[Journal of the Faculty of Science, Hokkaidô University, Japan, Ser. II, Vol. IV, No. 4 1953]

The Threshold Voltage of the G-M Counter

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(Received June 18, 1953)

In the G-M counter operation, the threshold voltage is important, but only a few empirical formula have been applied for the determination of the threshold voltage. Here I intend to calculate it in a more theoretical way and to compere that value with the experimental value. Good agreements are obtained at the relation with the counter geometries and pressures. But the relation with gas property cannot be explained satisfactorily, and this reason are discussed.

§ 1. Introduction

Theoretical and experimental investigations on the G-M counter behavior have already been made by many authors.[1)-8)] But the relations between the threshold voltage and the counter variables (such as radii of the cathode wall and wire, and various gas mixtures) has been studied empirically by a few authors.[4)7)8)]

Here an attempt will be made to calculate those relations theoretically. The word "threshold voltage" is used simply to identify the voltage which is necessary across a G-M counter so that it will "count" whenever a radiation goes across in the counter.

In calculating the threshold voltage, the following five conditions will be assumed for simplification of the discussion.

A). The threshold voltage is determined by the first avalanche only, i.e., it will be the voltage in which the number of electrons in the first avalanche reaches to some critical amount.

B). The shielding by positive ion field is negligible with respect to the formation of the first avalanche.

C). Initial avalanche will be produced by the electron collision with the counter gases only, and has no photo electron by photoelectric emission at cathode.

D). The second electron emission by positive ion at the cathode is neglected in initial avalanche.

E). Recombination and electron attachement to neutral molecule can be neglected.

On the spread of discharge, of course, the assumptions (B), (C), and (D) are not correct, the space charge ion field and photon emssion play very important roles[9]. But in the initial avalanche the space ion field is weak, and the first avalanche is made in advance of the secondary electron formation by photon, so they are neglected Recombination and electron attachment will also be unimportant under the conditions of high field and low pressure as in counters.

§ 2. The Formation of First Avalanche

The secondary electrons which formed in the counter gases are drawn toward the wire with the electric field and accelerated, and these electrons collide with gas atoms or molecules. If the electron energy is larger than the ionization energy of the collided gas atom or molecule, then that atom or molecule is ionized and a new electron becomes free, thus the electrons increase by degrees until they reach the wire, thus the electrons form an avalanche.

From a well-known law, the number of electrons of the first avalanche is

$$N(1) = \exp \int_a^{r_c} \alpha \, dr \,, \tag{1}$$

where α is the number of produced electrons when an electron goes unit length in the direction of the electric field, this is the so-called TOWNSEND's 1st Coefficient, a is the radius of counter wire, r_c is the distance from the counter center to the position where the first electron was produced in the counter.

Now assume that in an electron path of length λ in field direction, the electron gains an energy $X\lambda$ at the end of the free path λ in the field X. * At the end of the impact, the electron loses all its energy, that is, all energy is transfered to the collided atom or molecule. Thus the electron which has the energy $X\lambda \geq E_i$ can ionize, where E_i is the ionization energy of its atom or

* Here I have omitted the electronic charge so that all energies are measured in the same units at the counter voltage.

molecule. The probability that an electron would ionize depends upon the probability that an electron has a free path λ

$$\lambda \geqq \lambda_i = E_i/X. \tag{2}$$

Put the mean free path $\bar{\lambda}$, then the collision probability at x becomes $\frac{1}{\bar{\lambda}} e^{-x/\bar{\lambda}}$. With use of relation (2), α will be written as

$$\alpha(r) = \frac{1}{\bar{\lambda}(r)} e^{-\frac{E_i}{X\bar{\lambda}(r)}}. \tag{3}$$

In general $\bar{\lambda}(r)$ is the function of the electron's energy, in which the energy is determined by the field X only, where electric field X is

$$X = \frac{V}{r \log b/a}, \tag{4}$$

where V is applied voltage, a and b are radii of counter wire and wall, respectively, so $\bar{\lambda}(r)$ can be considered a function of position r.

