Simultaneous Observations of the Mass, Falling Velocity and Form of Individual Snow Crystals.*

By

Ukitiro NAKAYA and Tôiti TERADA, Jr.

(Plates I–III)

I. Introduction.

It is well known that snow falls in the state of flakes when the climate is not so cold. The snow flake consists of many snow crystals gathered together, and it is considered to be produced by the adhering of individual snow crystals in the course of the mutual collision while they are falling. At Sapporo the temperature is fairly low and usually separated crystals are observed more or less mixed with the flakes. At a spot half way up Mt. Tokati where the observation of snow crystals was carried out in the last two winters, the altitude being 1030 metres, the snow was usually observed to fall as individual crystals. There is little doubt that snow grows as individual crystals at a high altitude and falls through a considerable distance before forming a flake. There are some observations on the falling velocity of snow flakes of various dimensions, but the authors are not aware of any data on the velocity of individual crystals. In this experiment the falling velocity was measured for various types of crystals, moreover the mass of those crystals was measured simultaneously, the microphotograph of the crystal being taken at the same time. Most of the experiments were done at Mt. Tokati, the locality and climatic conditions of which place are described in the foregoing paper in this Journal.[1)]

II. Methods of measurement.

The measurements can be safely made when the temperature is considerably below the freezing temperature, say, below $-5°$C. The climatic condition at Mt. Tokati was very convenient for this purpose. The princi-

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* Investigations on Snow, No. 4.

1) U. NAKAYA and K. HASIKURA, This Journal, Vol. 1 (1934) 163.

ple of the measuring method is that all measurements must be done as quickly as possible. Even when the temperature is far below 0°C, the crystal undergoes a transformation, especially the sharp edges or the pointed tips of the crystal are very easily blunted by sublimation due to the action of the surface tension of crystal. The following methods were adopted in order to satisfy the requirement.

i) Method of measuring the mass of single crystal.

The mass of a crystal was obtained from the volume of water after melting the crystal. A snow crystal is received on a glass plate, the surface of which has been previously covered with a thin paraffin sheet of uniform thickness. The crystal is melted by touching the under side of the glass plate beneath the crystal with the tip of a finger. Usually the melting begins from the tips of the pointed branches and the melted water is attracted to the centre of the crystal by capillary action, the whole crystal being melted finally into a hemispherical drop. The drop thus formed takes a hemispherical form of a fair completeness and the volume is easily calculated by measuring the diameter under microscope. This point is confirmed by taking a microphotograph of the side view of the drop, two examples of which are reproduced in Photos. 1 & 2, Plate I. By measuring the diameter and height of the drop, one sees that the accuracy of this method is sufficient for the present purpose. Sometimes a crystal is melted into a small number of drops, especially in case of a plane dendritic crystal of large dimension. In such case the sum of volumes of the drops was taken.

ii) Method of measuring the falling velocity of single crystal.

The falling velocity was measured directly with a stop watch by letting the crystal fall down through a long closed cylinder that is held in a vertical position. The apparatus used is shown in Fig. 1, in which C and D are cylinders made of sheet-iron both of them 12 cm. in diameter. A and B are wooden boxes provided with three glass windows, one in front and two in the sides, the distance between the centres of these two boxes being kept at 2 metres. The falling crystal was observed by the naked eye at the two positions A and B by illuminating from the sides with two portable electric lanterns I_1 and I_2. The velocity was determined with fair accuracy by measuring with a stop watch the time taken for falling the distance AB. E is another wooden box with a hole F at the bottom. The crystal is made to fall through F, which is 1 cm. in diameter, into the long cylinder, and this hole is shut up as quickly as possible after letting the crystal fall, in order to prevent any upward air current. G is

the glass plate covered with a paraffin sheet, which receives the falling crystal. The whole system was set in the open air and the experiments were carried out at Mt. Tokati when the temperature was between −8°C and −15°C.

In case of a graupel or a snow pellet, this method is not suitable and a photographic method is used. For this purpose the apparatus used previously by the authors for the study of the electrical nature of snow particles[2] was again employed. The track of the graupel was photographed by illuminating from both sides with intense beams of arc lamps, and the velocity was determined from the time marks on the image of the track, which were made by rotating an electric fan in front of the lens. An example of those photographs is reproduced in Photo. 3, Plate I. The experiment was carried out at Sapporo.

