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Visco-elastic Property of the Granular Substance

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In this study, for the layer of the granular substance, the magnitude of depression at constant load was measured. The result showed that there exists the formal resemblance between the visco-elastic properties of the granular and amorphous substances. The data were analysed in an analogous manner to that used in the case of amorphous substance.

§ 1. Introduction

Experimental studies on the deformation of sand mass and soil have been carried out by many authors.[1),2)] In MIYABE's report,[2)] the mode of the deformation of soil has been explained by assuming that the visco-elastic properties of sand are expressed by MAXWELL's fundamental formula. However, the mechanism of the deformation,

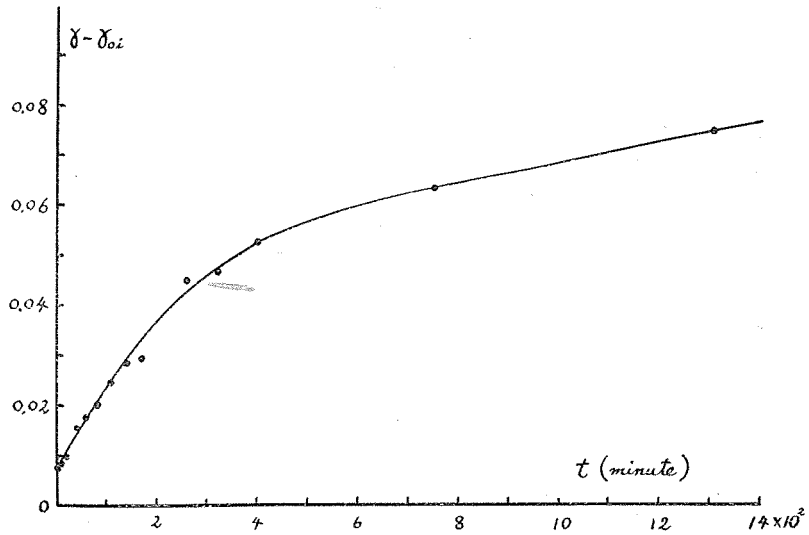

Fig. 1. Strain at constant load as function of time (the size of particles is smaller than 20 mesh).

especially the behavior of the individual particles of the soil or sand has been scarecely treated.

According to the present author's experimental result for the granular substance, the strain (the magnitude of the depression per unit thickness) at constant load is expressed by the following formula

$$\gamma = \gamma_{0i} + \gamma_0[1 + s(1 - e^{-qt}) + rt], \qquad (1)$$

where s, q and r are constants which can be determined from the $(\gamma\text{–}t)$ curve obtained experimentally, and γ_0 and γ_{0i} are the reversible and irreversible parts of the initial strain at the instant of the loading. Such a general feature of $(\gamma\text{–}t)$ curve as shown in Fig. 1 is the characteristic one observable in the case ofamorphous or granular substances. Moreover an elastic hysterisis can be observed also on both substances undercertain conditions. The formal resemblance perceptible between these two sorts of substances will give us an insight into the mechanical behavior of the granular substance.

In this paper, the experimental data on granular substance were analysed in an analogous manner to that used in the case of amorphous substance and from the result the mechanism of the mechanical behavior of the granular substance was discussed.

§ 2. Method of Experiment

The specimen contained in a vessel was depressed vertically by weight over a small area (diameter = 10 cm) on the surface (diameter = 25 cm), the thickness of the layer of the specimen being 5 cm. The magnitude of the depression was measured by means of an optical lever. The granular substance used in the present experiment was finely crushed quartz classified by sieving method.

§ 3. Theoretical Treatment

When the layer of the granular substance is acted upon by an external force, there takes place a deformation which causes a certain degree of strain. The strain reaches a certain value abruptly through elastic process, but, if the strain is kept con-

stant, the stress in the granular substance will gradually decrease. The stress energy will be expended by the motion of each individual particle which moves towards a more stable position through rotation or translation (sliding). From the formal correspondency which exists between the characters of (γ–t) curves obtained for the granular and the amorphous substances, the BENNEWITZ-RÖTGER[3] theory which concerns the visco-elastic properties of an amorphous substance may be also applicable as the special case for explaining the mechanical behavior of granular substances. During the experiment the load is kept constant. When the load is placed on the granular substance, the stress will increase abruptly to a certain value S_0 which corresponds to the elastic change. Hence

$$S_0 = E\gamma_0 \,, \tag{2}$$

where E is an elastic constant. But the stress energy will be expended in the granular substance by the motion (rotational and translational) of the particles.

Now let it be assumed that the phenomena can be treated separately referring to these two motions. Hence

$$S = S_1 + S_2 = const. \tag{3}$$

Denote the initial stresses at $t = t_0$ by S_{01} and S_{02}, then it follows

$$S_{01} = E_1\gamma_0 \,, \tag{4}$$

$$S_{02} = E_2\gamma_0 \,, \tag{5}$$

$$S_0 = S_{01} + S_{02} = (E_1 + E_2)\gamma_0 \,, \tag{6}$$

where E_1 and E_2 are two distinct elastic constants.

