D-D Scattering in Square Well Potential

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Phase shifts of D-D scattering are calculated with square well potentials and compared with the earlier analysis of J. M. Blair et al. and the velocity dependent forces seem to exist.

§ 1. Introduction

This author had performed the phase shifts analysis of D-D elastic scattering data given by J. M. Blair et al. and determined $K_o$ and $K_1$ accurately within a few % of mean deviations (max. 5.6% at 0.87 Mev and min. 0.3% at 2.49 Mev). Recently, a new more comprehensive table of Coulomb wave functions are given by I. Bloch et al., which this author utilized to calculate phase shifts assuming simple square well potentials and compared with the earlier analysis. Agreements between them are pretty good, but a slight deviations in the higher energy region may be considered as an energy dependent nuclear forces.

§ 2. Method of Calculations

The outline of the principle and the method of calculations are given in this author's another paper. So, only the $\eta$ and $\rho$ values needed here are given in Table 1.

<table>
<thead>
<tr>
<th>$E$ (Mev)</th>
<th>$\eta$</th>
<th>$\rho$ ($r_0=2.8 \times 10^{-15}$ cm)</th>
<th>$\rho$ ($r_0=7.10^{-15}$ cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.87</td>
<td>0.2595</td>
<td>0.4054</td>
<td>1.0134</td>
</tr>
<tr>
<td>1.43</td>
<td>0.1868</td>
<td>0.5197</td>
<td>1.2993</td>
</tr>
<tr>
<td>1.96</td>
<td>0.1506</td>
<td>0.6084</td>
<td>1.5210</td>
</tr>
<tr>
<td>2.49</td>
<td>0.1415</td>
<td>0.6858</td>
<td>1.7144</td>
</tr>
<tr>
<td>3.02</td>
<td>0.1295</td>
<td>0.7552</td>
<td>1.8881</td>
</tr>
<tr>
<td>3.52</td>
<td>0.1190</td>
<td>0.8154</td>
<td>2.0384</td>
</tr>
</tbody>
</table>
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Unfortunately, however, I. Bloch et al.'s table do not cover the region of \( \gamma = 0.1585 \), or above \( E = 2 \) Mev in this problem, so this author employed \( E_q(7.5) \) of B.C.P.\(^5\) for smaller radius of \( r_0 = 2.8 \times 10^{-15} \) cm case.

\[
\rho \frac{F'_e}{F'_i} = y \frac{X + (2 \ln 2y + f)}{y} \frac{\Phi'_o}{\Phi'_o + (2 \ln 2y + f) y \Phi'_o},
\]

where \( y = \rho \gamma \), \( X = (\Phi'_o + 2y \Phi'_o) / y \), \( f = -2 \ln \gamma + q \gamma + (C_0 / \gamma) \cot \theta \).

B.C.P. gives the expansion to the 3~4--th order terms, and are sufficient for the smaller radius. For the larger radius of \( r_0 = 7 \times 10^{-15} \) cm case, however, the situation is not so good due to the lack of convergence from \( \gamma \sim 1 \), so this author calculates directly with the following power series expansions to the order of 5~3-th order terms in the line of Yost, and Wheeler, and Breit.\(^6\)

\[
\psi_o = \sum_{j=1}^{\infty} A_j \rho^{j-1} = 1 + \gamma \rho + \left( \frac{\gamma^2}{3} - \frac{1}{6} \right) \rho^2 + \left( \frac{\gamma^3}{18} - \frac{\gamma}{9} \right) \rho^3
\]
\[
+ \left( \frac{\gamma^4}{5 \cdot 3^2} - \frac{\gamma^2}{36} - \frac{1}{5 \cdot 3 \cdot 2^2} \right) \rho^4
\]
\[
+ \left( \frac{\gamma^5}{5 \cdot 3^2 \cdot 2} - \frac{\gamma^3}{5 \cdot 3 \cdot 2} - \frac{17 \gamma}{10^2 \cdot 3^4 \cdot 2} \right) \rho^5 + \ldots,
\]

\[
\psi'_o = \sum_{j=1}^{\infty} jA_j \rho^{j-1},
\]

