ON SOME RELATIONS BETWEEN THE FORM OF LICHTENBERG'S FIGURE AND THAT OF ITS EXCITING SPARK

By

Ukitirō NAKAYA

With 4 Plates

INTRODUCTION

In our previous experiments on the form and structure of long electric spark(1), we found that an ordinary long electric spark of zigzag type is transformed into an almost straight spark consisting of three parts, which was then called a "three-part spark." The spark is excited by the discharge of Leyden jar, the capacity being 1 m. in most cases, and this three-part form of spark is obtained by attaching a needle point leakage to the positive side of the leads to electrodes. Some of the characteristic natures of this type of spark made out during the course of our previous experiments led us to examine and compare the nature of the Lichtenberg's figures excited by the ordinary zigzag spark and by the three-part spark respectively. The results are described here as a preliminary report.

I. EXPERIMENTAL PROCEDURE

Experimental arrangement is essentially the same as that used by many workers in this line, and the electric connection is shown in Fig. 1.

Two Leyden jars of capacity 1 m. are charged up gradually through two water resistances of the order of $10^6 \Omega$ to the potential $+ V_1$ and $- V_2$ respectively, and discharged at the gap $G$ in one of the two types

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of spark, that is the ordinary zigzag spark and three-part spark. For the latter case the needle point $N$ is attached to the positive side and for the former it is removed. The impulses produced by either of these two sparks are transmitted to two photographic plates $P_1$ $P_2$ and produce the positive and negative Lichtenberg's figures at the same time on these plates. $E$ is a Wommelsdorf's condenser machine of five plate type, the diameter of the disc being 55 cm., and the precautions for preventing leakage current and other troubles are the same as described in our former report\(^{(2)}\).

As was described repeatedly in our previous reports, it is necessary to work with a gap length much longer than the diameter of the electrode sphere, in order to get these types of spark form. For a longer gap, however, the potential required is too high and the Lichtenberg's figures are developed in the form of too much irregular brush discharge or canal formation. Thus, we confined the diameter of the electrode sphere to 1.5 cm., 2.0 cm. and 2.5 cm., the gap length being 3.6 cm., 4.6 cm. and 4.8 cm. respectively, which is the most favourable condition for getting those types of spark form.

Figs. 2a and 3a show the typical examples of the zigzag spark and the three-part spark obtained with gap length 3.6 cm. and diameter of electrode sphere 1.5 cm., and Figs. 2b, c and 3b, c are the Lichtenberg's

On Some Relations Between the Form of Lichtenberg's Figure

figures produced by their impulses. At first sight, the negative figure produced by three-part spark is decidedly smaller in size than the one produced by the zigzag type, corresponding to the same gap length, and the positive figure is also smaller but to a less extent. As for the nature of pattern, we do not notice much difference in the positive figures, but in the negative figure, comparing Figs. 2c and 3c, we notice three marked differences of characteristics; (i) The radiating stripes are much finer in the case of three-part spark; (ii) The central part with no stripe is larger in area and the whole portion looks as if illuminated uniformly; (iii) A faint trace of the positive figure of irregular form is developed in this central portion, as can be seen in Figs. 3c, 6c, 8c and shown by the annexed diagram (Fig. 4). This irregular positive figure in the central portion of negative figure is developed sometimes also in the case of figure excited by a zigzag spark, but to a decidedly less extent.

These characteristics both in the size and form of the discharge pattern are seen also in the case when we use the electrode spheres of 2.0 cm. in diameter, as shown in Figs. 5a-c and 6a-c. In this case we have sometimes one or two positive canals radiating from the centre of the positive figure (Fig. 6b).

With the electrode spheres of 2.5 cm. in diameter (Figs. 7 and 8), the zigzag spark gives always a canal formation both in positive and negative figures, when we work in a range of electrode distance where these types of spark form appear. But the three-part spark usually gives positive canals only, while the negative figure presents markedly the three characteristics described above. With the electrode spheres of 3.0 cm. in diameter and a suitable gap length, the potential seems
high enough, so that the three-part spark also gives both positive and negative canal formations, as shown in Figs. 9b and c. The negative canal of the figure excited by the three-part spark is of different nature than that excited by zigzag one (compare Figs. 7c and 9c). This point will be discussed later.

