[Journal of the Faculty of Science, Hokkaidô Uuiversity, Japan, Ser. II, Vol. VI. No. 1, 1963]

Application of the Williams-Weizsäcker Picture to the High Energy Nucleon-Nucleon Collisions.

Kanji FUJII, Hideaki NAGAI, Tetsurô SAKUMA
and Tetsuya TSUCHIDA

(Received Oct. 10, 1962)

An attempt is made to develop the Williams-Weizsäcker (W–W) method so as to be applicable to the nucleon-nucleon collisions at high energies. Under a certain condition, that is, in case of ⟨*no excitation*⟩ of one of the colliding nucleons, it is shown that the W–W picture can be applicable to nucleon-nucleon collisions due to a peripheral pion exchange. On the basis of our modified W–W method, the total cross section of the high energy nucleon-nucleon collisions, σ_T, is estimated, and it is found that σ_T is constant at high energies if the pion-nucleon total cross section is assumed to be constant. The total number of almost real pions surrounding a physical nucleon is also estimated and is consistent with that obtained by Blokhinčev.

Further, the correspondence between our W–W method and the perturbation theory is discussed, and results similar to those in electromagnetic processes are obtained.

§ 1. Introduction

Recently, super high energy phenomena have been investigated by many authors. Based on the experiences that we have found new aspects of the structure of matter with increasing energies, we can hope to get, by analysing super high energy phenomena, certain new informations which may be useful in constructing the future theory.

Experimental data of jet showers with two narrow cones show the importance of the interaction with a large impact parameter in both collisions of pions with nucleons and of nucleons with nucleons[1]. Niu[2] expleined this feature phenomenologically as the results of the peripheral collisions accompanied by productions of two excited states. The field-theoretical descriptions of this feature were given by Dremin and Chernavsky[3], and Salzman and Salzman[4]. They estimated the contributions of one ⟨*almost real*⟩ pion exchange interaction to the inelastic cross section of high energy nucleon-nucleon collisions. Their considerations seem, in some senses, to be very similar to Williams-Weizsäcker (W-W) picture in case of electromagnetic processes[5].

In this paper we will try to modify the original W-W method so as to be applicable to nucleon-nucleon collisions. By using this method, the contributions of an ⟨*almost real*⟩ pion exchange interaction to the total cross section of nucleon-nucleon collisions is calculated. Correspondence between our modified

W-W picture and the field theoretical consideration is also investigated.

Of course, straightforward application of the W-W method to strong interactions will lead to incorrect results, since in nucleon-nucleon collisions we should take into account following aspects: A) the pion-nucleon interaction is much stronger than the electromagnetic one, and, B) the pion field is a pseudoscalar one with nonzero mass instead of vector and massless one.

Because of A), in case of nucleon-nucleon collisions it does not give a good approximation to neglect the incident particle excitation. Here, the word '*incident*' is used in the sense of W-W picture. At present, however, one connot find how to deal with the '*incident*' nucleon excitation, so that we are obliged to develop our program by starting from the static Yukawa potential arround the '*incident*' nucleon, and therefore the excitation of '*incident*' nucleon is put out of question. Thus, in order to guarantee the little deflection of the incident nucleon, the magnitude of transverse momentum, $k_\perp$, of pions arround the incident nucleon should be very small compared to the incident energy. Because of B), high velocity of the incident nucleon does not necessarily guarantee the reality of pions sorrounding the incident nucleon. Corresponding to the condition which is needed to guarantee the reality of pions, we have to take an additional parameter. Since the relation $k_\perp^2 \approx k_\mu k_\mu$ (k_μ denotes the four momentum of pions around the incident nucleon) is established, we can take $k_\perp$ as this parameter. These situation teaches us not only the way how to modify the W-W picture for the strong interaction but also the limit of applicability of the modified W-W method to be developed in the following.

In § 2, the original W-W method is modified, from our point of view, so as to be applicable to the nucleon-nucleon collisions. The spectrum of almost real pions surrounding a nucleon with a high velocity is estimated, and contribution to the total cross section of nucleon-nucleon collisions due to such pions is also estimated. Further, the total number of almost real pions in the physical nucleon is calculated.

