DIFFRACTION OF FALLING DROP BY PLANE NET. PART I.

By

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CONTENTS.

INTRODUCTION.

Some systematic studies on the sun shaped pattern obtained letting fall a liquid drop on a solid surface have already been made by Mr. Tsutsui in a paper entitled "Rupture Phenomena of Liquid Drop"[1]. These sun shaped patterns and impulsive electric patterns known as Lichtenberg figures seem to belong to the same category in respect to their formation.

However, if a two dimensional plane net is placed at a suitable position between a liquid dropper and the surface of a paper, many splashes through the mesh of the plane net are distributed on it so regularly that is rather surprising. Hereafter we shall call a group of these regularly distributed splashes a "diffraction pattern" of liquid drop by two dimensional plane net. These diffraction patterns as reported in the preceding paper[2], are just analogous, at least in appearance, to those formed by X-rays and electrons passed through crystal.

The present experiments were undertaken with the intention of studying as fully as possible the inter-relation of the shape and size of mesh, volume and velocity of falling drop, and the distance between plane net and surface of paper.

I. ARRANGEMENT OF EXPERIMENTS

The main apparatus used in this experiment is shown in Fig. 1. The distance between the dropper and the plane net is 33.5 cm. in the most experiments. To facilitate the object of the experiment, the relative distances among dropper, net plane and surface of paper are arranged so as to be changeable at will. The water dropper is made from thin walled glass tubing drawn out to various sizes. The other end of the glass tube is connected to a needle valve to regulate the flow of water. The diameter of drop was determined by measuring the number of drops which are required to fill a 10 cc. vessel.

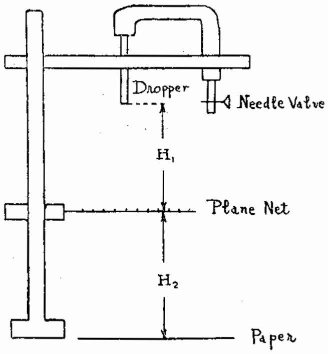


Fig. 1.

The diffracted splashes produced by a water drop are marked on a plane glass which is uniformly covered by fine powder such as lycopodium. The pattern thus obtained perfectly coincides with the one obtained by ink drop in shape and size. Therefore, in order to mark spots on paper a drop of ink is always used instead of a water drop.

The notations and symbols used in the present paper are summarized in the following.

H_1 = the height in cm. of dropper above the plane net.
H_2 = the height in cm. of plane net above the surface of paper.
S = opening of the mesh in cm.
d = diameter of mesh wire.
D = distance between two innermost spots.
d_p = diameter of sun shaped pattern showing the volume of drop.

II. EXPERIMENTAL DETAILS

1. Effects of the shape of mesh on the pattern.

In order to see how the shape of mesh affects the form of the pattern, various networks with different shapes of meshes were used.

(i) Square mesh.

The typical pattern obtained by the square mesh of brass wire is well analogous to the optical pattern of diffraction by four pin holes. Fig. 2. represents the comparison of these two patterns. Many patterns obtained by this mesh are shown in this paper.

(ii) Rhombic mesh.

Shifting the two oblique sides of square mesh of plane net within some certain range, we have obtained a diffraction pattern corresponding to each different form of mesh. Fig. 3 (a), (b), (c) respectively represent the diffraction patterns corresponding to angles of 90°, 65° and 45° between two sides of the mesh. It is remarkable that each spot of the pattern is marked out on the intersecting points of two groups of lines separated by some distance from each other parallel to the two sides of the mesh. Fig. 3 (d) shows the pattern obtained when the angle between these two sides of the mesh is comparatively small.

(iii) Hexagonal mesh.

Fig. 5 shows the pattern obtained with a net-work of cotton fabric with hexagonal mesh, of which one side of the hexagonal is about 1.3 mm. From this photograph it may be clear that the line connecting each spot and the central one of the diffraction pattern intersects perpendicularly the line parallel to one side of the hexagonal.

(iv) Parallel wires.

The diffraction pattern through the parallel wires always has a tendency to distribute perpendicularly to the parallel wires. Some examples of this case are illustrated in Fig. 6.

2. Effects of the opening of mesh and diameter of mesh wire on the pattern.

In the case of square meshes, the relation between D (the distance between two innermost spots of pattern) and S the opening of the mesh was investigated. D seems to increase with S, but the diameter of the wire of the plane net is also an important factor in the effect on the size of these patterns. In this experiment various sized mesh and different diamaters of wire of mesh were used. The data obtained are plotted in Fig. 7, and the relation between D and S is shown in the same figure. From these curves for a given opening the values of D against the diameter of wire are plotted in Fig. 8. (i) When the diameter of the wire is diminished, D is given as D_0. The relation between D_0 and S is perfectly straight as shown in Fig. 9. (ii) K the inclinations of these lines in Fig. 10. is

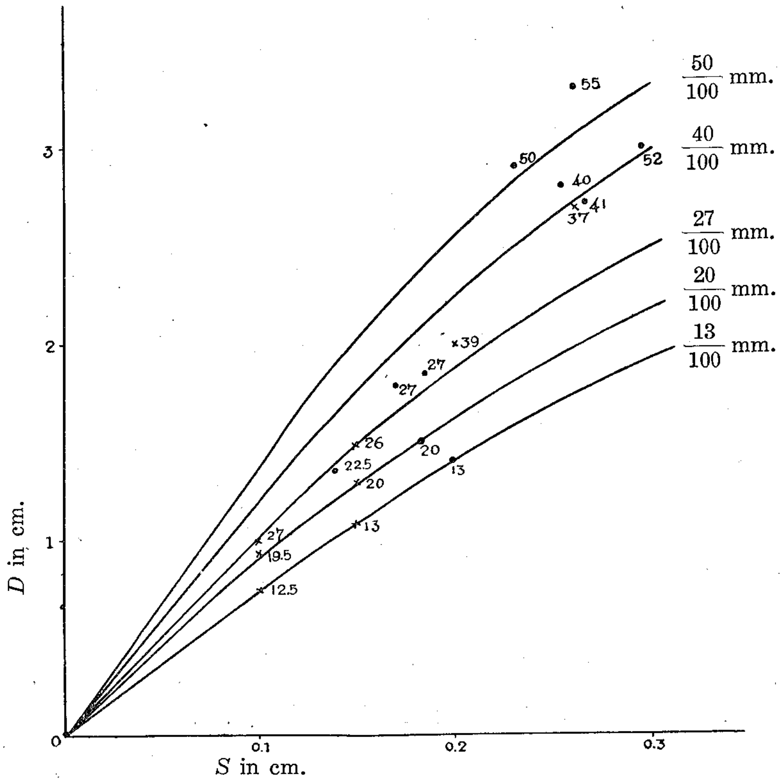
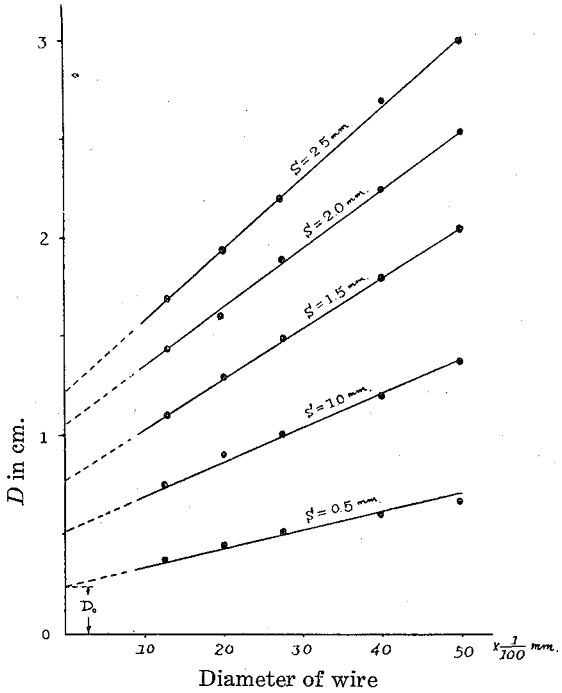
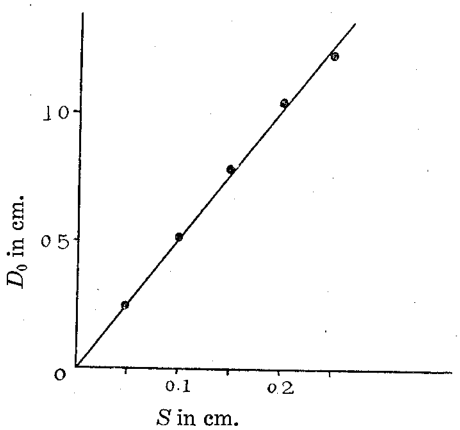