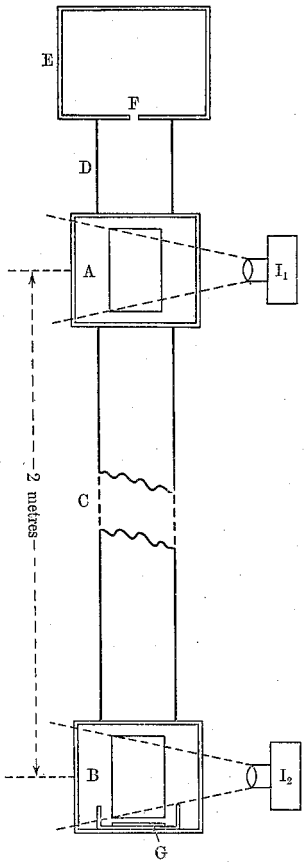

Fig. 1.

iii) The programme of observations.

The point of this experiment is that the microphotographing of a crystal and the determinations of its falling velocity and mass must be carried out within the shortest possible time. The programme of observations is as follows; a suitable snow crystal is chosen among many received on an ordinary glass plate and its microphotograph is taken, the exposure being a few seconds, then the crystal is lifted with a fine splinter and brought to the hole F in Fig. 1, and it is made to fall into the cylinder for the determination of the falling velocity, the crystal being received on a paraffin glass and then melted into a drop for measuring its mass. After a little practice, the whole programme could be carried out in about one minute or one and half. Thus the authors succeeded in the simultaneous observations of the mass, velocity and form of the crystal for various types of snow.

2) U. NAKAYA and T. TERADA, Jr., This Journal, Vol. 1 (1934) 181.

III. The Result of Observation.

The variety of the snow crystals is so great that one must classify them into more than ten types for the complete explanation. The general classification of snow particles will be published in the near future in this Journal.[3] In the present experiment, however, the data are not sufficiently numerous for such a complete classification, and the authors adopted a simpler one as described in the following.

1) Needle. Photo. 10, Plate III.
2) Plane dendritic crystal. The most ordinary plume-like or stellar type of crystal developed in one plane, Photo. 5 *a*, Plate II.
3) Spatial dendritic crystal. Dendritic branches are developed in a spatial distribution. There are two kinds of them; the one has dendritic branches extended in space from a stellar base crystal, Photo. 6 *a*, and the other has all branches radiating in space from the centre, Photo. 7 *a*. These two sorts are grouped as one kind in this paper.
4) "Powder" snow. In a more scientific classification as described in the later communication No. 8, powder snow implies an apparently amorphous type of snow particle, but in this paper "powder" snow means a small crystal of various types, except the plane dendritic form and the needle. For example, a small crystal consisting of a spatial assemblage of plates and branches as shown in Photo. 8 *a*, Plate III, is also included under this term.
5) Dendritic crystal with water droplets. All sorts of crystals, except the needle, with a considerable number of water droplets attached are grouped in this type. A thick plate as shown in Photo. 9 *a* also belongs to this kind.
6) Graupel (snow pellet). The authors consider a graupel as an advanced state of the crystal belonging to 5). A microphotograph representing two graupels is shown in Photo. 11.

The dimension of a crystal is also measured, which means that the diameter is measured of a sphere or circle just enclosing the crystal except needles, in which case the length is meant. The relation between the dimension of a crystal and its mass or falling velocity was examined respectively for the six types of crystals above classified.

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3) U. NAKAYA and Y. SEKIDO, This Journal, Investigations on Snow, No. 8.

a) The relation between the mass and the dimension of crystal.