In § 3, a discussion of the correspondence between the modified W-W method and the perturbational treatment is given, and it is shown that, in case of ⟨*no excitation*⟩ of one of the colliding nucleons, this correspondence is established just as in case of electromagnetic processes[6,8].

§ 2. Extension of the Williams-Weizsäcker method to nucleon-nucleon collisions.

In the rest frame of a nucleon, the pion potential of the nucleon is given at a point $\boldsymbol{r}'$ by

$$\varphi(\boldsymbol{r}')=\frac{G}{4\pi}\cdot\frac{\exp(-\mu r')}{r'};\ r'=|\boldsymbol{r}'|,\text{ and }\mu\text{ is pion mass.}\tag{1}$$

Transforming $\varphi(\boldsymbol{r}')$ into the center of mass system of two colliding nucleons (the quantities referring to this system is denoted by the sign U), we obtain

$$\varphi_U(\boldsymbol{r})=\frac{G}{4\pi}\cdot\frac{\exp(-\mu\gamma R)}{rR},\tag{2}$$

where $\boldsymbol{R}\equiv\left(\frac{x}{\gamma},\frac{y}{\gamma},z-vt\right)$, $R\equiv|\boldsymbol{R}|$, $\gamma\equiv\sqrt{1-\left(\frac{v}{c}\right)^2}^{\,-1}$ and $v=(0,\ 0,\ v_z)$ is the velocity of one of the colliding nucleons relative to the center of mass system. From Eq. (2), the following relation is easily derived

$$\frac{1}{b}\left(\frac{\partial\varphi_U}{\partial b}\right)\Big/\left\{\frac{1}{z-vt}\left(\frac{\partial\varphi_U}{\partial(z-vt)}\right)\right\}=\frac{1}{\gamma};\ b\equiv\sqrt{x^2+y^2},\tag{3}$$

which shows that the potential φ_U is strongly contracted if $\gamma\gg1$. In contrast to the vector and massless field, the condition $\gamma\gg1$ means only the contraction of the proper field around the incident particle, but does not necessarily guarantee its reality.

We are now concerning the case of $|\boldsymbol{v}|\cong c=1$ and the transvers momentum of the pion, $|\boldsymbol{k}_\perp|\lesssim\alpha\mu$, α being a parameter of order 1. This restriction for $|\boldsymbol{k}_\perp|$, as will be shown in the next section, means not only the little deflection of the incident nucleon but also the small "virtuality" of pions around the nucleon.

Following the procedures similar to those of the W-W method, we will calculate the energy spectrum of the ⟨*almost real*⟩ pion in the proper field of a nucleon. Making use of Fourier transform of $\varphi(\boldsymbol{r}')$,

$$\varphi(\boldsymbol{r}')=\frac{G}{(2\pi)^3}\int d\boldsymbol{k}\ \exp(i\boldsymbol{k}\boldsymbol{r})/(k^2+\mu^2),$$

we obtain

$$\varphi_U(\boldsymbol{b},\ t)=\frac{2G}{(2\pi)^3\gamma v}\int_{w_U>0}dw_U\int d\boldsymbol{k}_\perp\frac{e^{-iw_Ut+i\boldsymbol{k}_\perp\boldsymbol{b}}}{\left(\frac{w_U}{\gamma v}\right)^2+k_\perp^2+\mu^2},\tag{4}$$

where $\boldsymbol{k}=(k_{\perp x},k_{\perp y},k_{||})=(\boldsymbol{k}_\perp,k_{||})$, $w_U=\gamma|k_{||}|v$ ann, $\boldsymbol{k}_\perp$ and $k_{||}$ mean, respectively, the perpendicular and parallel components of the $\boldsymbol{k}$ to the transmission line of the incident nucleon, i.e. z-direction. The energies per unit area in the x-y plane when the incident nucleon with a collision distance $\boldsymbol{b}$ passes are given by

$$\int_{-\infty}^{\infty}dt\ G_z(\boldsymbol{b},t)=-\int_{-\infty}^{\infty}dt\frac{\partial\varphi_U^*}{\partial t}\cdot\frac{\partial\varphi_U}{\partial z}\bigg|_{z=0}=\frac{1}{v}\int_{-\infty}^{\infty}dt\left|\frac{\partial\varphi_U(\boldsymbol{b},t)}{\partial t}\right|^2.\tag{5}^{*)}$$

*) Throughout this paper, factors of the isotopic spin space are not taken into account.