Fig. 7.



3
2
1
0
D in cm.
$s = 2.5$ mm.
$s = 2.0$ mm.
$s = 1.5$ mm.
$s = 1.0$ mm.
$s = 0.5$ mm.
D_0
10
20
30
40
50
$\times \frac{1}{100}$ mm.
Diameter of wire

Fig. 8.



1.0
0.5
0
D_0 in cm.
0.1
0.2
S in cm.

Fig. 9.

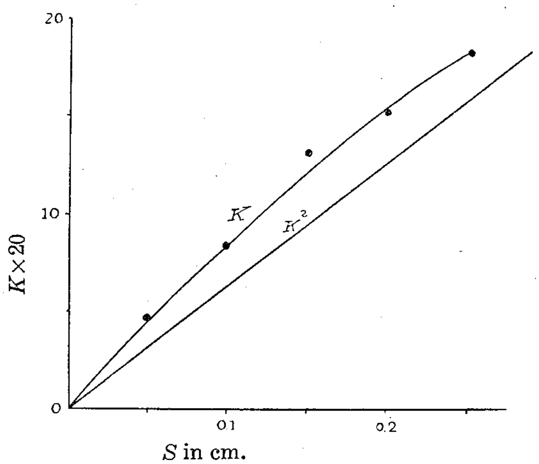


Fig. 10.

propotional to $\sqrt{S}$. Therefore, for given size of drop, D is expressed in the following formula:

$$D = D_0 + Kd$$
$$= kS + K_1\sqrt{S}\,d.$$

However, k is independent of d and K_1 but is related to d_p in $K_1 = a_1 - b_1 d_p$.

Therefore, the formula is rewritten as

$$D = kS + (a_1 - b_1 d_p)\sqrt{S}\,d \quad \dots\dots\dots\dots \quad (1)$$

where d is the diameter of mesh wire and k, a, b, are positive real constants independent of S, v, d_p and d.

Putting $d = 0$ and $D = D_0$,

the expression is simply written

$$D_0 = k\,S \quad \dots\dots\dots\dots \quad (2)$$

This agrees with the fact of (i).

Next, putting d_p = constant, the expression (1) becomes

$$\frac{D - D_0}{k} = K = \text{constant}\,\sqrt{S} \quad \dots\dots\dots\dots \quad (3)$$

This agrees with the fact of (ii).

Lastly, putting $S=$ constant and $d=$ constant, (1) becomes as follows

$$D = \text{constant}_1 - \text{constant}_2\, d \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

This expression indicates the effect of the volume of drop.

3. Effect of the volume of falling drop on the pattern.

When the volume of the drop is changed while the height of fall remains constant, the distance D varies as is shown in Fig. 11. The volume of the falling drop is used conveniently to scale of the diameter of the sun shaped pattern made by the water drop on paper when it was allowed to fall freely through the atomosphere at the same height. The relation[1] $V = k\, d_p^2$ between the diameter of this sun shaped pattern and volume of falling drop is known and the constant k is emperically determined by experiment.

The relation between D and d is linear in table I and it is plotted in Fig. 13. These curves show that D is decreased with the increase of the volume. This relation is fairly expressed by equation (4) of the preceding section. The tendency of this decrease in the case of square mesh is very remarkable when S lies in the range of 0.06 to 0.16 cm. Without this range, this tendency becomes rather small.

In the case when rhombic mesh is used, this variation of D similarly occurs in the pattern.

TABLE I.

Table of value D (cm.).

Square mesh (brass wire) $H_1 = 33.5$ cm.
$H_2 = 9$ cm.

Volume of Drop in d_p	$S_{(mm)}$	2.6	1.59	1.0	0.77	0.5
	$d_{(mm)}$	38/100	15/100	13/100	14/100	9/100
8.5 mm.		3.3	2.4	2.0	1.8	0.95
10.0 mm.		3.1	2.3	1.85	1.7	0.9
16.0 mm.		2.8	1.6	1.3	1.1	0.7
22.0 mm.		2.4	1.0	0.7	0.6	0.5

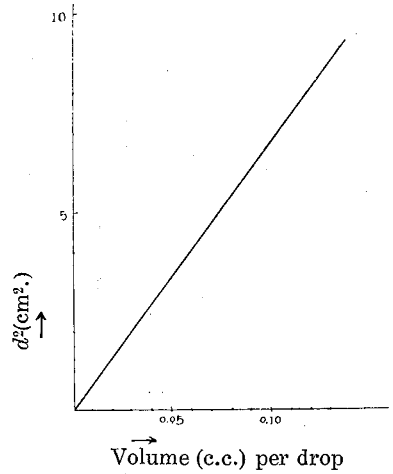

10
5
0.05
0.10
d^2(cm^2.) ↑
Volume (c.c.) per drop

Fig. 12.

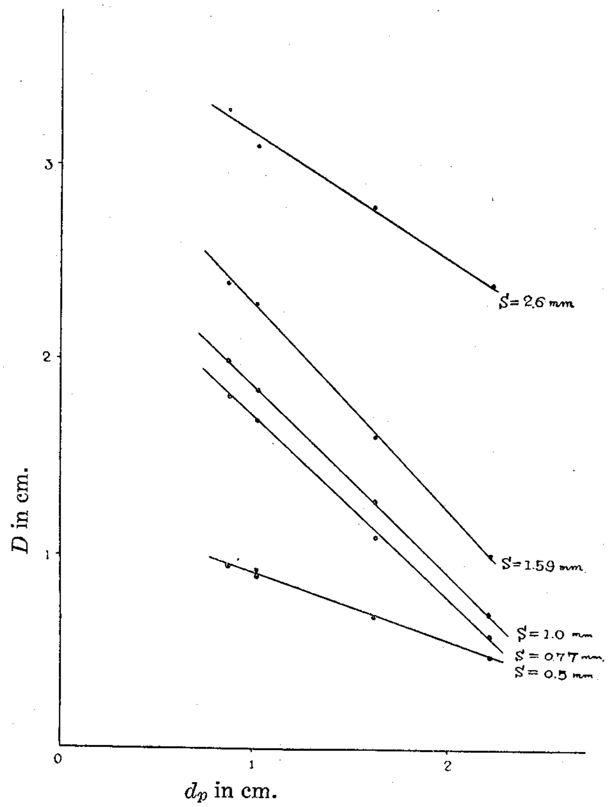

3
2
1
0
1
2
D in cm.
d_p in cm.
S = 2.6 mm
S = 1.59 mm
S = 1.0 mm
S = 0.77 mm
S = 0.5 mm

Fig. 13.