The radius of drop r was first plotted with respect to the dimension of crystal d. The diagram thus obtained showed that the curve took a form like a parabola in the case of the dendritic crystals and powder snow, while the relation was represented by a straight line in the case of the graupel. In the latter case one can interpret that the mass of a graupel is proportional to the cube of dimension, that is the density of a graupel is nearly constant without regard to its dimension. As for dendritic crystals, it can be assumed that the thickness of crystal is constant for various crystals. In such case the mass is proportional to the square of the dimensions, so that $r^{\frac{3}{2}}$ is proportional to d. This relation holds not only

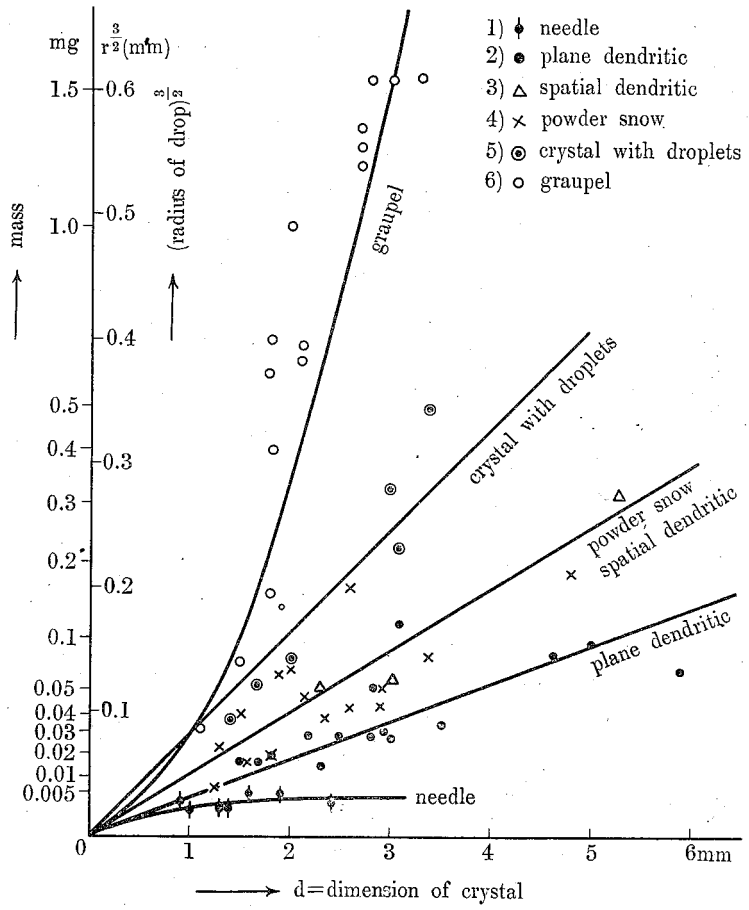

Fig. 2

for plane dendritic crystals but also for spatial ones under the assumption above mentioned. The diagram representing the relation between $r^{\frac{3}{2}}$ and d is shown in Fig. 2. The points are scattered to a considerable extent, but a general tendency is observed so that the points belonging to one kind of crystal are represented as a first approximation by a straight line passing the origin, except for the graupel and the needle. In the figure one can see clearly the order of magnitude of the mass for various types of crystals. The powder snow and the spatial dendritic crystals are of the same order in their masses. The curve for graupel is concave upwards, which is naturally expected from the fact that the mass is proportional to d^3 in this case. The one for the needle-form is concave downwards and this shows that the mass is proportional to d, that is, the thickness is nearly constant for various samples and the mass is proportional to its length. Of course the needles of various sizes are observed, but at present the authors deal with only the needles of ordinary size which are commonly observed in this country.

From the inclinations of the straight lines, the empirical formulae representing the relation between m and d are deduced respectively for the four types of crystals. For graupel the formula is deduced from the diagram of r and d, and for needle it is obtained from that of r^3 and d. The results are as follows;

$m=0.065\ d^3$ for graupels,
$m=0.027\ d^2$ for crystals with water droplets,
$m=0.010\ d^2$ for "powder" snow and spatial dendritic crystals,
$m=0.0038\ d^2$ for plane dendritic crystals,
$m=0.0029\ d$ for needles,

in which m is measured in mg and d in mm. Of course the variety in the form of crystal is so wide-spread that these formulae hold only as a first approximation, but one can know from them the order of magnitude of mass when he measures the dimension of the crystal, which means as above in the case of the ordinary snow and graupel the diameter of the circle or sphere just enclosing the crystal and in case of needle its length.

The density of graupel, which is constant for samples of various sizes, was calcualted from the formula above described and was found that

$$\rho(\text{graupel})=0.125,$$

which indicates that there is a considerable interspace inside a graupel.

b) The mean thickness of a plane dendritic crystal.