Substituting the Eq. (4) into Eq. (5) and integrating over x-y plane with $|\boldsymbol{b}| \gtrsim \frac{1}{\alpha\mu}$, we get the total energy carried by pions which are accompanied by incident nucleon, i.e.

$$\int_{|b|\geqslant\frac{1}{\alpha\mu}} d\boldsymbol{b}\int_{-\infty}^{\infty} dt\, G_z(\boldsymbol{b}, t) \cong C\int_{w_U>0} dw_U\cdot w_U^2$$

$$\times\int_{|k_\perp|\leqslant\alpha\mu} d\boldsymbol{k}_\perp \frac{1}{\left[\left(\frac{w_U}{\gamma}\right)^2+k_\perp^2+\mu^2\right]^2}, \tag{6}$$

where $C\equiv\frac{4\,G^2}{(2\pi)^3\gamma^2}\beta$, and β is a parameter of order 1 and independent of r and w_U (see Appendix I). As we are concerning with the almost real pion spectrum, the region of integration of $|\boldsymbol{k}_\perp|$ should be limited to $|\boldsymbol{k}|\leqslant\alpha\mu$, $\alpha\approx1$. In this point our method of calculation is different from CHERNAVSKY's[9]. They performed the integration of $\boldsymbol{k}_\perp$ over the range $(-\infty, +\infty)$ but it can not be a good approximation for W–W picture. When we take into account the relation

$$\int dw_U\int d\boldsymbol{k}_\perp\, w_U\cdot p(w_U\cdot\boldsymbol{k}) = \int_{|b|\geqslant\frac{1}{\alpha\mu}} d\boldsymbol{b}\int dt\, G_z(\boldsymbol{b}, t),$$

where $p(w_U, \boldsymbol{k}_\perp)$ is the number spectrum of almost real pions accompanied by passing of the fast incident nucleon, we get

$$p(w_U, \boldsymbol{k}_\perp) = C\frac{w_U}{\left[\left(\frac{w_U}{\gamma}\right)^2+k_\perp^2+\mu^2\right]^2};\quad |\boldsymbol{k}_\perp|\leqslant\alpha\mu. \tag{7}$$

Thus, we can write, according to the spirits of W–W method, the total cross section of nucleon-nucleon collisions due to a peripheral pion exchange interaction in terms of the pion-nucleon total cross section as follows.

$$\sigma_{NN}^{WW}(E_U) \cong 2\int dw_U\int_{|k|_\perp\leqslant\frac{1}{\alpha\mu}} d\boldsymbol{k}_\perp\, p(w_U, \boldsymbol{k}_\perp)\cdot\sigma_{\pi N}^{tot}(w_U, \boldsymbol{k}_\perp), \tag{8}$$

where $\sigma_{\pi N}^{tot}$ is the observable pion-nucleon total cross section.

Here it seems to be natural to define the total number of almost real pions surrounding a phpsical nucleon, n, by the following relation.

$$\int_{E_U>w_U>0} dw_U\int d\boldsymbol{k}_\perp\, p(w_U, \boldsymbol{k}_\perp) \equiv n. \tag{9}$$

Performing this integration, we get

$$n = \frac{\pi}{2} C \cdot \ln(\alpha^2 + 1) \cdot \left(\frac{E_U}{m}\right)^2,$$

where E_U is the total energy of the incident nucleon in the U-system, and m is the nucleon mass. On the other hand, from the definition of C we get, putting $\frac{G^2}{4\pi} \approx 0.1$, $C \cong \frac{\beta}{50}\left(\frac{m}{E_U}\right)^2$. We can therefore see that n is independent of E_U and obtain $n \cong \frac{\beta}{50}$ for $\alpha = 1$, and $\beta/20$ for $\alpha = 2$. These numerical values of our n are not inconsistent of that obtained on the basis of the extended source theory.[7]