4. The distance between net plane and surface of paper.

It is important to knew the passage of each splash. There are two methods for determining that point; viz., the passage may be photographed by using a high speed cinematograph or, otherwise, it may be judged easily from the series of patterns on paper at different distances under the plane net. The method used in the present paper is the latter one. The relation between D and H_2 is expressed in curve of Fig. 14. The relation between D and H is linear within

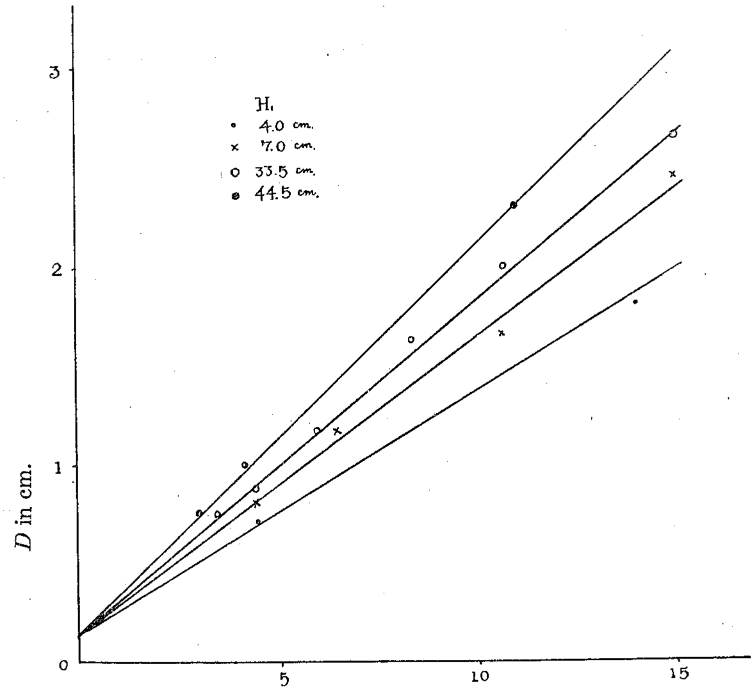


Fig. 14.

the small range of H_2. The value of D at $H_2 = 0$ agrees well with the opening S. Therefore opening S will be determined by the value of D at $H_2 = 0$. Fig. 15 show a good example in this case. Therefore the opening S is easily determined from the pattern. This fact is also analogous to the determination of the gitter constant of given

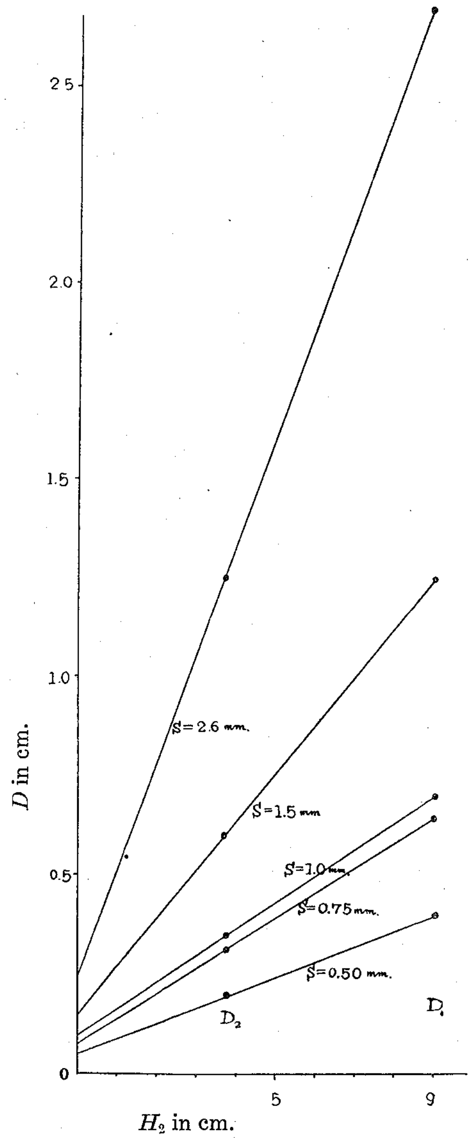


Fig. 15.

crystal from Laue spots in the case of X-ray. From the results of experiments an empirical relation has been deduced, which may be written as follows:

$$D = S + a_1 H_2 - b_1 H_2^2$$

where a_1 is a constant depending upon the velocity of drop for the same liquid and b_1 is a constant depending upon the gravity and air resistance. It will be seen that the above empirical relation agrees fairly well for the distance D within the range of distance H_2 investigated.

TABLE II.

Determination of Opening of Meshes.
$d_p = 20$ mm.

S mm.	D_1 mm.	D_2 mm.
2.6	27	12.5
1.5	12.5	6
1.0	7	3.5
0.75	6.5	3
0.50	4	2

The break-up of a drop into smaller particles may alter the direction of the fall and cause a large decrease in the rate of the fall.

Under the assumption that the air resistance to a particle is proportional to the square of the velocity, one may easily obtain the relation between H_2 and $\frac{D_2-S}{2} = D_1$.

$$H_2 = a_2 D_1 + b_2 D_1^2$$

Curve I in Fig. 17 shows the observed values and curve II in the same Fig. the calculated ones using constants at part A of observed curve. This discrepancy between observed and calculated values is considered to be due to the internal vortex of drop.

If H_2 is comparatively large, one spot is decomposed into several spots as shown in Fig. 18.

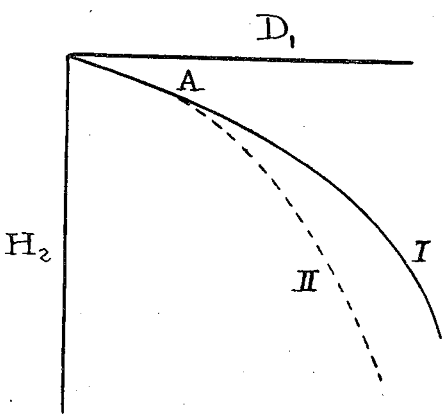


Fig. 17.

5. Effects of the velocity of drop on the pattern.

The velocity of the falling drop can be easily changed by changing the height of its fall. Fig. 19 shows the series of change of pattern with the change of velocity of liquid drop. In order to obtain a fine pattern, it is necessary to have a suitable velocity for drop. However, a drop with very high velocity decreases the degree of fineness and even distorts the form of the figure. Some pattern by high velocity drop is fairly analogous to the pattern by electron beam as shown in Fig. 20. This electron diffraction pattern is obtained by S. Kikuchi and is known as the pattern of thin mica at the intermediate state from L-type to N-type.

