Ionic Distribution and Lattice Distorsion of Magnetic Spinels

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§ 1. Introduction

In the crystal lattice of spinels two sites A and B are distinguished for the cation positions, the former is surrounded by oxygen atoms tetrahedrally and the latter octahedrally. The case where the divalent cations are at tetrahedral sites is called normal and the case where the divalent cations are at octahedral sites is called inverse.\textsuperscript{13} Beside the above two cases the intermediate type is also found. One can give a parameter $x$ for the proportion of the divalent cations on A sites. It is clear that the normal and inverse spinels are characterized respectively $x=1$ and $x=0$.

One of the aims of present study is to criticize the experiments to determine the parameter $x$ for magnetic spinels and to find a rule for the cation distribution.

Besides, it is well known that some magnetic spinels deform their lattices slightly into the ones having lower symmetry in lower temperatures; e.g. magnetite transforms into orthorhombic structure at about 119°K, and hausmannite, copper ferrite, copper chromite etc. transform into tetragonal structure at lower temperature. The second aim of the study is concerned to the above distorsion.

§ 2. Methods to determine the cation distribution

Several methods are applicable to determine the cation distribution, but each method is, by itself, not enough accurate to determine quantitatively the parameter $x$.

1) Lattice parameter

If the ions are thought to be rigid spheres, the lattice parameters would be different for the normal and the inverse spinels; then we can determine the cation distribution by the measurement of the lattice parameter.
VERWEY et al. studied the cation distribution by this method. This is convenient, however, the applicability is limited because of the assumption of rigid sphere ions.

2) Magnetic moments

Magnetic spinels are thought to be ferrimagnetic, then the total magnetic moment is expressed as difference of the magnetic moments of A and B sublattices. Hence, we can deduce the cation distribution from the measurement of saturation magnetization. Though the accuracy of the magnetic measurement is considerably high, we can hardly decide the distribution, when spinel have several sorts of cations. For spinel other than ferrites, it is likely, that the theory of ferrimagnetism does not always hold.

3) Intensity of X-ray diffraction lines.

BERTAUT showed a method to determine quantitatively the parameter x by the intensity measurement of X-ray diffraction lines. This is theoretically the most reliable method; however, the measurement of intensity is not so easy, and the experimental accuracy is not always enough high to determine quantitatively the parameter x.

4) Other methods

Beside the above methods, infra-red absorption spectra, internal friction, electrical conductivity etc. are also related to the cation distribution. But, these are not useful to the aim to determine the parameter x except a few which might be developed in future.

§ 3. Theories for the cation distribution

The cation distribution of a spinel is theoretically decided to minimize its free energy $F = U - TS$, where $U$ means the internal energy, which consists of MADELUNG’s energy, repulsive energy and the electronic energy.

Magnetic ferrites are thought to be ionic crystals, then it is naturally to assume MADELUNG’s energy $V$ is the most important. For a spinel lattice $V$ is given by

$$ V = -\frac{1}{2} \frac{e^2}{a} (pM_A + 2qM_B + 8M_0) = -M \frac{e^2}{a} $$

where $M_A$, $M_B$ and $M_0$ means respectively MADELUNG’s constants of A, B and oxygen sublattices, and $p$ and $q$ mean the average charges of A and B sites respectively.

VERWEY et al. calculated MADELUNG’s constants $M$ of normal
and inverse 2–3 and 4–2 spinels as functions of the oxygen parameter \( u \), which indicate the position of oxygen ions. (Fig. 1). VERWEY tried to explain the cation distribution by MADELUNG's energy. However, some spinels fail to be explained; e.g. CuFe,O, and ZnFe,O, have equal \( u \) value (\( u = 0.380 \)), nevertheless, the former is inverse and the latter is normal.

At the calculation of MADELUNG's energy the ions are assumed as electric point charges. Actually, however, the ions have repulsive force. It is difficult to take the repulsive force exactly into account, we make the following neglect. Since repulsive potential vanishes rapidly against the ionic distances, we assume the ions as a rigid sphere, whose parameters are decided empirically and take only electrostatic energy.

By electronic energy one means the energy of electron cloud of individual ion. Naturally electronic energy in crystal differs from those of free ions. The crystalline field having cubic symmetry removes degeneracy of \( d \)-level into two levels. DUNITZ, ORGEL \(^1\) and MCCLURE \(^2\) studied the separation of the levels by spectroscopic data and calculated the B site preference energy, i.e. the difference of electronic energies of A site and B site, of various ions in magnetic compounds, which is tabulated in Tab. 1.

