On the Time Lag of the Piezoelectric Effect of Rochelle Salt

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In this work, the writers have verified that the phase difference between the stress and the produced piezoelectric potential of Rochelle Salt has anomaly at Curie Temperature by means of the phase difference method and have shown that it is qualitatively explained by MUeller's theory if the inverse dielectric susceptibility for X crystallographic direction is assumed to be a complex value, $\chi_1 + j\chi'$. 

§ 1. Introduction

Recently H. B. HUNTINGTON\textsuperscript{1}) and W. J. PRICE\textsuperscript{2}) have studied the anomalous temperature dependence of elastic properties of Rochelle salt by means of a pulsed ultrasonic technique and have shown that the anomalies are explained by MUeller's theory in the same way as for other properties of Rochelle salt if the inverse dielectric susceptibility for the X crystallographic direction is assumed to be a complex value, $\chi_1 + j\chi'$. If the inverse dielectric susceptibility is that complex value, the anomalous temperature dependence of the phase difference between the stress and the produced piezoelectric potential may also be anticipated from MUeller's theory. In this work, the writers have verified that idea.

§ 2. Experimental Method

A block diagram of the experimental apparatus that was used is shown in Fig. 1. Vibrational stress was given to specimen with the fork whose characteristic frequency was 100 cycles per second and which was driven by 50 c.p.s. alternating current. The specimens were thin plates of 45° X-cut Rochelle salt crystal whose width in a-axis was about 1 mm. The a-faces were fold and the potential difference between the foils was led to an oscilloscope through an
amplifier. The time axis of the oscilloscope was connected to the 50 c.p.s. alternating current which drove the fork. The figure produced on the oscilloscope is shown in Fig. 2. If \( a \) is the amplitude of the time axis and \( x \) is the distance between the center of the curve and the point where the curve intersects with the grand line, the angle of the phase difference \( \alpha \) is calculated as follows:

\[
\tan \alpha = -\frac{2\left(\frac{x}{a}\right)\sqrt{1-\left(\frac{x}{a}\right)^2}}{1-2\left(\frac{x}{a}\right)^2}.
\]

(1)

As the angle obtained from Eq. (1) is the phase difference between the produced piezoelectric potential and alternating current, the value must be corrected as in § 4. below in order to obtain the phase difference between the produced piezoelectric potential and the stress. The temperature of the crystal specimens was controlled to 0.5°C and the alternating current which drove the fork was kept at constant value in order to make constant the amplitude of the stress.

§ 3. Results of Experiments

This experimental method was applied to two specimens. The
results in one case are graphed in Fig. 3. The curve has two anomalies, at 21° and at 30°C. Below we shall show that these anomalies are related to the anomalous elastic phenomena of Rochelle
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salt at Curie point. Fig. 4 shows the temperature dependence of the vertical amplitude of the oscilloscope, which is proportional to the piezoelectric constant $d_{14}$ at constant stress. This figure shows that $d_{14}$ has its maximum value at Curie temperature, as has already been reported by other authors.

§ 4. Discussion

According to Mühlner's theory,\textsuperscript{3)} electric, polarization $P_x$, electric field $E_x$, and stress $Y_z$ are related, as shown in the following equation

$$P_x = \varepsilon_1 E_x + d_{14} Y_z,$$

where $\varepsilon_1$ is dielectric susceptibility and $d_{14}$ is inverse piezoelectric constant. When $F$ is a force applied at 45° angle to b-axis, and $V$ is a potential difference between the foils on the faces of the specimens

$$Y_z = \frac{1}{2ac} F,$$  \hfill (3)

$$E_x = \frac{1}{a} V,$$  \hfill (4)

$$V = \frac{bc}{C_f} P_x,$$  \hfill (5)

where $a$ is the width of the specimen in the direction of a-axis, while $b$, and $c$ are the lengths of the other two sides. $C_f$ is electric capacity between the foils and

$$C_f = \frac{(1+4\pi\varepsilon_1) bc}{4\pi a}.$$  \hfill (6)