TABLE III.

$$D = 0.143 + H_2 a - b\ H_2^2$$

H_1	$\sqrt{H_1}$	a	b
4	2	0.1237	0.93×10^{-3}
7	2.65	0.1497	0.84×10^{-3}
33.5	5.8	0.1687	0.865×10^{-3}
44.5	6.67	0.1967	0.75×10^{-3}

If H_2 is small, a is almost equal to $\frac{D}{H_2}$.

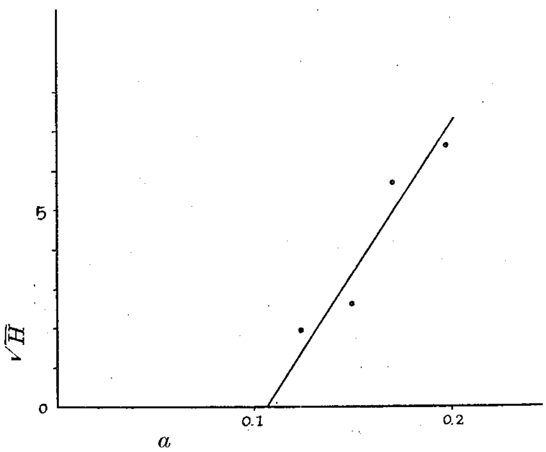


Fig. 21.

Generally D becomes larger with increase of falling velocity of drop. The ratio $\frac{D}{H_2}$ seems to increase with the increase of falling velocity within some experimental range. Neglecting the air resistance, the velocity of drop can be simply calculated from its distance of fall. The relation between $\frac{D}{H_2} = a$ and $\sqrt{H_1}$ is plotted in Fig. 21.

6. The case of mercury drop.

It is easy to suppose that the mode of this diffraction phenomena depend upon the physical properties such as viscosity and surface tension as used in the present experiment. As first example of these effects, a mercury drop was used with diameter of 0.27 cm. and falling height of 33.5 cm. The surface tension and density of mercury were 470 and 13.6 respectively, while those of water were 77 and 1.

A mercury drop is let fall through meshes of plane net onto a thin glass plate which is uniformely strewn with a lycopodium powder. The splashes of the mercury drop blow off the powder at the points when they strike. The patterns thus obtained are almost the same as the case of water in form and are shown in Figs. 22 (a) and 22′ (a). Patterns obtained by water drops of the same volumes are also shown in Figs. 22 (b) and 22′ (b). The comparison in size is shown in the following Table.

Diameter of wire	Opening S	D		$\frac{D_1}{D_2}$
		Mercury drop (D_1)	Water drop of same volume (D_2)	
20/100 mm.	2.6 mm.	34 mm.	22 mm.	1.48
20/100 mm.	1.0 mm.	22 mm.	15 mm.	1.47

III. GENERAL CONSIDERATIONS

1. Classification of patterns.

In the case when a drop is let fall on squre meshes, the patterns obtained are chiefly classified in the following three different types:

(i) Odd type.
(ii) Even type.
(iii) Combined type.

(i) Odd type.

A typical example of this type is shown in Fig. 23. It is obtained when the centre of the drop falls on the centre of one of the meshes. The dotted circles of the left side in Fig. 24 indicate the size of the drop and full lines the mesh of the net. If the size of the drop is equal to circle 1, the spots obtained is of course only spot 1 on the right hand in that figure. If the volume of the drop is increased to circle 2, the spots of pattern obtained become spot 1 and the spots numbered 2. Generally, if the volume of drop is increased to circle n, the obtained pattern consists of spot 1, spots 2, ..., spots n. Of course the distance between the two innermost spots decreases with the increase of volume of drop.

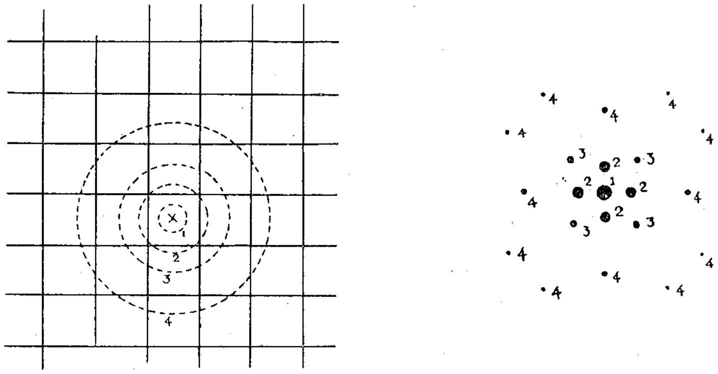


Fig. 24.

(ii) Even type.

This type is obtained when the centre of drop alights on the intersecting point of meshes. The mode of developing of pattern in this type is shown in Fig. 15. The numbers of circle and spots have the same meaning as described in the odd type. The example of this type is shown in Fig. 26.

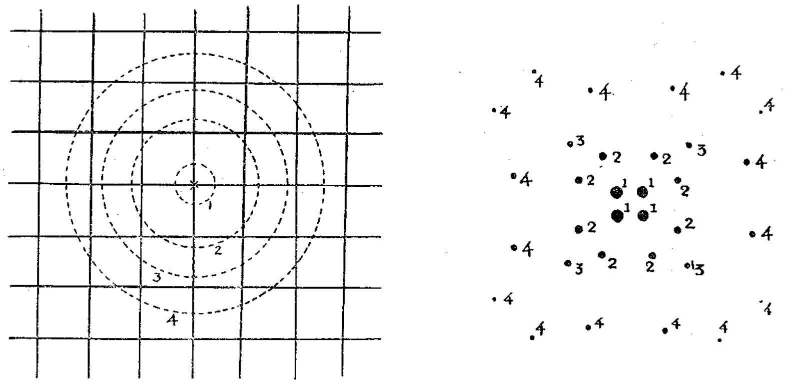


Fig. 25.

(iii) Combined type.

This type is obtained when the centre of the drop falls on the cross of mesh as in Fig. 27. The mode of developing of pattern in this type is shown in Fig. 27· Some examples of this type are shown in Fig. 28.

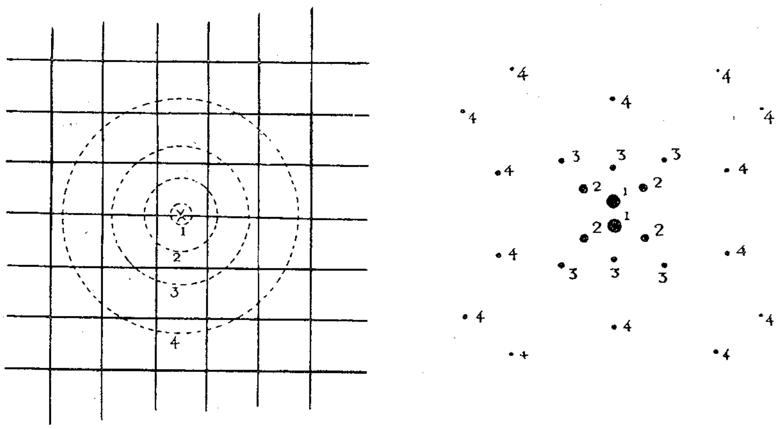


Fig. 27.