According to SMITH,[10] in the energy region such as that considered in our counter, the ionization cross section increases linearly with its energy and we can write for the cross section $\sigma(\epsilon) = k\epsilon(r)$,[11] where k is a constant depending on the nature of the gas, and $\epsilon(r)$ is its energy. Then the mean free path is

$$\bar{\lambda}(r) = \frac{1}{N\sigma(\epsilon)} = \frac{1}{kN\epsilon(r)}, \tag{5}$$

where N is the number of atoms per unit volume. Use the energy relation $\epsilon(r) = X\bar{\lambda}(r)$ and (5), can be written as

$$\bar{\lambda}(r) = \sqrt{\frac{1}{kNX}}. \tag{6}$$

With (4), (5) and (6), the integral part of equation (1) is

$$\int_a^{r_c} \alpha dr = \int_a^{r_c} \frac{1}{\bar{\lambda}(r)} e^{-\frac{E_i}{X\bar{\lambda}(r)}} dr$$

$$= \int_a^{r_c} \left(\frac{kNV}{r \log b/a}\right)^{\frac{1}{2}} \exp\left\{-E_i\left(\frac{kNr \log b/a}{V}\right)^{\frac{1}{2}}\right\} dr,$$

which becomes after integration

$$\int_a^{r_c} \alpha dr = -\frac{2V}{E_i \log b/a} \Bigg| \exp\left\{-E_i\left(\frac{kNr \log b/a}{V}\right)\right\}\Bigg|_a^{r_c}. \tag{7}$$

In general r_c will be much larger than a, i.e., $r_c \gg a$. Substituting this limit into (7), the exponential at r_c becomes very small. It can be neglected in comparison with the term a, so equation (1) can be written with (7) as

$$\log N(1) = \frac{2V}{E_i \log b/a} \exp\left\{-E_i\left(\frac{kNa \log b/a}{V}\right)^{\frac{1}{2}}\right\}. \qquad (8)$$

This equation gives the threshold voltage. Now I attempt the numerical calculation of this equation.

In our counter, the filling gases are 10 mm Hg Alcohol and 90 mm Hg Argon, counter condition is $2b = 45$ mm $2a = 0.1$ mm, $V = 1160$ V, and gas condition is $E_i = 15$ eV, $k = 1.8 \times 10^{-17}$cm^2/V.[11] Then from equation (8). One obtains

$$\log N(1) = 13.6 .$$

Therefore

$$N(1) \simeq 10^6 . \qquad (9)$$

This number of electrons is about in agreement with the expectation.[4]

§ 3. Threshold Voltage vs. Counter Dimensions

The relation of threshold voltage with counter dimensions is obtained from equation (8) as

$$\frac{V}{\log b/a} \exp\left\{-K\left(\frac{a \log b/a}{V}\right)^{\frac{1}{2}}\right\} = \text{const}, \qquad (10)$$

where K is a constant.

Fig. 1. shows several curves of this relation under different conditions, and also shows the experimental value. ○ is our counter data, △ is MINAGAWA's data,[12] (his counters have two different wire diameters, one 0.1 mm and the other 0.3 mm, symbolized there by △ and ▲ respectively), □ is the experimental data of WEISS,[7] whose counter has 0.003 inch tungsten wire and filling gases are 15 mm Hg Methane and 135 mm Hg Argon.

Because the experimental data are chosen under several different conditions, a good agreement was not obtained, but the tendency of the curve is in agreement with the experimental values.

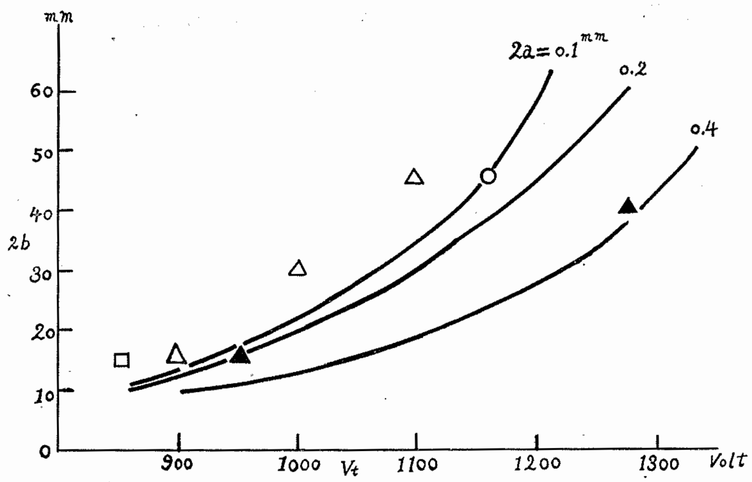

Fig. 1.