\[
\Phi_o = \sum_{j=0}^{\infty} a_j \rho^j = 1 - \left( 3\gamma^2 + \frac{1}{2} \right) \rho^2 + \left( \frac{\gamma}{9} - \frac{14}{9} \gamma \right) \rho^3
\]
\[
+ \left( \frac{1}{24} + \frac{43}{108} \gamma^2 - \frac{35}{108} \gamma^3 \right) \rho^4
\]
\[
+ \left( - \frac{2}{225} \gamma + \frac{77}{540} \gamma^3 - \frac{151}{2700} \gamma^5 \right) \rho^5 + \ldots,
\]

\[
\Phi'_o = \sum_{j=1}^{\infty} jA_j \rho^j,
\]

\[
\theta_o = \Phi_o + \rho (2 \ln 2 \rho + q_o) \psi_o,
\]

\[
q_o = 2 \gamma \left[ \gamma - \frac{1}{1 + \gamma^2} + (S_3 - 1) \gamma^2 - (S_5 - 1) \gamma^4 + (S_7 - 1) \gamma^6 - \ldots \right],
\]

\[
\gamma = 0.5772 \ldots,
\]

\[
S_3 = 1.2021, \quad S_5 = 1.0369, \quad S_7 = 1.0083, \]

\[\text{Eq. (1)}\]

\[\text{Eq. (2')}\]

\[\text{Eq. (3')}\]

\[\text{Eq. (4)}\]

\[\text{Eq. (5)}\]
Thus obtained values needed are as follows. Here, 1.93 (interp.) are given from interpolation of I. Bloch et al.'s table for comparison and this table shows that the accuracy they call is certainly within a few %.

Table II. (for \( r_0 = 7.1 \times 10^{-13} \text{cm} \))

<table>
<thead>
<tr>
<th>( E ) (Mev)</th>
<th>1.96 (interp.)</th>
<th>1.95</th>
<th>2.49</th>
<th>3.05</th>
<th>3.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_0^2/\Phi_0 )</td>
<td>0.4182</td>
<td>0.4005</td>
<td>0.1272</td>
<td>-0.1769</td>
<td>-0.4916</td>
</tr>
<tr>
<td>( \Phi_0^2 )</td>
<td>0.2189</td>
<td>0.2007</td>
<td>0.0583</td>
<td>-0.0433</td>
<td>-0.1124</td>
</tr>
<tr>
<td>( F_0 )</td>
<td>0.9922</td>
<td>0.9919</td>
<td>1.0278</td>
<td>1.0243</td>
<td>0.9953</td>
</tr>
</tbody>
</table>

Further, for reference's sake in the region not tabulated in their table, the Coulomb wave functions derived from (1) are also given in Table III. (though this author did not use these values, but the relation (1) directly).

Table III. (for \( r_0 = 2.8 \times 10^{-13} \text{cm} \))

<table>
<thead>
<tr>
<th>( E ) (Mev)</th>
<th>1.96 (interp.)</th>
<th>2.49</th>
<th>3.02</th>
<th>3.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_0^2/\Phi_0 )</td>
<td>0.9730</td>
<td>0.9331</td>
<td>0.9247</td>
<td>0.8531</td>
</tr>
<tr>
<td>( \Phi_0^2 )</td>
<td>0.7831</td>
<td>0.7464</td>
<td>0.7030</td>
<td>0.6650</td>
</tr>
<tr>
<td>( F_0 )</td>
<td>0.4800</td>
<td>0.5493</td>
<td>0.5891</td>
<td>0.6609</td>
</tr>
</tbody>
</table>