From this classification of patterns, one can easily conclude that a drop of water is split into smaller splashes by the mesh wire and pattern is produced.

2. The Distance-Area relation of pattern.

The distance between each spot and the central point of the pattern which corresponds to the centre of the drop seems to depend on the area A of each spot. Therefore, in the case of square mesh this distance is plotted against A in Fig. 29. If A is replaced by $\sqrt{A}$,

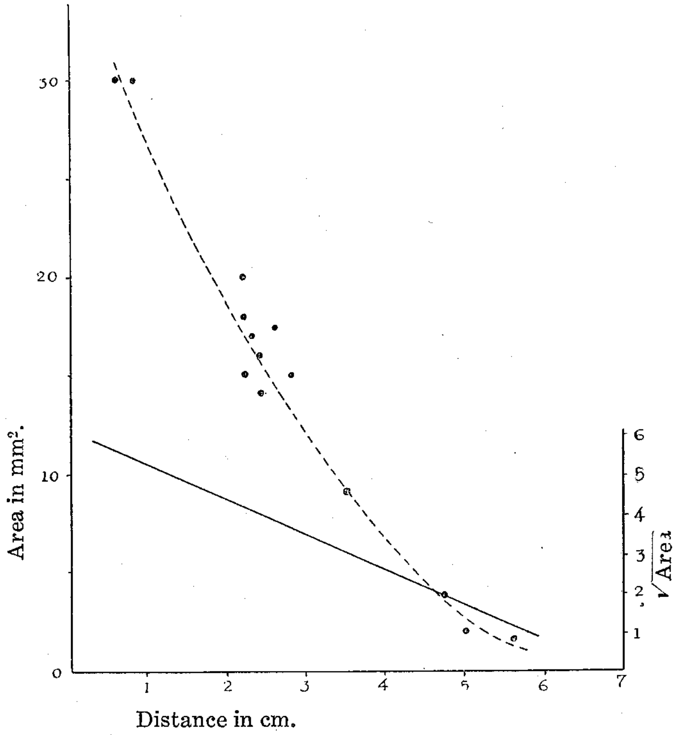


Fig. 29.

the dotted curve will be changed into a full straight line. From this curve, it is evident that the position of spots will be so distributed that the smaller the volume of the splash becomes, the greater is the distance of the spots from the centre of the pattern. Next, examining this relation as to the outtermost spots of the pattern in the case of square mesh one obtains the same results.

3. Mechanism on the formation of the pattern.

The decrease of D with increase of volnme chiefly depends on the decrease of ratio of magnitude of horizontal momentum to that of vertical momentum acting on the water drop at net position. The vertical momentum is esteemed to be approximately that due to the gravity of the falling drop while the horizontal momentum is considered as the sum of (i) reaction of drop on the surface of mesh wire, (ii) viscosity and surface tension of drop and partially (iii) internal vortex of drop. If the several meshes of net are previously covered with a thin film of water, no fine pattern has been obtained. From this fact, the surface tension of liquid must be an important factor. Though the effect is not so large, the existence of the internal vortex of drop was recently confirmed by Ahlborn.(3) However, the mechanical reaction on wire seems to be the most important one as the origins of horizontal momentum. We can roughly explain the mechanism of the formation of pattern by this reaction in the following manner. If the surface of drop comes into contact with No. 1 wire, the reaction on that wire will take an upward direction. Therefore, the horizontal momentum in liquid drop is produced by the combination of this reaction and falling vertical momentum. We assumed that the reaction on a point of surface of wire is represented by a function that is increased with the thickness of the liquid drop on that point. If a falling drop comes into contact with No. 2 wire to the left side of No. 1 wire, the reaction is produced on the surface of No. 2 wire. Therefore the reaction produced by No. 1 and No. 2 wires will determine the splitting direction of the splashes with combined vertical momentum of drop.

Firstly, if the distance between No. 1 and No. 2 wires or the opening S is enlarged while the volume of drop is kept constant, the difference between the horizontal components of the reactions produced by No. 1 and No. 2 wires become greater and therefore the spots will be farther separated from the centre of pattern. Next, if the volume per drop is increased while the opening S is kept constant, the difference between the horizontal components of reaction at these two points is decreased and the distance between the two innermost spots is shortened. Thirdly, if the diameter of mesh wire is increased, the distribution of reaction is broadened and therefore the increase of horizontal component of momentum makes more widely separate the distance between the two innermost spots of the pattern. All these effects are confirmed by the experiments that are precisely explained in 2 and 3 of article II of the present paper. The writer does not think that the mechanism above described explains sufficiently the formation of pattern but it seems to represent approximately the true process of these phenomena.

The photograms of phenomena at moment of the destruction of falling drop by plane net, which can be obtained by high speed cinematograph, will give probably definite clues to the mechanism of the formation of these patterns.

When the net and falling drop respectively are positively or negatively charged, somewhat different phenomena are expected.

SUMMARY

Some interesting results obtained in the present experiment are briefly summarized as follows:

1. It is possible to determine the shape of mesh from an obtained pattern.

2. The relation between D (distance between two innermost spots) and size of mesh is found.

3. The distance D is decreased with the increase of volume and it is linearly related to the square root of volume of drop.

4. The velocity of drop also has important effects on the form and fineness of the pattern.

5. It is easy to determine the opening of mesh of net from two patterns at two different small distances below the plane net.

6. The Distance-Area relation in the distribution of spots of he pattern is a very interesting fact.

7. In the case of square mesh, many patterns are easily classified into three types.

8. The mechanism of the formation of these patterns is given in the last section of the present paper.

In conclusion the writer wishes to express his best thanks to Prof. Y. IKEDA under whose kind guidance and encouragement the present experiments were carried out.

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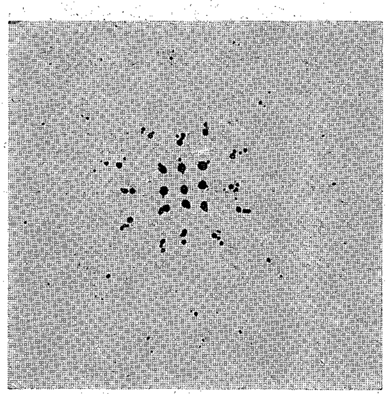
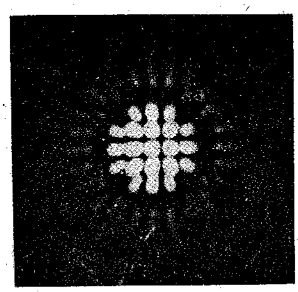
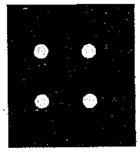
(3) Ahlborn: Zur Dynamik des Regens. Phy. Zeit. 32 Jahr. Nr. 3. 1931, S. 139

$\times\frac{1}{3}$



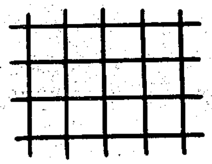


Optical Pattern.



Pin Holes.

Fig. 2.