The thickness of a plane dendritic crystal is so slight that it is difficult

to measure directly. The mean thickness, however, is easily obtained from the mass and the area of the crystal. The latter was measured by a planimeter on an enlarged photograph and the mean thickness was calculated from the mass for seven samples. The result is shown in Table I.

Table I.

Dimension mm.	Mean thickness mm.	Mean
2.35	0.009	
2.5	0.012	
2.8	0.011	
2.9	0.015	0.011±0.0015
3.0	0.009	
3.5	0.011	
5.0	0.010	

The mean thickness of this type of crystal is nearly constant, being independent of the dimension. The assumption described in the foregoing section is supported by this result. Thus the plane dendritic crystal of snow, at least, observable at Mt. Tokati may be considered to be of the thickness of 0.01 mm.

c) The relation between the falling velocity and the dimension of crystal.

The relation between the velocity and the dimension of a crystal is shown in Fig. 3. In the figure one sees the remarkable phenomenon that the velocities of plane dendritic, powder and spatial dendritic crystals are each independent of their dimensions; that is, the velocity is nearly 30 cm/sec for all dimensions of plane dendritic crystals, 50 cm/sec for powder snows and 57 cm/sec for spatial dendritic ones. When water droplets are attached to the crystals, the velocity increases to the order of 100 cm/sec and in this case the velocity tends to increase with the dimensions. This tendency is most markedly observed for graupels; the velocity of a small particle is about 100 cm/sec, while that of a large one is more than 250 cm/sec. In the case of the needles also a similar phenomenon is observed; that is, the longer needle falls more rapidly than the shorter one, the velocity ranging from 30 cm/sec to 70 cm/sec. The data are not sufficiently numerous but one will be able to estimate from this figure the order of the falling velocity for all sorts of snow crystals.

d) The mean values of d, m, and v.

In order to get a rough estimation of the relative values of these

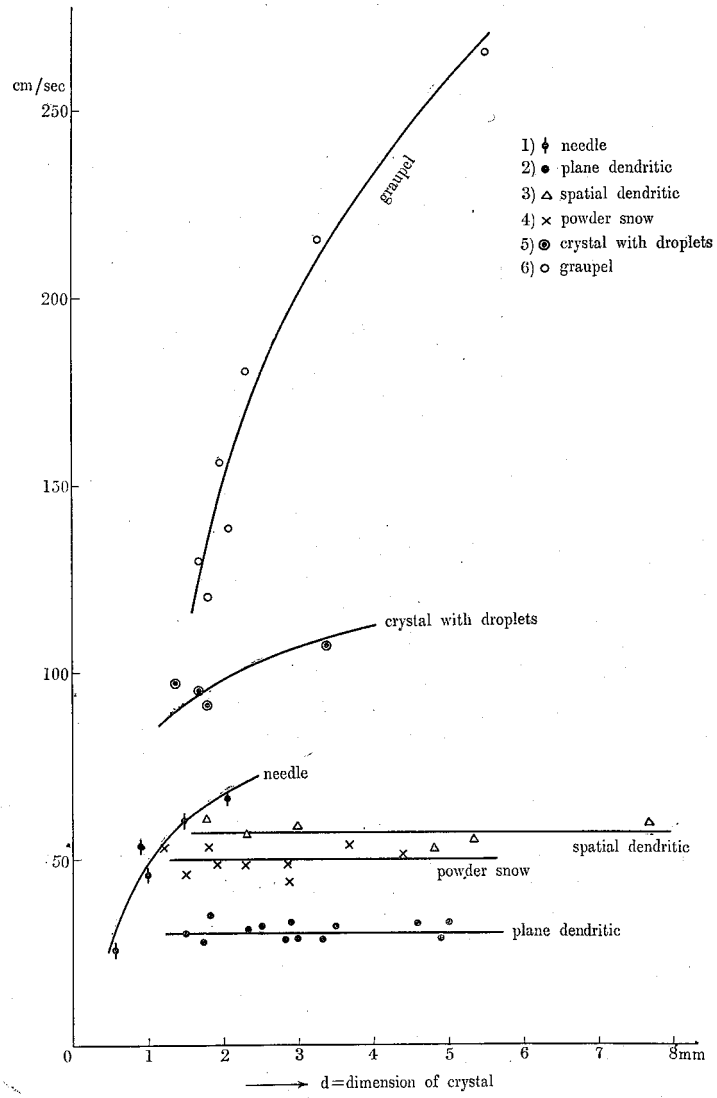

Fig. 3

properties for various types of crytsals, the mean values of d, m and v were compared for six kinds of crystals without regard to the distribution of frequency. The results are tabulated in Table II.