Again let it be assumed that for $t > t_0$, both stresses S_1 and S_2 satisfy MAXWELL's fundamental equations:

$$\frac{dS_1}{dt} = E_1\frac{d\gamma}{dt} + \beta_1 S_1 \,, \tag{7}$$

$$\frac{dS_2}{dt} = E_2\frac{d\gamma}{dt} + \beta_2 S_2 \,, \tag{8}$$

where β_1 and β_2 are the inverses of the relaxation times, when γ is constant.

Integrating equations (7) and (8) by using condition (3), one ob-

tains the following result:

$$\gamma = \frac{S_0}{E}\left[1+s^*(1-e^{-q^*t})+r^*t\right], \tag{9}$$

where

$$r^* = \frac{\beta_1\beta_2}{\beta_1\varepsilon_2+\beta_2\varepsilon_1}, \tag{10}$$

$$q^* = \varepsilon_1\beta_2 + \varepsilon_2\beta_1, \tag{11}$$

$$s^* = \frac{\varepsilon_1\varepsilon_2(\beta_1-\beta_2)^2}{(\beta_1\varepsilon_2+\beta_2\varepsilon_1)^2}, \tag{12}$$

in which

$$\varepsilon_1 = E_1/(E_1+E_2), \quad \text{and} \quad \varepsilon_2 = E_2/(E_1+E_2).$$

Now compare equation (1) with (9). It will be found that these two formulae are identical to each other if the first term of (1) is excluded. s^*, q^* and r^* can be measured from the (γ–t) curve obtained from experiment. Accordingly, the constants E_1, E_2, β_1 and β_2 can be calculated from γ_0, s^*, q^* and r^* by formulae (6), (10), (11) and (12). The results are arranged in table (I).

TABLE (I)

size of particle (mesh no.)	$\frac{d\gamma}{dt}_{t=\infty}$ cm/sec	γ_0	s	q	r	$\lambda_1(=1/\beta_1)$ sec	$\lambda_2(=1/\beta_2)$ sec	E_1 dyne/cm²	E_2 dyne/cm²
20–28	0.94	5.21	1.52	1.095	0.181	3.50	1.44	1.7	0.9
36–48	4.45	8.30	0.40	1.266	0.536	5.08	0.29	0.7	0.9
48–65	11.70	8.53	8.0	0.439	1.371	1.93	0.86	1.5	0.1
65–100	2.53	8.08	1.57	0.540	0.313	6.27	0.94	1.2	0.5
100–150	2.91	8.08	8.27	0.433	0.361	2.32	2.75	1.5	0.2
smaller than 200	3.66	7.63	6.2	0.617	0.480	2.06	1.64	1.5	0.2
lycopodium	1.37	12.6	0.75	7.2	0.109	0.78	1.63	4.8	0.6

§ 4. Discussion

(1) Relaxation time: According to MAXWELL's fundamental equations (7) and (8), it is clear that the stress at constant deformation will gradually decrease with time and the relaxation times

will be inverses of the quantities β_1 and β_2.

In the case of the granular substance, such relaxation of stress may arise from the translational and rotational motions of the particles which remove to more stable positions. So that, it will probably be pertinent to regard that the relaxation time is the time interval required to change the position of particle by translational or rotational motion from one stable position to another more stable position. As will be seen in table (I), one of these relaxation times is in the order of 10^3 sec. and another is in the order of 10^5 sec.

(2) The effect of packing: The $(\gamma-t)$ curve is greatly affected by the state of packing. In this study, the initial packing of the granular substance is very loose. In the case of the particles larger than 80 mesh, if the state of packing becomes sufficiently compact by repeated loading and unloading by turns, the elastic after-effect and plastic deformation almost disappear. Then $(\gamma-t)$ curve is shown by Fig. (2)–A.

In this case the phenomenon is compared to the elastic, so it corresponds to the one with infinitely large relaxation time, $(\lambda_1=\lambda_2=\infty)$. On the contrary in the case of smaller particles or lycopodium, the plastic deformation gradually decreases with increasing of the compactness of packing and at last almost disappears, while the elastic after effect is still observed as shown in Fig. (2)–B. Such a feature of $(\gamma-t)$ curve corresponds to that of the phenomena in which two relaxation times exist and the one is infinitely large while the other is finite.

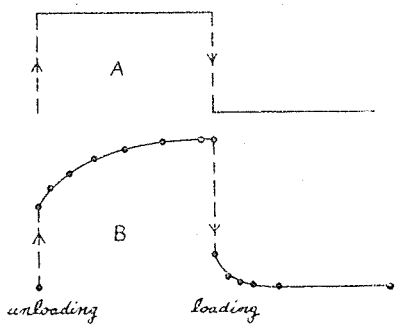

Fig. 2.
Strain at constant load as function of time when the packing is compact.

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