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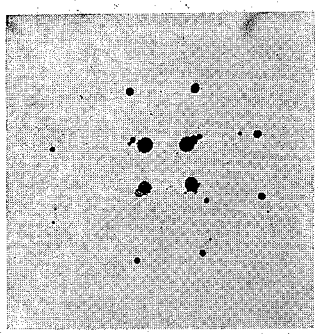
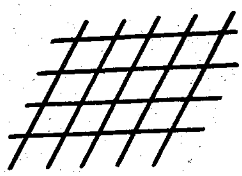


(*a*)



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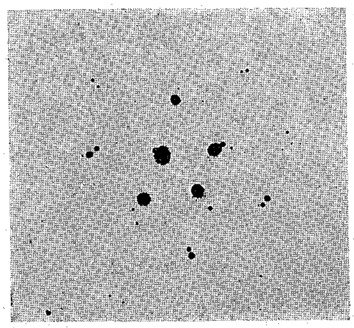

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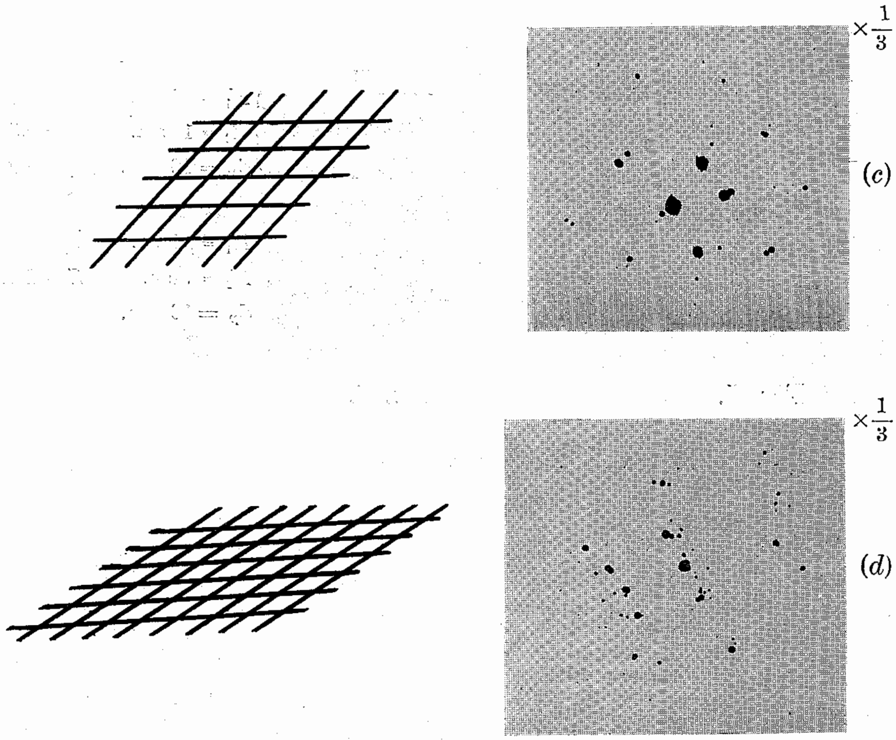


Shape of Mesh. Pattern.

$H_1 = 33.5$ cm. $H_2 = 11$ cm. $S = 1.45$ mm. $d_p = 16$ mm.
Brass wires.

Fig. 3.

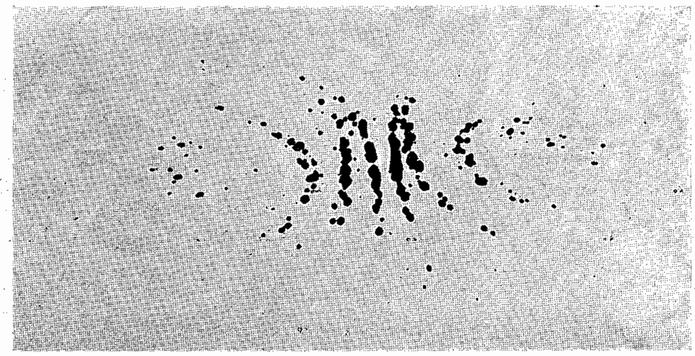


Fig. 4.

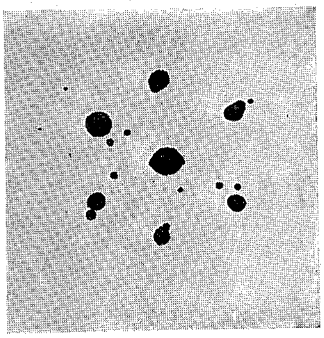
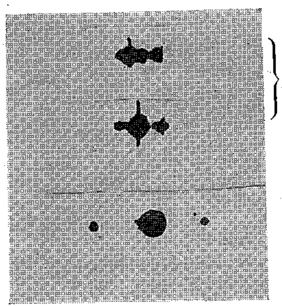


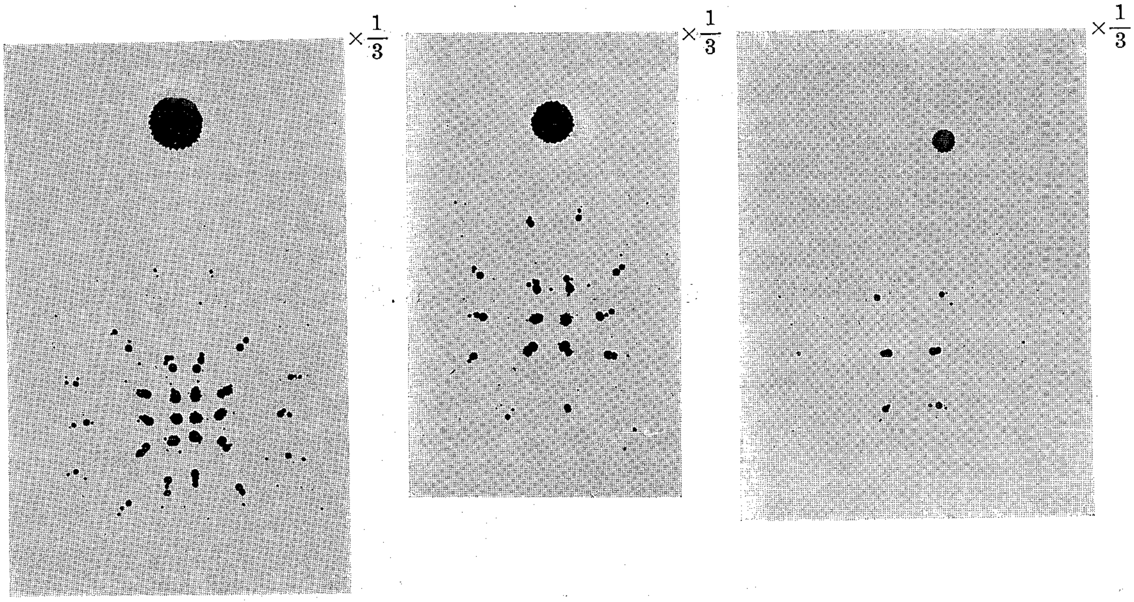
Fig. 5.



$\left.\begin{array}{l} H_1 = 15\text{ cm.} \\ H_2 = 15\text{ cm.} \\ d_p = 16\text{ mm.} \end{array}\right\}$ $S = 0.5$ mm.