When we assume that $\sigma_{\pi N}^{tot}$ tends to a constant value, i.e. about 30 mb, at high energies, Eq. (8) reduces to

$$\sigma_{NN}^{WW}(E_U) \cong 2n\ \sigma_{\pi N}^{tot}(\infty) \approx \begin{cases} 1.2\,\beta\, mb & \text{for } \alpha = 1. \\ 3\,\beta\, mb & \text{for } \alpha = 2. \end{cases} \tag{10}$$

Thus, we conclude that in high energies our model predicts a constant cross section of nucleon-nucleon collisions, where one of the colliding nucleons is not excited. Our conclusion is in contrast to SALZMAN's result which predicts the total cross section due to a peripheral collision increasing logarithmically with respect to the incident energy.

§ 3. Comparison between the modified W–W method and the perturbational treatment.

In case of electromagnetic processes (for instance, Bremsstrahlung in electron-proton collisions), it has been shown that results from the W–W method are essentially equivalent to those obtained on the basis of Feynman-Dyson method under the same approximations as taken in W–W method[8]. In the following these situations in nucleon-nucleon collisions are investigated and similar correspondence will be shown to exist.

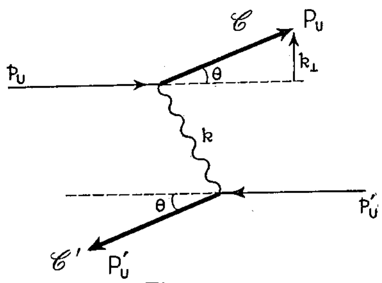

Fig. 1.

In order to compare our W–W method with perturbational treatment, it is convenient to rewrite Eqs. (7) and (8) in terms of a variable set $(\mathscr{M}, \delta^2)$, defined in the following.

As shown in Fig. 1, colliding nucleons with four momentum p_U and p'_U exchange a pion with momentum $(\boldsymbol{k}, k_4)$ (its virtuality is defined by $\delta^2 \equiv k_\mu k_\mu =$

$\boldsymbol{k}^2-k_0^2$), producing two groups of particles $\mathscr{C}$ and $\mathscr{C}'$ with total momenta P_U and P'_U, respectively. Defining $\sin\theta\equiv|\boldsymbol{k}_\perp|/|\boldsymbol{P}_U|$, we get the following relation from energy-momentum conservation law,

$$\left(\frac{k_\perp}{P_U}\right)^2=\frac{1}{P_Up_U}\Big(2P_Up_U-2E_U^2+\frac{\mathscr{M}^2+\mathscr{M}'^2+2m^2+2\delta^2}{2}\Big)\qquad\text{for}\quad\theta\ll1,\tag{11}$$

where $\mathscr{M}$ and $\mathscr{M}'$ are defined by $P_U^2=-\mathscr{M}^2$ and $P_U'^2=-\mathscr{M}'^2$. By expanding $P_U=E_U\sqrt{1-\dfrac{\mathscr{M}^2+\mathscr{M}'^2}{2E_U^2}+\left(\dfrac{\mathscr{M}^2-\mathscr{M}'^2}{2E_U^2}\right)^2}$ with respect to the power series of $(\mathscr{M}/E_U)^2$ and $(\mathscr{M}'/E_U)^2$, we obtain from Eq. (11) the relation between virtuality δ^2 and $\boldsymbol{k}_\perp^2$ as follows.

$$\boldsymbol{k}_\perp^2\cong\delta^2\left(1-\frac{\mathscr{M}'^2+\mathscr{M}^2-2m^2}{4E_U^2}\right)-\frac{(\mathscr{M}^2-m^2)(\mathscr{M}'^2-m^2)}{4E_U^2}.\tag{12}$$

Second term in Eq. (12) vanishes if one of the colliding nucleons is not excited i.e. $\mathscr{M}'=m$. Here, it should be noted that this relation (12) indicates, as already mentioned in the introduction, the two conditions under which the W–W picture is applicable to nucleon-nucleon collisions, that is, the small deflection of colliding nucleons and the reality of an exchanged pion, can be represented by only one parameter, i.e. δ^2. Therefore δ^2 and $\boldsymbol{k}_\perp^2$ should be the order of μ^2.