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Fig. 1.
They attempted to explain the cation distribution from the electronic energy, and their attempts seem to be successful (see Tab. 2), but the following questions still remain.

1) Why Mg ferrite is inverse and Mn, Zn, Cd ferrites are normal?
2) Why NiAl₂O₄ is inverse and other aluminates are normal?

### TABLE 2.

<table>
<thead>
<tr>
<th></th>
<th>Fe²⁺</th>
<th>Cr³⁺</th>
<th>Mn²⁺</th>
<th>Al³⁺</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg²⁺</td>
<td>0</td>
<td>N(37.7)</td>
<td>N(22.8)</td>
<td>0</td>
</tr>
<tr>
<td>Mn²⁺</td>
<td>0</td>
<td>N(37.7)</td>
<td>N(22.8)</td>
<td>0.66</td>
</tr>
<tr>
<td>Fe²⁺</td>
<td>1.40</td>
<td>N(33.7)</td>
<td>N(18.8)</td>
<td>0</td>
</tr>
<tr>
<td>Co²⁺</td>
<td>1.74</td>
<td>N(30.3)</td>
<td>N(15.4)</td>
<td>0.81</td>
</tr>
<tr>
<td>Ni²⁺</td>
<td>2.6</td>
<td>N(17.1)</td>
<td>N(2.2)</td>
<td>0.5</td>
</tr>
<tr>
<td>Cu²⁺</td>
<td>1.52</td>
<td>N(22.5)</td>
<td>N(7.6)</td>
<td>1.52</td>
</tr>
<tr>
<td>Zn²⁺</td>
<td>0.95</td>
<td>N(37.7)</td>
<td>N(22.8)</td>
<td>1</td>
</tr>
<tr>
<td>Cd²⁺</td>
<td>1.28</td>
<td>N(37.7)</td>
<td>N(22.8)</td>
<td>0</td>
</tr>
</tbody>
</table>

* neutron
§ 4. A theoretical consideration for the cation distribution

To explain the cation distribution completely, one should calculate simultaneously the lattice structure itself, esp. oxygen parameter. We might calculate the oxygen parameter by the rigid sphere assumption. In fact, however, we do not know the exact information about ionic radii in spinels.

If oxygen ions construct close-packed structure, the parameter \( u \) is equal to 0.375. Cations on A site or B site change the position of oxygen by \( \delta \), then \( \delta \) be expressed approximately lineally to \( r_\lambda \) as

\[
\delta = ar_\lambda + b (r_m, r_o)
\]

At the normal 2-3 spinel, \( r_\lambda \) is the radius of divalent ion and \( r_m \) and \( r_o \) are radii of trivalent ions and oxygen respectively. The constants \( a \) and \( b \) are chosen in the following way. In MgFe\(_2\)O\(_4\), ZnFe\(_2\)O\(_4\), electronically, Mg and Zn are equally favorable to A or B sites, because both ions are in singlet state. Then, the ionic distribution would be determined by MADELUNG’s energy. If we suppose that inverse spinel has, at least, short range order, then from the calculation by VERWEY, the normal
structure is stable for \( u > 0.81 \), and inverse is stable for \( u < 0.81 \). The fact, that \( \text{MgFe}_2\text{O}_4 \) is inverse and \( \text{ZnFe}_2\text{O}_4 \) is normal, leads to the conclusion that \( u = 0.81 \) corresponds to the atomic radius between \( \text{Zn}^{++} \) and \( \text{Mg}^{++} \). The other fact to decide the constants is distortion of \( \text{CdAl}_2\text{O}_4 \). \( \text{Cd} \) is the largest ion observed in stable spinel structure. The largest deformation allowable for a stable spinel is geometrically given: \( \delta = 0.0125 \). From these two facts the relation between \( \delta \) and \( r_A \) is given, which is shown in Fig. 2. Observed oxygen parameters \( u \) agree satisfactorily with the above relation. By using this relation we can easily obtain MADELUNG's energy by mean of VERWEY's Figure.

If we take both MADELUNG's energy and electronic energy simultaneously, the cation distribution is quite well explained. For instance \( \text{NiMn}_2\text{O}_4 \) must be normal by electronic energy only (2.2 Kcal/mol), however, if one take MADELUNG's energy together into account, normal structure would lose 120 Kcal/mol, and \( \text{NiMn}_2\text{O}_4 \) must be inverse. Another example is \( \text{NiAl}_2\text{O}_4 \). It gains 19.2 Kcal/mol in MADELUNG's energy for normal structure, however, it loses 20.6 Kcal/mol electronically, and \( \text{NiAl}_2\text{O}_4 \) must be inverse. On the other hand \( \text{CuAl}_2\text{O}_4 \) gains more energy electrostatically than the electronic energy lost in normal structure, and the compound is normal.