Table II.

	$\bar{d}$ mm	$\bar{m}$ mg	$\bar{v}$ cm/sec	$\bar{v}/v_r$
Needle	1.53	0.004	50	1/2
Plane dendritic	3.26	0.043	31	1/6
Spatial dendritic	4.15	0.146	57	1/5
Powder snow	2.15	0.064	50	1/4
Crystal with droplets	2.45	0.176	100	1/5
Graupel	2.13	0.80	180	1/2.5

It will be understood that a crystal of snow is very small in its mass and unexpectedly slow in the rate of falling; for example, one million particles, as a mean, make 40 c.c. of water in case of plane dendritic crystals and only 4 c.c. of water in case of needles, and a plane dendritic crystal will take about one hour for falling a distance of 1 km.

The relation between the mass and the velocity of a crystal is understood clearly by comparing the velocity of snow fall with that of a rain drop, the mass of which is equal to the mass of the crystal. For the velocity of a rain drop v_r, the value according to SCHMIDT[4] is used. The ratio $\bar{v}/v_r$ is also shown in Table II. The velocity of a plane dendritic crystal is smallest, being only one sixth of that of the corresponding rain drop, while a graupel or a needle falls with a velocity one half that of the rain drop.

e) The size of water droplets attached to the snow crystal.

The photograph of a plane dendritic crystal with water droplets attached was enlarged and the diameter of the droplet was measured

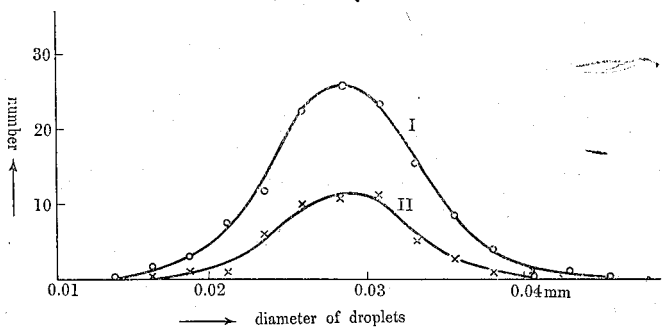

Fig. 4

4) W. SCHMIDT, Sitz. Ber. Wien, **118** (1909) 71.

directly with a Messlupe. One example of the photographs used for this purpose is reproduced in Photo. 4, Plate I. The frequency curve of the diameter is constructed for two crystals, as shown in Fig. 4. The number of droplets used for the construction of curve I was 125 and that for curve II was 50. Both curves show a typical form of a probability curve and the most frequent value of the diameter is about 0.03 mm. for both cases. This value agrees well with that obtained by WAGNER[5] as the diameter of cloud particle, which was 0.033 mm. He did not measure the size directly but calculated it from the diameter of the halo produced in the cloud. Pernter and others also measured the size of cloud particles with this optical method and got more or less similar results. This type of crystal will be considered to be produced by direct attaching of cloud particles on the crystal.

IV. Summary.

A simple method suitable for the observations of the mass, the falling velocity and the form of a snow crystal within a short time interval is described. The crystal is received on a glass plate covered with a thin paraffin sheet and is melted into a hemispherical drop; the mass then may be calculated from the diameter of the drop. The snow crystals are classified into six kinds and for each of them an empirical formula is obtained, representing the relation between the mass and the dimensions of the crystal. The density of graupel is found to be constant for various sizes of the samples, being 0.125. The thickness of plane dendritic crystals is more or less the same for all crystals and is about 0.01 mm. The falling velocity of a plane dendritic crystal is constant, being independent of the dimension. A similar fact is also observed in the cases of spatial dendritic crystals and powder snow. The velocities are also measured for graupels and needles, in which cases they are found to increase with the dimensions of the sample. The relation between the mass and the velocity of a crystal is discussed for various kinds of crystals, by comparing the actual velocity of snow with that of a rain drop having the same mass as the snow crystal. The size of water droplets attached to some crystals is taken and found to measure about 0.03 mm., which agrees well with the size of cloud particles measured by an optical method.