Using the relation (12) and neglecting the excitation of one of the colliding nucleons, we obtain $p(w,\boldsymbol{k}_\perp)$ and σ_{NN}^{WW} with new variables as follows.

$$p(w,\boldsymbol{k}_\perp)=\frac{C}{(\delta^2+\mu^2)}\cdot\frac{\mathscr{M}^2-m^2}{4E_W},\tag{13}$$

and

$$\sigma_{NU}^{WN}(E_U)=\frac{C\pi}{2E_U^2}\int d\mathscr{M}\cdot\mathscr{M}^2\cdot p_{\mathscr{M}}\int d\left(\frac{\delta^2}{\mu^2}\right)\cdot\frac{\mu^2\sigma_{\pi N}(\mathscr{M},\delta^2)}{(\delta^2+\mu^2)^2},^{*)}\tag{14}$$

where $p_{\mathscr{M}}$ represents the four momentum of an exchanged pion in the rest system of the particles of group $\mathscr{C}$. In deriving Eq. (14), a Lorentz invariant relation $v\,E_U\,w_U\,\sigma_{\pi N}^{tot}(w_U,\boldsymbol{k}_\perp)=\mathscr{M}\,p_{\mathscr{M}}\,\sigma_{\pi N}^{tot}(\mathscr{M},\delta^2)$ and $v\approx c=1$ were used.

According to Feynman-Dyson method, the nucleon-nucleon total cross section, due to one almost real pion exchange, $\sigma_{NN}^{tot}(E_U)$, can be written as

---

*) $dw_U\,d\boldsymbol{k}_\perp=\dfrac{\pi}{2}\mu^2\dfrac{\mathscr{M}}{E_U}d\mathscr{M}\,d\left(\dfrac{\delta^2}{\mu^2}\right)$.

$$v_{NN}\,\sigma_{NN}^{tot}(E_U)=\sum_{n,n'=0}\int d^4k\int d\Omega^{(n+1)}d\Omega^{(n'+1)}(2\pi)^4\cdot\delta\,(\sum_{i=1}^{n}p_i+p_0-p+k)$$
$$\cdot\delta(\sum_{j=1}^{n'}p'_j+p'_0-p'-k)\cdot\left(\frac{m}{E_U}\right)^2|M^{(n)}(-k,p)|^2$$
$$\cdot|M^{(n')}(k,p')|^2\frac{2}{(k^2+\mu^2)^2},\tag{15}$$

where p_0 and p'_0 are the four momenta of the final nucleons and

$$d\Omega^{(n+1)}=\frac{dp_0}{(2\pi)^3}\left(\frac{m}{E(p_0)}\right)\prod_{i=1}^{n}\frac{dp_i}{(2\pi)^3(2w_i)}$$

is an invariant phase volume. Eq. (15) is essentially regarded as a product of the two pion-nucleon cross section, since $\sigma_{\pi N}^{tot}$ is written as

$$v_{\pi N}\sigma_{\pi N}^{tot}(w_k)=(2\pi)^4\sum_{n=0}\int d\Omega^{(n+1)}\frac{m}{E(p')}$$
$$\cdot\frac{1}{2w_k}|M^{(n)}(k,p')|^2\cdot\delta(\sum_{j=1}^{n}p_j+p'_0-p'-k).\tag{16}$$

If the two matrix elements in Eq. (15) are replaced by $\sigma_{\pi N}^{tot}$ according to Eq. (16), then the expression derived already by F. SALZMAN and G. SALZMAN is obtained.[4] We, however, are interested in the case of only one nucleon excitation processes, so only one matrix element is replaced by $\sigma_{\pi N}^{tot}$ and the other one, $M^{(n=0)}$ is calculated by perturbation technique. Setting $M^{(n=0)}\equiv\bar{u}(p_0)\,F(\delta^2)\,i\gamma_5\,u(p)$, where $\delta^2\equiv(p_0-p)^2$ and using $\frac{1}{2}\sum_{\rm spin}|M^{(n=0)}|^2=\frac{F^2\delta^2}{4m^2}$, the following expression is obtained from Eq. (15)

$$\sigma_{NN}^{tot}(n=0,\text{ or }n'=0)\cong\frac{\mu^2F^2(0)}{2(2\pi)^2E_U^2p_U^2}\int d\left(\frac{\delta^2}{\mu^2}\right)\frac{\delta^2}{(\delta^2+\mu^2)^2}$$
$$\times\int d\mathcal{M}\,\mathcal{M}^2\,p_{\mathcal{M}}\,\sigma_{\pi N}^{tot}(\mathcal{M},\delta^2).\tag{17}$$

Here, we assumed $F(\delta^2)\cong F(0)$.