Thus, the cation distribution of all spinels hitherto studied are explained without exception.

§ 5. Distorsion of crystal lattice of spinels

Various spinels exhibit tetragonal distorsion, and its origin was discussed
by several authors.* GOODENOUGH17 explained the distortion of spinels by covalent bond between ions. For instance, in CuFe₂O₄ valence electrons of copper make \( (dsp^2) \) orbital of square type, and it causes \( c/a \) to increase. This explanation, however, is not appropriate for CuCr₂O₄, since where \( c/a \) decreases.** DUNITZ and ORGEL19 explained the distortion by the JAHN-TELLER effect, i.e. lowering electronic energy by deformation to lower crystal symmetry.

Crystal distortion of the following compounds were experimentally studied. We exclude magnetite, because it is believed that its distortion is caused from different origin. (Low temperature transformation of magnetite, cubic \( \rightarrow \) orthorhombic, is considered as order-disorder transformation of \( Fe^{++} \) and \( Fe^{+++} \) ions on B sites.)

1) Spinels including \( Cu^{++} \) ion

\( Cu^{++} \) is a ion responsible to crystal distortion, and various compounds are studied. Mixed spinel of \( CuFe_2O_4 \) and \( CuCr_2O_4 \) are studied by MIYAHARA, OHNISHI28 and TERANISHI. DELORME19 studied GeCo\(_{1-t}\)Cu\(_t\)O, Cu\(_{1-t}\),Me\(_t\)Fe\(_2\)O\(_4\) and Cu\(_{1-t}\),Me\(_t\),Cr\(_2\)O\(_4\) (Me=Ni, Co, Mg, Zn, Cd). In CuFe\(_2\)O\(_4\) - CuCr\(_2\)O\(_4\) series \( Fe \)-side deforms to tetragonal structure: \( c/a > 1 \), while Cr-side: \( c/a < 1 \). In GeCo\(_{1-t}\)Cu\(_t\)O, series tetragonal distortion is observed in the region \( 0.25 < t < 0.6 \), whose maximum axial ratio is 1.034.

2) Spinels including \( Mn^{+++} \) ion

Hausmannite and zinc manganite are studied by MASON3,5 and found, that it deforms tetragonally below 1170°C and 1025°C resp. Recently SINHA3,5 et al. studied various manganites and lattice parameters were measured. Manganites except NiMn\(_2\)O\(_4\) and CuMn\(_2\)O\(_4\) exhibit tetragonal distortion. FINCH20 et al. studied the manganite-ferrite system (Me=Ni, Mn, Co, Fe). MURAMORI4,12 studied Cu and Mg ferrite-manganite system. From the above experiments it is commonly concluded that the tetragonal distortion takes place when content of Mn ions exceeds 50%, and volume change is hardly observed between cubic and tetragonal phases.

3) Other spinels

FRANCOMBE4,5 studied \( Fe^{++}Fe^{+++}_{1-t}Cr_tO_4 \) system and found that the

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* Note added in proof: The author has an opportunity to read the Thesis (Grenoble 1956) by C. DELORME, who studied lattice distortion of various mixed spinels including \( Cu^{++} \) ion. He stated that deformation is caused by \( Cu^{++}, Ni^{++} \) and \( Mn^{+++} \) ions and their neighbors.

** Goodenough insists that his theory also holds for this case.
crystal at low temperatures is cubic for \( t < 1.0 \), tetragonal for \( 1.0 < t < 1.4 \) and orthorhombic for \( 1.4 < t < 2.0 \). McGuire et al. studied \( \text{NiFe}_{2-t}\text{Cr}_t\text{O}_4 \) systems and found that the crystal is tetragonal \((c/a < 1)\) for \( 1.25 < t < 1.75 \), orthorhombic for \( 1.75 < t < 1.85 \) and tetragonal \((c/a > 1)\) for \( t > 1.85 \). Rooksby et al. found tetragonality \( c/a = 0.9988 \) in cobalt ferrite at the liquid nitrogen temperature.

These results are shown in Tab. 3.