In conclusion, the authors wish to express their best thanks to Nippon Gakuzyutu-Sinkôkai for financial aid in this research.

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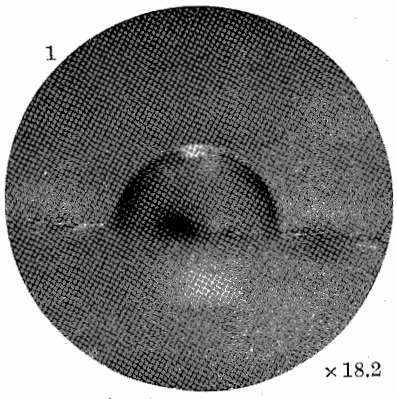
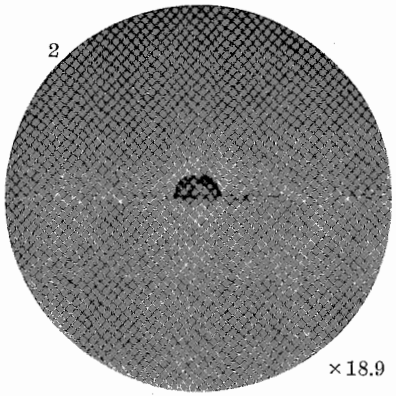
5) WAGNER, Met. Zeit., **26** (1909) 371.