In order to make a closer study of these photograms, $D_+$, $D_-$, $d_+$, $d_-$ marked in Fig. 4 were measured, and also the number $n$ of the stripes cutting a circle of which the diameter is of the mean value of $D_-$ and $d_-$ was counted. The results are tabulated in Table I.

**Table I.**

(a) Dia. of electrode sphere = 1.5 cm., gap length = 3.6 cm.

<table>
<thead>
<tr>
<th></th>
<th>$D_+$</th>
<th>$d_+$</th>
<th>$D_-$</th>
<th>$d_-$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zigzag spark</td>
<td>4.8</td>
<td>1.8</td>
<td>8.1</td>
<td>0.62</td>
<td>66</td>
</tr>
<tr>
<td>4.4</td>
<td>2.1</td>
<td>3.0</td>
<td>0.8</td>
<td>73</td>
<td></td>
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<tr>
<td>4.5</td>
<td>1.9</td>
<td>2.8</td>
<td>1.0</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>Three-part spark</td>
<td>3.9</td>
<td>1.7</td>
<td>2.4</td>
<td>1.15</td>
<td>99</td>
</tr>
<tr>
<td>3.85</td>
<td>1.8</td>
<td>2.3</td>
<td>1.4</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>3.6</td>
<td>1.8</td>
<td>2.3</td>
<td>1.3</td>
<td>98</td>
<td></td>
</tr>
<tr>
<td>3.8</td>
<td>1.8</td>
<td>2.4</td>
<td>1.2</td>
<td>91</td>
<td></td>
</tr>
<tr>
<td>Zigzag spark</td>
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<td>1.9</td>
<td>3.0</td>
<td>0.8</td>
<td>75</td>
</tr>
<tr>
<td>Three-part spark</td>
<td>3.8</td>
<td>1.8</td>
<td>2.35</td>
<td>1.3</td>
<td>99</td>
</tr>
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(b) Dia. of electrode sphere = 2.0 cm., gap length = 4.6 cm.

<table>
<thead>
<tr>
<th></th>
<th>$D_+$</th>
<th>$d_+$</th>
<th>$D_-$</th>
<th>$d_-$</th>
<th>$n$</th>
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<tr>
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<td>2.3</td>
<td>3.3</td>
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<td>78</td>
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<tr>
<td>5.0</td>
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<td>3.1</td>
<td>1.0</td>
<td>77</td>
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<tr>
<td>5.4</td>
<td>2.5</td>
<td>3.3</td>
<td>1.2</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>Three-part spark</td>
<td>4.6</td>
<td>1.6</td>
<td>2.6</td>
<td>1.3</td>
<td>82</td>
</tr>
<tr>
<td>4.4</td>
<td>2.1</td>
<td>2.7</td>
<td>1.6</td>
<td>93</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>2.0</td>
<td>2.6</td>
<td>1.5</td>
<td>106</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>1.6</td>
<td>2.4</td>
<td>1.2</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td>4.4</td>
<td>2.1</td>
<td>2.7</td>
<td>1.1</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>Zigzag spark</td>
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<td>2.3</td>
<td>3.2</td>
<td>1.0</td>
<td>89</td>
</tr>
<tr>
<td>Three-part spark</td>
<td>4.4</td>
<td>1.9</td>
<td>2.6</td>
<td>1.3</td>
<td>88</td>
</tr>
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</table>
II. PROVISIONAL DISCUSSION

In most cases, when we excite a spark at a gap, one of the electrodes is connected to earth so that the potential of the other electrode is definite and equal to the potential difference between those two electrodes. Again, when we use a transformer as a source, we can put one point in the source to earth so that the potential distribution of the electrodes is definitely determined. But in our present case, referring to Fig. 1, the equal quantity of positive and negative electricity separated at $E$ charge up gradually two condensers $C_1$ and $C_2$. Although $C_1$ is equal to $C_2$, the leakage currents of positive and negative sides of the condenser machine and the leads are not the same owing to the unsymmetry of machine and the condition of circuit, then $V_1$ is not equal to $V_2$ and their absolute values cannot be determined by the length of the gap. This difficulty itself, however, is one of the reasons why a statical machine gives such a copious variety of types of spark, unravelling of which was our original aim.