Neglecting $P_x$ and $E_x$ from Eq. (2), (3), (4), (5) and (6), we obtain

$$V = \frac{2\pi}{c} d_{14} F,$$  \hfill (7)

when $\chi_1$ is inverse dielectric susceptibility, $c_{ii}$ is the elastic constant and $f_{ii}$ is the piezoelectric constant, $d_{14}$ is given as follows:

$$d_{14} = \frac{f_{14}}{c_{ii}\chi_1 - f_{14}^2}.$$  \hfill (8)

If $\chi_1$ is assumed to be a complex number $\chi_1^* = \chi_1 + j\chi''$, the following
relation is obtained from Eq. (8)

\[ d_{ii} = \frac{f_{ii}}{\sqrt{(c_{ii} \gamma_{ii} - f_{ii})^2 + (c_{ii} \xi')^2}} e^{-j\alpha} \quad (9) \]

\[ \tan \alpha = \frac{c_{ii} \xi'}{c_{ii} \gamma_{ii} - f_{ii}} \quad (10) \]

Mueller has pointed out that \( c_{ii} \) and \( f_{ii} \) are nearly independent of temperature but \( \gamma_{ii} \) varies with temperature in such a manner as to cause \( c_{ii} \gamma_{ii} - f_{ii} \) to vanish at Curie point,\(^{4}\) and above Curie point \( T_c \)

\[ c_{ii} \gamma_{ii} - f_{ii} = \frac{c_{ii}}{178} (T - T_c) \quad (11) \]

below Curie point \( T_c \)

\[ c_{ii} \gamma_{ii} - f_{ii} = \frac{2c_{ii}}{178} (T_c - T) \quad (12) \]

From Eq. (7), (9), (10), (11) and (12), potential difference \( V \) and the angle of the phase difference \( \alpha \) are maximum at Curie temperature \( T_c \). Fig. 4 shows that the potential difference \( V \) is largest at \( T_c \).

Concerning the phase difference, one must take account of the phase difference which was produced by the apparatus. When the phase difference of apparatus was \( \beta \), the observed phase difference became \((\beta - \alpha)\), and

\[ \tan (\beta - \alpha) = \frac{\tan \beta - \tan \alpha}{1 + \tan \beta \tan \alpha} \quad (13) \]

In the present case, \( \beta \) was \(-70^\circ\). Then substituting (10), (11) and (12), in (13), we obtain

above \( T_c \)

\[ \tan (\beta - \alpha) = \frac{-2.75(T - T_c) - 178 \chi'}{(T - T_c) - 490 \chi'} \quad (14) \]

below \( T_c \)

\[ \tan (\beta - \alpha) = \frac{-2.75(T_c - T) - 89 \chi'}{(T_c - T) - 245 \chi'} \quad (15) \]

The theoretical curve calculated from Eq. (14) and (15) is plotted
with dotted line in Fig. 4, inasmuch as the figures on the os­cil­loscope are the same at the angles $(\beta-a), (a-\beta), (\pi-\beta+a)$ and $(\pi+\beta-a)$. $\mathcal{X}'$ is determined under the condition (14) is maximum at $21^\circ$, and (15) at $30^\circ$ and becomes 0.0125.

§ 5. Conclusion

In this work, the writers have verified the temperature de­pendence of the phase difference between the stress and produced piezoelectric potential and shown that it is qualitatively explained by MUELLER's theory. In this case $\mathcal{X}'$ calculated, using their results, is a little greater than the value given by MASON by means of measuring the dielectric-loss at the frequencies of several kilo­cycles. This is perhaps due to the large amplitude of the stress.

Using the phase difference method, the value of $\mathcal{X}'$ can be obtained at low frequency, but other methods are difficult to apply to the measurement of frequency below several k. c.

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References

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4) H. MUELLER, Phys. Rev. 57 (1940), 829.