1



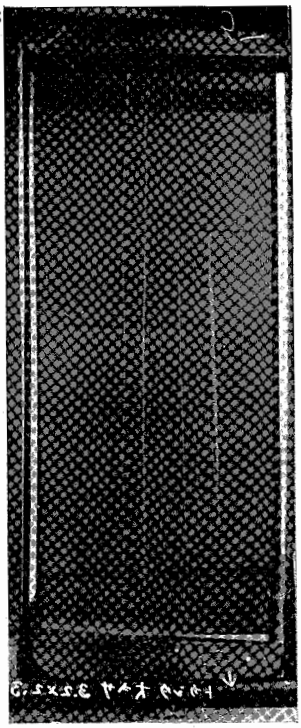
×18.2

2



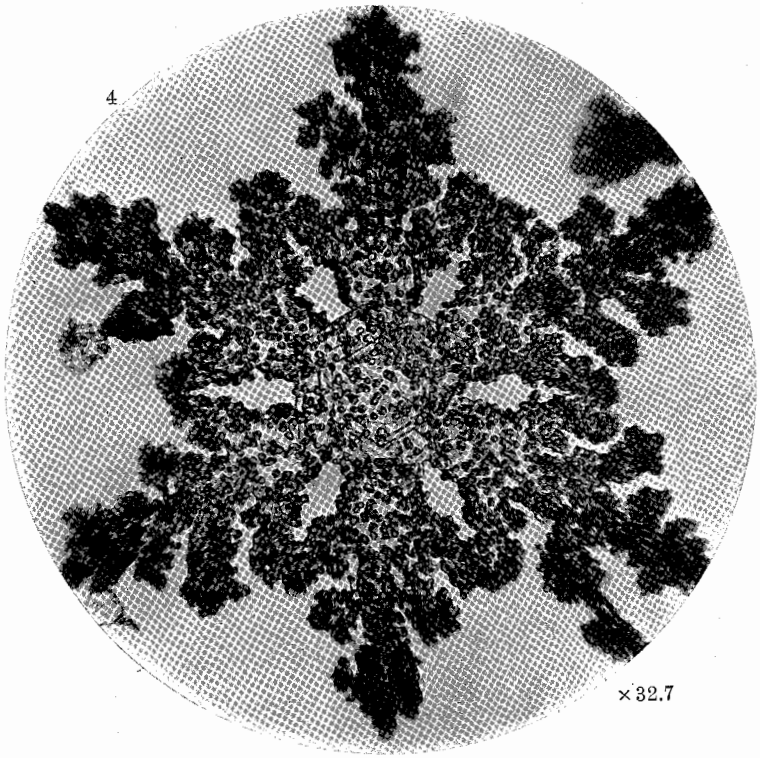
×18.9

3



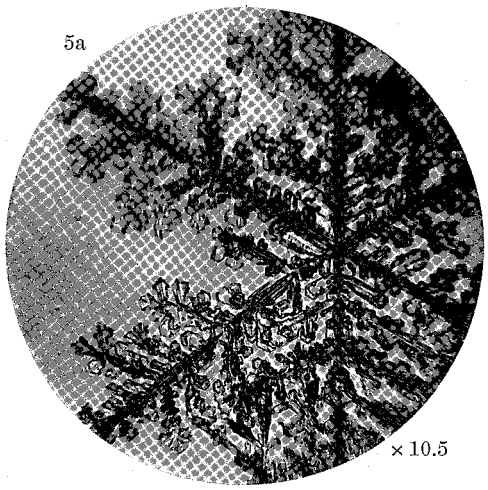
×0.11

4



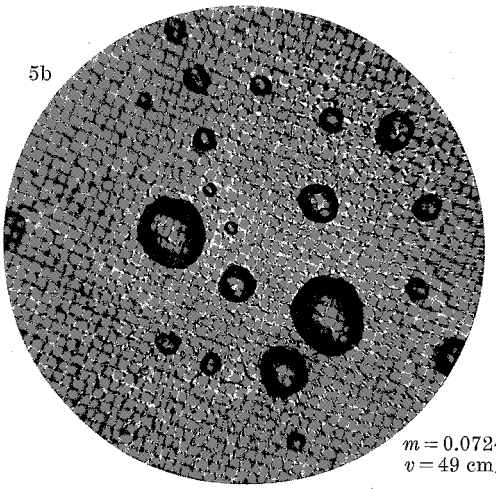
×32.7

5a



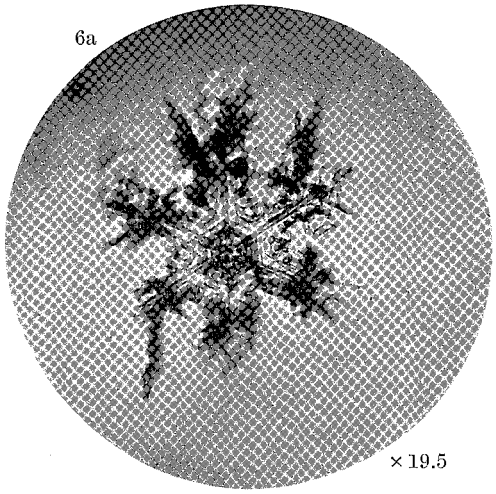
× 10.5

5b



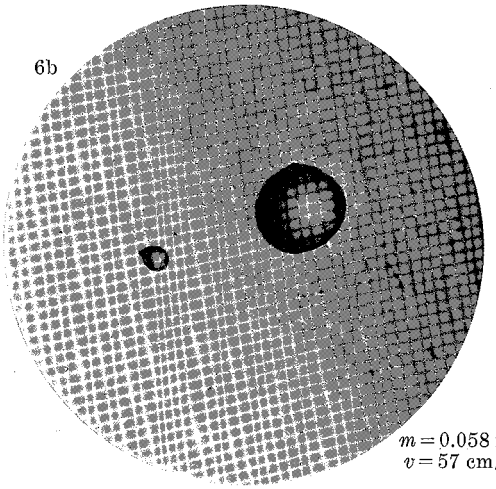
$m = 0.0724$ mg
$v = 49$ cm/sec.