Similar expansions for \( l = 1 \) case are as follows,

\[
\phi_1 = \sum_j A_j \rho^{j-2} = 1 + \gamma \rho + \left( \frac{\gamma^2}{5} - \frac{1}{10} \right) \rho^2 + \left( \frac{\gamma^3}{5 \cdot 3} - \frac{\gamma}{15} \right) \rho^3
\]

\[
+ \left( \frac{\gamma^4}{7 \cdot 5 \cdot 3^2} - \frac{\gamma^2}{7 \cdot 4 \cdot 3} + \frac{1}{280} \right) \rho^4
\]

\[
- \left( \frac{\gamma^5}{8 \cdot 7 \cdot 5^2 \cdot 3} - \frac{29 \gamma^3}{8 \cdot 7 \cdot 5 \cdot 3^3 \cdot 2} - \frac{31 \gamma}{8 \cdot 7 \cdot 5^3 \cdot 3^2} \right) \rho^5 + \cdots, \quad (8)
\]

\[
\phi_1^s = \sum_j j A_j \rho^{j-2}. \quad (8')
\]

But, this author tried with values extrapolated graphically from I. Bloch's table for \( K \) case for convenience's sake.
§ 3. Results and Discussions

Comparison between the observed and the theoretical phase shifts are shown in Fig. 1 ($K_0$) and Fig. 2 ($K_1$). For $K_0$, the next sets of values are probable,

<table>
<thead>
<tr>
<th>$E_q$(MeV)</th>
<th>0.88</th>
<th>1.48</th>
<th>1.96</th>
<th>2.49</th>
<th>3.02</th>
<th>3.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1) $r_0 = 2.8 \times 10^{-13}$ cm, $D = 18.35$ Mev
(2) $r_0 = 7.1 \times 10^{-13}$ cm, $D = 7.245$ Mev

![Fig. 1 D-D Scattering $K_0$](image1)

For $K_1$,

<table>
<thead>
<tr>
<th>$E_q$(MeV)</th>
<th>0.88</th>
<th>1.48</th>
<th>1.96</th>
<th>2.49</th>
<th>3.02</th>
<th>3.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1) $r_0 = 2.8 \times 10^{-13}$ cm, $D = 27.48$ Mev
(2) $r_0 = 7.1 \times 10^{-13}$ cm, $D = 11.96$ Mev

![Fig. 2 D-D Scattering, $K_1$](image2)

$r_0 = 2.8 \times 10^{-13}$ cm with $D = 18.35$ Mev
$r_0 = 7.1 \times 10^{-13}$ cm with $D = 7.245$ Mev,

and for $K_1$,

$r_0 = 2.8 \times 10^{-13}$ cm with $D = 27.48$ Mev
$r_0 = 7.1 \times 10^{-13}$ cm with $D = 11.96$ Mev.

Agreements are pretty well in the smaller force range, in which lower energy 0.87 Mev point deviates slightly, but this is not serious, because (i) in this energy point the statistical error is maximum (5.6%), (ii) it becomes nearer to the former analysis by Sueoka, Kurosumi, and Toya (1940) of Heydenburg and Roberts's $D-D$ scattering data at 0.72 Mev, i.e. $K_0: -11^\circ -13^\circ$.

On the other hand, in $r_0 = 7.1 \times 10^{-13}$ cm case, the behavior in the higher energy range above 2 Mev is not good due to the flatness
of the theoretical. These are seen in both \( K_0 \) and \( K_1 \). So, if we choose the suitable changing depth energy, they come to as given in Table 4, for \( l = 0 \) case.

| \( E \) (Mev) | 1.48 | 19.6 | 2.49 | 2.92 | 3.52 |
| \( D \) (Mev) | 7.24 | 7.11 | 6.96 | 6.82 | 6.65 |

Namely, we may say that the observed phase shifts are velocity dependent and this effect can be avoided by reducing nuclear force range.

Similar situations are reported in \( p-p \) scattering discussed by Powell et al.\(^9\) as the effect of the so-called Pais' "\( f \)-interaction (C-meson field)" by which the incident waves are already distorted as done by Coulomb field. But, in this \( D-D \) scattering case, the situation is probably not so at all, however, other interaction e.g. spin-orbit coupling as needed in \( D-D \) reaction theory\(^10\) may be effective in this problem, that must be investigated in future.

References

1) Y. Nakano, "Phase shifts analysis in \( D-D \) scattering up to 3.5 Mev", Journal of the Faculty of Science, Hokkaido University, Japan, Ser. II. Vol IV No. 3 1952, 173.
3) I. Bloch et al., Rev. Mod. Phys. 23 (1951), 147.