$H_1 = 15$ cm.
$H_2 = 7$ cm. $S = 2$ mm.
$d_p = 16$ mm.

Fig. 6.



$d_p = 21$ mm. $d_p = 16$ mm. $d_p = 8.5$ mm.

$H_1 = 33.5$ cm. $H_2 = 9$ cm. $S = 1.0$ mm. $d = 13/100$ mm.

Brass wires.

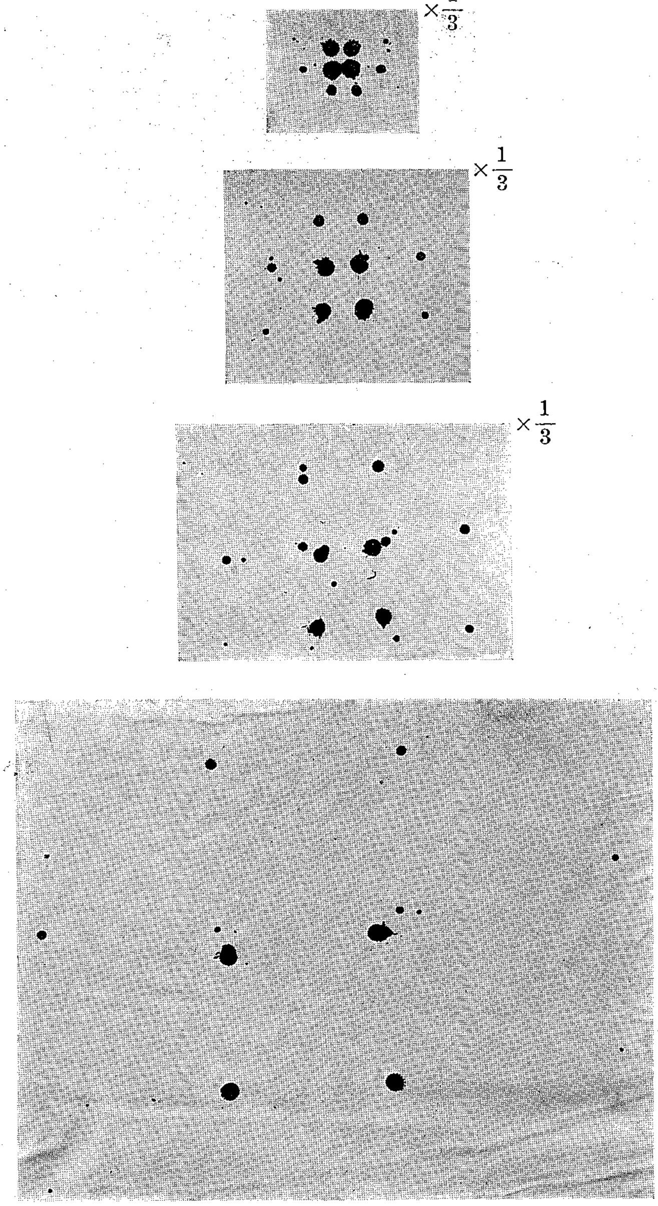
Fig. 11.

$H_2 = 3$ cm. $\times \frac{1}{3}$

$H_2 = 8.4$ cm. $\times \frac{1}{3}$

$H_2 = 15$ cm. $\times \frac{1}{3}$

$H_2 = 37$ cm. $\times \frac{1}{3}$

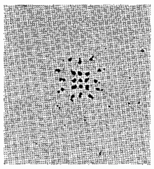


diameter of drop = 4.14 mm. ($d_p = 16$ mm.)

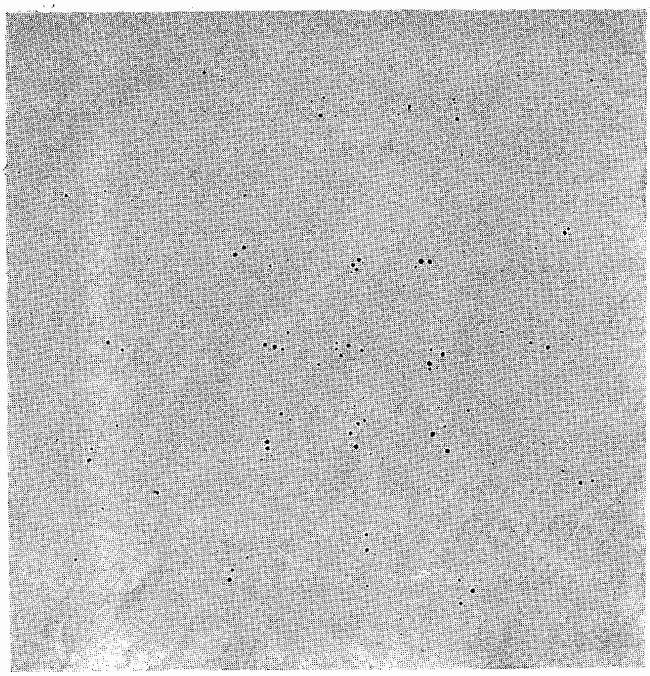
$S = 1.43$ mm. $d = 15/100$ mm. $H_1 = 33.5$ cm.

Brass wires.

Fig. 16.



$\times\frac{1}{3}$



$\times\frac{1}{5.5}$

$H = 4.4$ cm. $H_2 = 83$ cm.

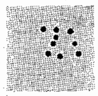
diameter of drop $= 4.14$ mm. ($d_p = 16$ mm.)

$S = 0.83$ mm. $d = 14/100$ mm. $H_1 = 33.5$ cm.

Brass wires.

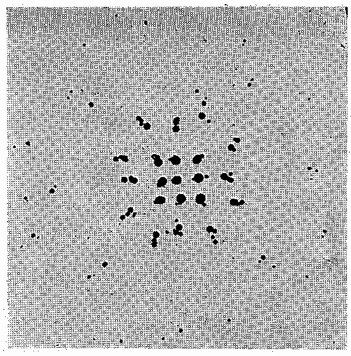
Fig. 18.

$H_1 = 3.5$ cm.



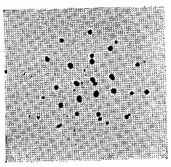
$\times\frac{1}{3}$

$H_1 = 33.5$ cm.



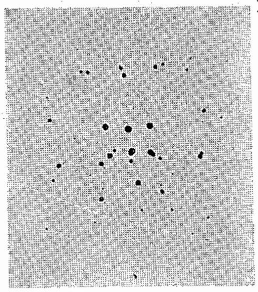
$\times\frac{1}{3}$

$H_1 = 6.5$ cm.



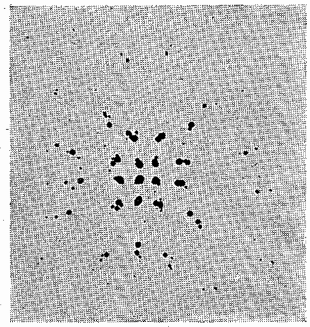
$\times\frac{1}{3}$

$H_1 = 17$ cm.



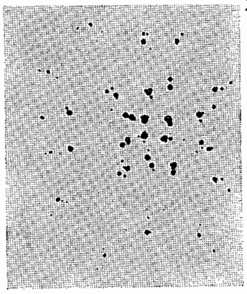
$\times \frac{1}{3}$

$H_1 = 37.5$ cm.



