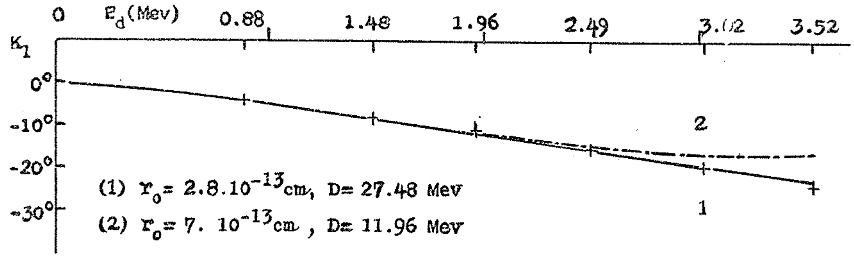

Fig. 2 *D-D* Scattering, K_1

$r_0 = 2.8.10^{-13}$ cm with $D = 18.35$ Mev

$r_0 = 7.10^{-13}$ cm with $D = 7.24_5$ Mev ,

and for K_1 ,

$r_0 = 2.8.10^{-13}$ cm with $D = 27.48$ Mev

$r_0 = 7.10^{-13}$ cm with $D = 11.96$ Mev .

Agreements are pretty well in the smaller force range, in which lower energy 0.87 Mev point deviates slightly, but this is not serious, because (i) in this energy point the statistical error is maximum (5.6%), (ii) it becomes nearer to the former analysis by Sueoka, Kuronuma, and Toya (1940)[8] of Heydenburg and Roberts's *D-D* scattering data at 0.72 Mev, i. e. $K_0 : -11° \sim 13°$.

On the other hand, in $r_0 = 7.10^{-13}$ cm case, the behavior in the higher energy range above 2 Mev is not good due to the flattness of the theoretial. These are seen in both K_0 and K_1. So, if we choosethe suitable changing depth energy, they come to as given in TABLE 4, for $l = 0$ case.

TABLE VI. (*energy dependent depths*)

E (Mev)	1.48	19.6	2.49	3.02	3.52
D (Mev)	7.24_5	7.11	6.96	6.82	6.65

Namely, we may say that the observed phase shifts are velocity dependent and this effect can be avoided by reducing nuclear force range.

Similar situations are reported in p-p scattering discussed by POWELL et al.[9] as the effect of the so-called Paiss' "f-interaction (C-meson field)", by which the incident waves are already distorted as done by Coulomb field. But, in this D-D scattering case, the situation is probably not so at all, however, other interaction e.g. spin-orbit coupling as needed in D-D reaction theory[10] may be effective in this problem, that must be investigated in future.

References

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