When we have a spark at $G$, the impulses of $-V_1$ and $+V_2$ will produce negative and positive figures respectively. In this case if the capacity of the photographic plate, or capacity between $M$ and point lead, is negligible compared with $C$, then we can estimate the values of $V_1$ and $V_2$ from the size of negative and positive figures. The capacity of the photographic plate, however, is not constant but varies to a considerable extent during the course of formation of the figure. It is better to say that we can easily estimate the potential if the quantity of electricity consumed during the formation of figure is negligible compared with that stored in the Leyden jar, which is the condition used by almost all workers in this line. Unfortunately, this is not the
present case. According to Przibram(3) a positive figure of 1.7 cm. radius consumes 230 e.s.u. and a negative figure of 0.65 cm. radius needs 88 e. s. u., both of them corresponding to 11.4 kV. This quantity $Q$ increases with $V$ and is proportional to $V^k$, where $k$ is between 2 and 3. From a determination of absolute potential $V_1, V_2$ by a modified gold-leaf electrometer(4) we learn that they are of the order of 30 kV. Then in our present case it will be between $5 - 2 \times 10^3$ e. s. u. for the positive figure. The electricity stored in the Leyden jar is about $10^4$ e. s. u., compared to which the electric quantity consumed by those figures tabulated in Table I is by no means negligible. This point is clearly seen also from Töpler's studies on canal formation(5). Replacing the Leyden jar with that of 30 m. capacity, we obtained well developed canals with a gap length less than half the present case, the electrode spheres being the same.

From these considerations it is undoubtedly simpler to use Leyden jars of much larger capacity than the present case and work under lower potentials. The sparks, to be examined, however, are difficult to be obtained with Leyden jars of large capacity, because the power of the source is not high enough for that purpose. Then we must understand that, for example, Figs. 2, 3, 5 are the figures in which canal formation was suppressed by the lowering of the impressed voltage due to the capacity of the figure. Comparing Figs. 6b with 5b, the former is smaller in size but has a canal. In Table I (a) we see that in three examples of positive figure excited by a zigzag spark there is no case of canal formation, while in case excited by a three-part spark two out of four have canal formation, although the size of $D_+$ is about 20% smaller. In Table I(b) the former case has one example of canal

(3) PRZIBRAM: Handbuch der Physik XIV. p. 400.
(4) Will be published in Sparks, Part VIII, in which the case of $d=3.0$ cm. was chiefly examined.
(5) Töpler, Ann. d. Phys. 21 (1906) 193. According to his results, a canal formation begins at a critical voltage $U_0$, when the capacity is large enough, and

$U_0(+) = 45\sqrt{a}$ kV, $U_0(-) = 48.5\sqrt{a}$ kV

where $a$ is a thickness of glass in cm. With our photographic plate, $U_0(+) = 18$ kV, $U_0(-) = 20$ kV which are much lower potentials than our present case.
formation out of three, and the latter case three out of five, the size $D_+$ being about 15% smaller. In these cases, therefore, the canal formation is not a problem of a critical voltage, but a question of the mode of suppression of the impressed voltage and a concentration of discharge in a particular path.

An experiment was made relating to this point. With a similar arrangement but a rectangular pole on the photographic plate, two sets of positive figures were obtained. One set is in ordinary air and the other set is in the air saturated with the vapour of acetylene-trichloride at room temperature, all the arrangements being kept the same for one pair of experiments. Using electrode sphere of 2.5 cm. diameter at the exciting gap, which is always in atmospheric air, the length of the gap $l$ was varied. With $l=1.8$ cm., both of the figures had no canal, although the potential exceeded $U_0(+)\text{ for large capacity, }$. Increasing $l$ to 2.42 cm. the figure obtained in the air saturated with the vapour had three positive canals, but in the figure obtained in atmospheric air there was no trace of canal formation. Thus we had no canal formation in atmospheric air even if we increase the exciting gap length to 4.02 cm., while the corresponding figure in the other case represented more than ten canals (Figs. 10 a, b). This will be explained by assuming that the organic molecules adsorbed on the film give rise to a heterogeneous character on the film, thus facilitating a concentration of discharge in a particular path.