Comparing Eqs. (14) and (17), we can see that there is no essential difference between them, provided $\delta^2\sim\mu^2$. This situation is quite similar to that in electromagnetic processes. Finally, by equating (14) to (17) we can reestimate n. For $F^2/4\pi\approx15$, the numerical values of n thus obtained are

$$n=\begin{cases}\dfrac{1}{100} & \text{for}\quad \alpha=1\\[2ex] \dfrac{1}{10} & \text{for}\quad \alpha=2,\end{cases}$$

which are not inconsistent with those obtained in the previous section.

§ 4. Discussions and final remarks.

It should be noted that the constancy of $\sigma_{NN}^{WW}(E\rightarrow\infty)$, as obtained in § 2, can be understood by a simple consideration. $p(w, \alpha)$, the spectrum of pions, may behave with respect to w as shown in Fig. 2., where α is a set of parameters, such as $q_{\perp}^2$, and δ^2. The total number of peripheral pions, n, is represented by

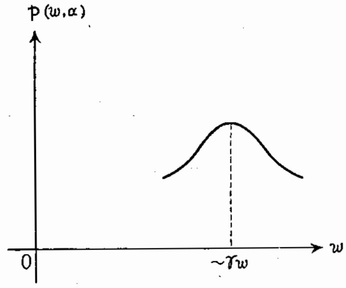

Fig. 2.

$$\sum_{\alpha}\int dw\, p(w, \alpha)=n$$

It $\sigma_{\pi N}^{tot}(w\rightarrow\infty)$ is constant and depends little on α, one can get

$$\sigma_{NN}^{WW}(E\rightarrow\infty\ ;\ \textit{peripheral one pion exchange})$$
$$\cong 2\sum_{\alpha}\int dw\, p(w, \alpha)\, \sigma_{\pi N}^{tot}(w, \alpha)\cong 2n\ \sigma_{\pi N}^{tot}(w\rightarrow\infty),$$

provided n is independent of the incident energy. Thus the total cross section of high-energy nucleon-nucleon collisions due to one peripheral pion exchange becomes independent of the incident nucleon energy.

The difference between our method and SALZMAN's field theoretical one[4] lies in that in the latter, the excitation of both colliding nucleons are treated on equal footing, while in our's the only one nucleon excitation is taken into account. At present we can say nothing about the possibility of including the excitations of both nucleons in our method.

Our model predicts large probability of the ⟨*asymmetric excitation*⟩[9] of colliding nucleons. Therefore, it cannot explain all the experimental data at high energies. Surely two pion exchange and pion-pion interactions will also play important roles.

Correspondence between our method and perturbation theory in the case of pion production in pion-nucleon collisions can be also obtained. Of course, as for the cross section both methods give essentially the same result. It may be also interesting to investigate, for example, pion production in electron-nucleon collisions.

The authors would like to express their sincere thanks to Prof. D. ITÔ and Dr. A. KANAZAWA for their discussions. Thanks are also due to Prof. Y. ÔNO and Dr. K. IWATA for their interests.

Appendix

Substituting the expression (4) into (5), we get

$$\int_{-\infty}^{\infty} dt\, G_z(\boldsymbol{b}, t) = \frac{4G^2}{(2\pi)^5 \gamma^2 v} \int dw_U \iiiint_{-\alpha\mu}^{+\alpha\mu} d\xi\, d\eta\, d\xi'\, d\eta'$$

$$\times \frac{w_U\, e^{i(\xi'-\xi)|\boldsymbol{b}|}}{\left[\left(\frac{w_U}{\gamma}\right)^2 + \xi^2 + \eta^2 + \mu^2\right]\left[\left(\frac{w_U}{\gamma}\right)^2 + \xi'^2 + \eta'^2 + \mu^2\right]}, \qquad \text{(A-1)}$$