§ 4. Threshold Voltage vs. Total Pressure

Rewriteing equation (8), one obtain

$$V \exp\left\{-A\,(p/V)^{\frac{1}{2}}\right\} = \text{const}\,. \tag{11}$$

This equation gives the relation of the threshold voltage to the pressure. Several curves are calculated and plotted in Fig. 2; the

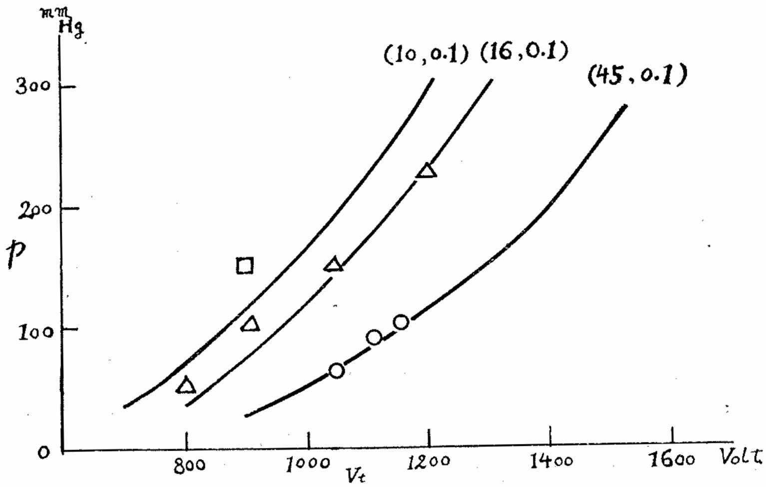

Fig. 2.

experimental values are also shown. The symbols are the same as in Fig. 1.

The numbers at the end of curves show the diameters of the counter wall and wire in mm, respectively. The gas filling condition, in this case, was restricted to 9 : 1, Argon Ethyl-alcohol mixture. Other concentrations and mixtures will be discussed in the following paragraph.

§ 5. Threshold Voltage vs. Gas Property

In equation (8) the quantities which are considered to be related to the gas properties are E_i and k. It is a very important point how to determine the effective ionization potential E_i and effective cross section constant σ in gas mixture. Herein I used the mixing value $K_{AB}=\dfrac{K_A\cdot K_B}{K_A n+K_B(1-n)}$ as in mobility of mixed gases,[13] where n is the concentration of K_B.

The values of k are considered to be related to the molecular size, but a functional form of k is not clear. From the experiment, when the quench gas concentration is increased from 10% to 15% the threshold voltage increases by about 100V in our counter. In order to explain this, it is necessary that the value of k increases by about 25%, at the same value of E_i. This increase of k can not be considered from the change of the cross section.

The relation of the threshold voltage to the gas concentration cannot be explained satisfactorily from equation (8).

§ 6. Discussions

The above calculation of threshold voltage seems to be permissible with respect to counter dimensions and total pressures, but the relation with the gas nature can scarcely be explained from this calculation.

This unagreement is, firstly, due to over simplified assumptions. The mechanism which relates to the spread of discharge may have a relation to the determination of threshold voltage.

When the molecule is changed, the mobility of ion becomes different, so shielding effect will be changed. According to KORFF and PRESENT,[9] the shielding effect is almost due to polyatomic heavy quenching molecule ions.

When heavy molecules increase, the shielding effect becomes larger. So threshold voltage will be higher. Then assumption (A) also must be reconsidered. The probabirity of the formation of the second avalanche will be different with gas property. The number of electrons in the first avalanche should be changed when the gas nature is changed; in general the left side of equation (8) is not constant, it is a function of the gas nature. So if the gas concentration is changed, the critical amount of electrons in the first avalanche shall be changed. But that cannot be calculated accurately.

Generally, the above described calculation is correct to determine the several properties of G-M Counters. The present writer is considering the more exact calculation of the discharge of counter.

It is a pleasure to acknowledge indebtedness to Prof. J. FURUICHI for his kind encouragements.

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