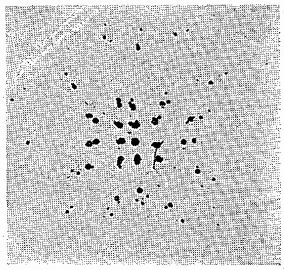
$\times \frac{1}{3}$

$H_1 = 26{\cdot}5$ cm.



$\times \frac{1}{3}$

$H_1 = 43.5$ cm.

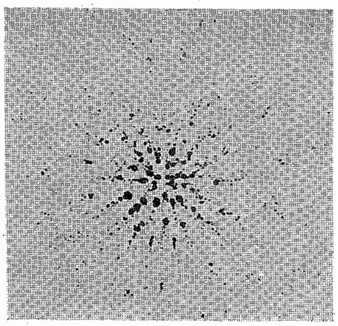


$\times \frac{1}{3}$

diameter of drop = 4.14 mm.

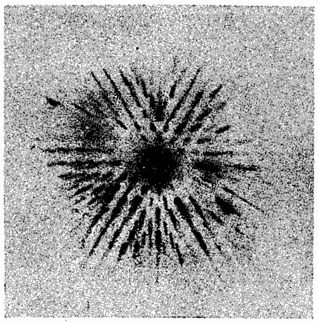
$S = 0.8$ mm. $H_2 = 7$ cm. Brass wires.

Fig. 19.



$\times \frac{1}{3}$

$S = 0.5$ mm.
$H_1 = 222$ cm.
$d_p = 16$ mm.
Brass wires.

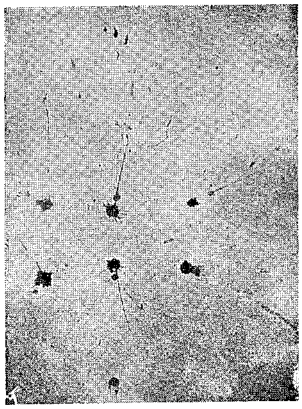


Kikuchi's pattern.

Fig. 20.

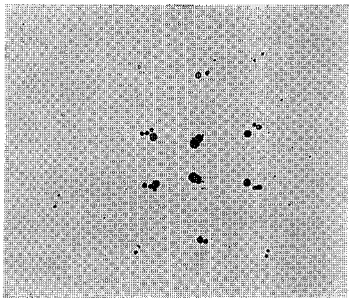
$S = 1.0$ mm. Brass wires.
$H_1 = 33.5$ cm.
$H_2 = 9$ cm.

diameter of drop $= 2.7$ mm.



$\times \frac{1}{3}$

(a) mercury drop



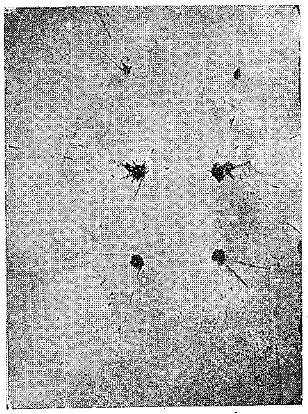
$\times \frac{1}{3}$

(b) water drop

Fig. 22.

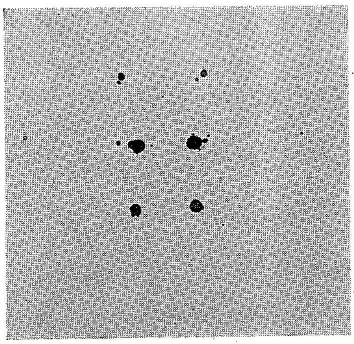
$S = 2.6$ mm. Brass wires.
$H_1 = 33.5$ cm.
$H_2 = 9$ cm.

diameter of drop $= 2.7$ mm.



$\times \frac{1}{3}$

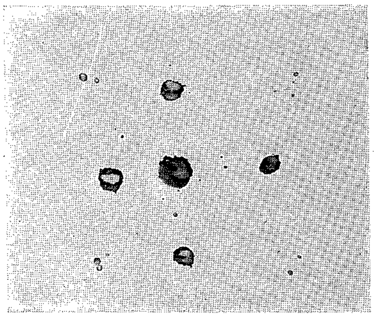
(a) mercury drop



$\times \frac{1}{3}$

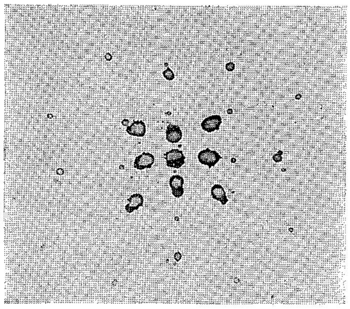
(b) water drop

Fig. 22′.



$\times\frac{1}{3}$

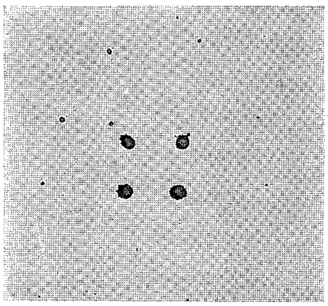
$S = 2.6$ mm. Brass wires.
$d_p = 22$ mm.



$\times\frac{1}{3}$

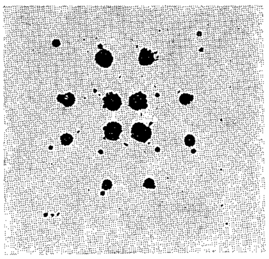
$S = 1.5$ mm. Brass wires.
$d_p = 22$ mm.

Fig. 23.



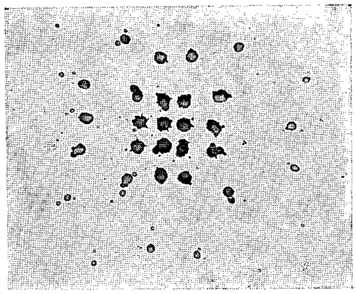
$\times\frac{1}{3}$

$S = 1.5$ mm. Brass wires.
$d_p = 13$ mm.



$\times\frac{1}{3}$

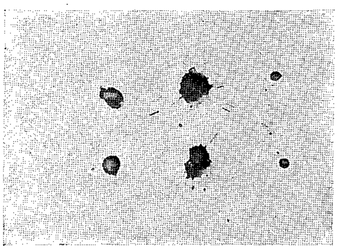
$S = 1.0$ mm. Brass wires.
$d_p = 21$ mm.



$\times\frac{1}{3}$

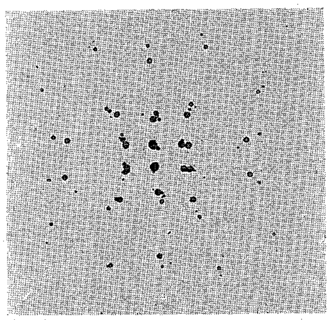
$S = 1.0$ mm. Brass wires.
$d_p = 26$ mm.

Fig. 26.



$\times\frac{1}{3}$

$S = 2.6$ mm. Brass wires.
$d_p = 22$ mm.



$\times\frac{1}{3}$

$H_1 = 33.5$ mm.
$H_2 = 9$ cm.

$S = 0.8$ mm. Brass wires.
$d_p = 12$ mm.

Fig. 28.