where (ξ, ξ') $((\eta, \eta'))$ represents the momentum components parallel (perpendicular) to $\boldsymbol{b}$. Integrating (A-1) over the $x-y$ plane with $|\boldsymbol{b}| \geqslant \frac{1}{\alpha\mu}$, we obtain

$$\int_{|\boldsymbol{b}|\geqslant\frac{1}{\alpha\mu}} d\boldsymbol{b} \int_{-\infty}^{\infty} dt\, G_z(\boldsymbol{b}, t) = \left[\int_{-\infty}^{\infty} d\boldsymbol{b} - \int_{|\boldsymbol{b}|\leqslant\frac{1}{\alpha\mu}} d\boldsymbol{b}\right] \int dt\, G_z(\boldsymbol{b}, t)$$

$$\approx \frac{4G^2}{(2\pi)^5\gamma^2 v} \int dw_U\, w_U^2 \int_{|k_\perp|\leqslant\alpha\mu} d\boldsymbol{k} \frac{1}{\left[\left(\frac{w_U}{\gamma}\right)^2 + k_\perp^2 + \mu^2\right]^2}$$

$$- \frac{8G^2}{(2\pi)^4\gamma^2 v} \int dw_U\, w_U^2 \iint_{-\alpha\mu}^{+\alpha\mu} d\xi\, d\eta$$

$$\times \frac{1}{\left[\left(\frac{w_U}{\gamma}\right)^2 + \xi^2 + \mu^2\right]\left[\left(\frac{w_U}{\gamma}\right)^2 + \eta^2 + \mu^2\right]}, \qquad \text{(A-2)}$$

where we have used approximate relations such as

$$\tan^{-1} \frac{\alpha\mu}{\sqrt{\left(\frac{w_U}{\gamma}\right)^2 + \xi^2 + \mu^2}} \approx \frac{\alpha\mu}{\sqrt{\left(\frac{w_U}{\gamma}\right)^2 + \xi^2 + \mu^2}} .$$

Second term in Eq. (A-2) has essentially the same structure as the first term so we may renormalize the second one into the first term through parameter β. Thus Eq. (6) in the text is deduced.

References

1) For example, P. Ciok, T. Coghen, J. Gierula, R. Hoynski, A. Jurak, M. Miesowicz, T. Saniewska, O. Stanisz and J. Pernegger: Nuovo Cimento **8** (1958), 166 and **10** (1958), 741; G. Cocconi: Phys. Rev. **111** (1958) 1699; K. Niu: Nuovo Cimento **10** (1958), 994; V. I. Veksler: Ninth Annual Conference on High Energy Physics at Kiev (1959).
2) K. Niu: ibid.
3) I. M. Dremin and D. S. Chernavsky: Zh. Dks. Teor. Fiz. **38** (1960), 227.

4) F. SALZMAN and G. SALZMAN: Phys. Rev. Lett. **5** (1960), 377.
5) E. J. WILLIAMS: Kgl. Danske Vid. Selsk. **13** (1935), No. 4.
C. F. von WEIZSÄCKER: Zeit. f. Phys. **88** (1934), 612.
6) H. J. BHABHA: Proc. Roy. Soc. A **152** (1935), 559; H. J. BHABHA: Proc. Camb. Phil. Soc. **31** (1935), 394; Y. NISHINA, S. TOMONAGA and M. KOBAYASHI: Sci. Pap. Inst. Phys. Chem. Research, Japan, **27** (1935), 137; T. MUROTA, A. UEDA and H. TANAKA: Prog. Theor, Phys. **16** (1956), 482.
7) D. I. BLOKHINCEV, V. S. BARASENKOV and B. M. BARBASOV: Nuovo Cimento **12** (1959), 602.
8) T. MUROTA and A. UEDA: Prog. Theor. Phys. **16** (1956), 497.
9) D. S. CHERNAVSKY: Nuovo Cimento Supple. **8** (1958), 775.
E. L. FEINBERG: Ninth Annual Conference on High Energy Physics at Kiev (1959); Yu. A. ROMANOV. and D. S. CHERNAVSKY: Zh. Eks. Teor. Fiz. **38** (1960), 1132.