As for the size of the figure, Table II shows that the negative figure is about 25% and the positive figure about 20% smaller in case

<table>
<thead>
<tr>
<th>Table II.</th>
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<tbody>
<tr>
<td>$D_-$ (zigzag) + $D_-$ (three-part)</td>
</tr>
<tr>
<td>$D_+$ (zigzag) + $D_+$ (three-part)</td>
</tr>
<tr>
<td>$D_+$ (zigzag) + $D_+$ (three-part)</td>
</tr>
</tbody>
</table>

of three-part spark. As described above, we cannot consider in this case the diameter of the figure to be proportional to the voltage, but
the sparking voltage $V_1 + V_2$ must be decidedly lower in case of threepart spark, which is in accord with our previous result.\(^{(6)}\) This lowering of potential results to greater part from the lowering of the positive potential, which also agrees with our separate measurement of positive and negative potentials by a special electrometer.\(^{(7)}\)

The manner in which the potential difference falls off during the course of formation of spark may be inferred from the figures excited by the spark. In our previous studies of the three-part spark with rotating plate and quartz-fluorite lens\(^{(8)}\) we found that it is preceded by a preliminary positive and negative brush discharge of pencil form, the duration of which is of the order of $10^{-2}$ sec. or $10^{-3}$ sec., while the preliminary discharge of the zigzag spark was estimated to be shorter than $2 \times 10^{-6}$ sec. Then, the value of $dV/dt$ applied on the photographic plate must be decidedly smaller in the case of the three-part spark. The negative canals seen in Fig. 7c and Fig. 9c are in good agreement with Töpler's classification\(^{(9)}\) of rapid and slow application of potential. With our present arrangement of electric connection it is verified that the characteristics of a rapidly applied voltage (feather form, marked by an arrow in Fig. 7c) was more enhanced when we use the electrode spheres of 6 cm. in diameter at the gap $G$ and a gap length 3.3 cm., which gives more disruptive spark. The spark gives the "split" structure in this case.

As for the irregular positive figure developed in the central portion of a negative figure, it is generally believed that it is due to an oscillation of the circuit. Toriyama considers, however, that it will be due to an after discharge of the surface charge of the insulator plate to the electrode.\(^{(10)}\) This irregular positive figure seems to develop more easily in case when the potential was applied more slowly, as can be seen in Fig. 8 of Müller-Hillebrand's report.\(^{(11)}\) The fact that they

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\(^{(7)}\) Loc. cit., Part VIII.
\(^{(9)}\) Töpler: Archiv f. Elektrot. 10 (1921), 167.
\(^{(11)}\) Müller-Hillebrand: Siemens Zeitschrift, 7 (1927) 605.
appear more markedly in cases when excited by three-part spark is not contradictory to what we might expect.

It is an interesting point that the positive figure of the three-part spark, which is excited by its negative brush at the gap, is smaller in size but is liable to have canal formation. The photograph of the negative brush taken by a rotating plate\(^{(12)}\) shows sometimes a striated form, which is a sign of some intermittent nature of the discharge. It may be supposed that a heterogeneity of discharge in time easily gives rise to a canal formation as in case of a heterogeneity of conductivity of the surface.

This experiment was carried out in The Institute of Physical and Chemical Research, Tokyo, under the guidance of Prof. T. Terada, to whom the writer expresses his hearty feeling of gratitude for his kind suggestion and advice.

\(^{(12)}\) Loc. cit. Sparks, Part VII. Photo. 129 d.
Fig. 2.
\[d = 1.5 \text{ cm}, \quad l = 3.6 \text{ cm}. \quad \text{Zigzag Spark.}\]

Fig. 3.
\[d = 1.5 \text{ cm}, \quad l = 3.6 \text{ cm}. \quad \text{Three-part Spark.}\]

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Fig. 5.
\[ d = 2.0 \text{ cm}, \quad l = 4.6 \text{ cm}. \quad \text{Zigzag Spark.} \]

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Fig. 7.
\[ d = 2.5 \text{ cm}, \quad l = 4.8 \text{ cm} \]
Zigzag Spark.

Fig. 8.
\[ d = 2.5 \text{ cm}, \quad l = 4.8 \text{ cm} \]
Three-part Spark.
Fig. 9.
\[ d = 3.0 \text{ cm}, \quad l = 5.8 \text{ cm} \]
Three-part Spark.

Fig. 10 (a).
\[ d = 2.5 \text{ cm}, \quad l = 4.02 \text{ cm} \]
In ordinary air. \( \times \frac{2}{3} \)

Fig. 10 (b).
\[ d = 2.5 \text{ cm}, \quad l = 4.02 \text{ cm} \]
In air saturated with the vapour of acetylene-tetrachloride. \( \times \frac{2}{3} \)

U. Nakaya: On Some Relations Between the Form of Lichtenberg's Figure.