<table>
<thead>
<tr>
<th></th>
<th>( \text{Fe}^{3+} )</th>
<th>( \text{Cr}^{3+} )</th>
<th>( \text{Mn}^{3+} )</th>
<th>( \text{Al}^{3+} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Mg}^{2+} )</td>
<td>theor exp.</td>
<td>theor exp.</td>
<td>theor exp.</td>
<td>theor exp.</td>
</tr>
<tr>
<td>( \text{Mn}^{2+} )</td>
<td>( c/a &gt; 1 )</td>
<td>( c/a = 1.15(3) )</td>
<td>0 cubic</td>
<td>0 cubic</td>
</tr>
<tr>
<td>( \text{Fe}^{2+} )</td>
<td>weak orth. (-160^\circ C)</td>
<td>weak ( c/a = 0.968(5) )</td>
<td>( c/a &lt; 1 )</td>
<td>( c/a = 1.05 )</td>
</tr>
<tr>
<td>( \text{Co}^{2+} )</td>
<td>( c/a = 0.9988(3) )</td>
<td>0 cubic</td>
<td>( c/a &gt; 1 )</td>
<td>( c/a = 1.12 )</td>
</tr>
<tr>
<td>( \text{Ni}^{2+} )</td>
<td>0 cubic</td>
<td>( c/a &gt; 1 )</td>
<td>( c/a = 1.056(2) )</td>
<td>( c/a = 1.20(5) )</td>
</tr>
<tr>
<td>( \text{Cu}^{2+} )</td>
<td>( c/a &gt; 1 )</td>
<td>( c/a = 1.056(2) )</td>
<td>( c/a &lt; 1 )</td>
<td>( c/a = 1.14(5) )</td>
</tr>
<tr>
<td>( \text{Zn}^{2+} )</td>
<td>0 cubic</td>
<td>0 cubic</td>
<td>( c/a &gt; 1 )</td>
<td>( c/a = 1.20(3) )</td>
</tr>
<tr>
<td>( \text{Cd}^{2+} )</td>
<td>0 cubic</td>
<td>0 cubic</td>
<td>( c/a &gt; 1 )</td>
<td>( c/a = 1.20(3) )</td>
</tr>
</tbody>
</table>

(3.4) H. Ohnishi: unpublished value.
(3.5) S. C. Abraham and R. A. Calhoun: Acta Cryst. 6 (1953) 105.
(3.9) B. Mason: Amer. Mineral. 32 (1947) 426.

Tetragonal distortion of these compounds are explained by the Jahn-Teller effect as mentioned above. But some spinels should be explained in connexion with cation distribution. In the followings some problems of special interests are discussed.

1) \( \text{NiMn}_2\text{O}_4 \) and \( \text{CuMn}_2\text{O}_4 \)

The reason that the above two manganites are cubic, is explained as
follows: NiMn₂O₄ is known as an inverse spinel and B sites include only a half of manganese ions. As study of mixed manganites suggests, distortion is caused when the content of Mn ions in B sites exceeds 50%. Hence, inverse nickel manganite remain cubic. In CuMn₂O₄, copper ion in B sites which would cause c/a>1 and manganese ion in A site which would cause c/a<1 compensate each other and no distortion take place.

2) CuAl₂O₄ and FeAl₂O₄

The above compound, of which one expects some distortion, have not been studied in detail. It is reported that CuAl₂O₄ does not exhibit any distortion at ordinary temperature. In this case Cu ions on A-sites which would cause an opposite distortion compensate the effect of Cu ions on B-sites.

3) Tetragonality and transformation temperature

It is noteworthy that all spinels which deforms tetragonally at lower temperature exhibit a relation between tetragonality \( \sigma = |c-a|/a \) (at low temperature) and the transformation temperature \( T_c \). The relation is
expressed in a curve for ferrites, chromites, manganites and aluminates (Fig. 3). This fact suggests that deformation of magnetic spinels is caused from the same origin, i.e. the JAHN-TELLER effect.

4) From the above discussed facts we have some questions about FRANCOMBE's analysis (3.6) for the crystal structure of FeFe$_{2-x}$CrO$_4$ system, where the compounds of $t > 1.0$ are thought to be normal, hence it is not naturally to think $(c-a)/a$ reverse its sign in the normal region. On the other hand we know that in some copper ferrite-chromites cubic and tetragonal phases co-exist. If we re-analyze FRANCOMBE's data, so-called
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orthorhombic phase is explained as cubic and tetragonal phases in co-existence. FRANCOMBE’s analysis is shown in Fig. 4 a, c and the re-analyzed results in Fig. 4 b, d.

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21) McGUIRE, HOWARD and SMART, Ceramic Age 60 (1952) 22.