6a



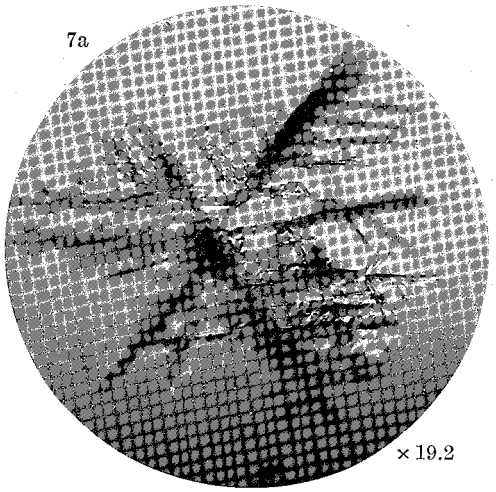
× 19.5

6b



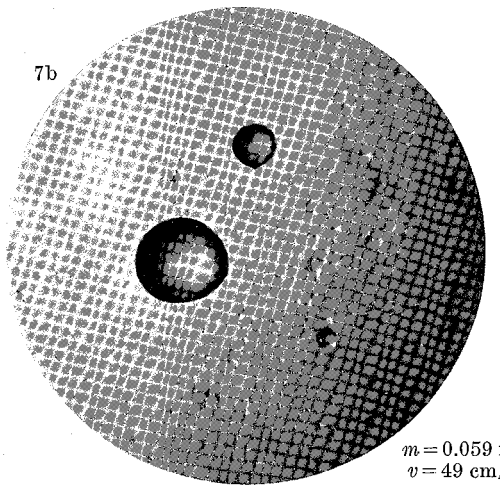
$m = 0.058$ mg
$v = 57$ cm/sec.

7a



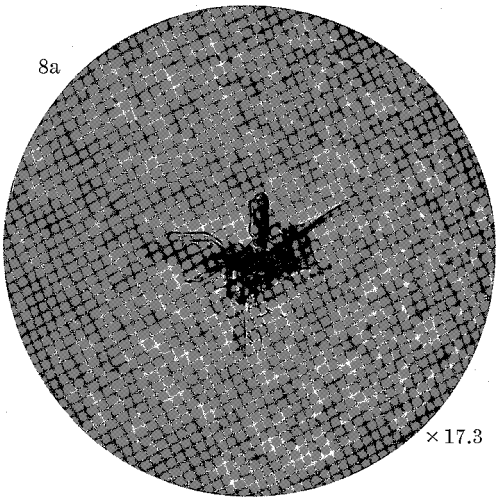
× 19.2

7b



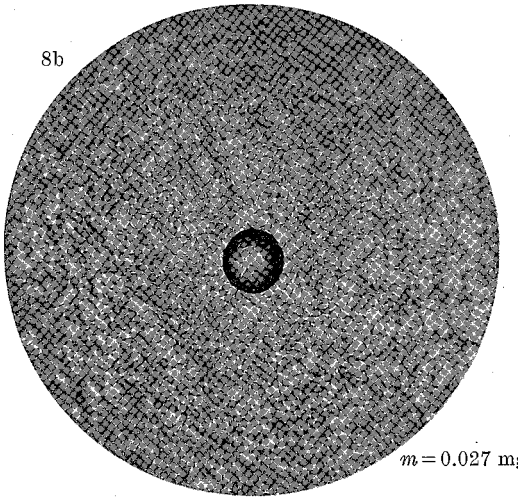
$m = 0.059$ mg
$v = 49$ cm/sec.

8a



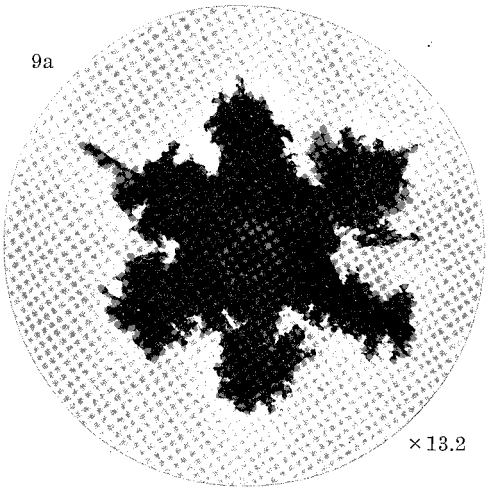
×17.3

8b



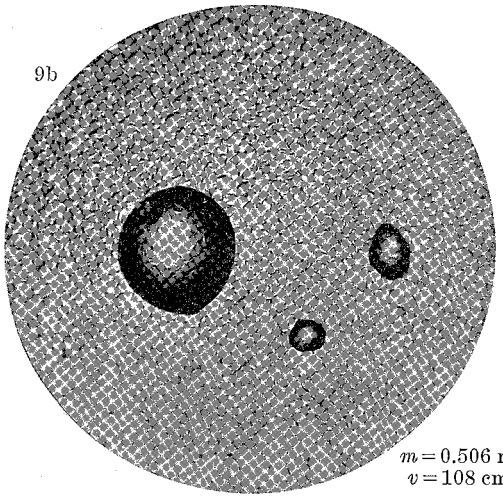
$m = 0.027$ mg

9a



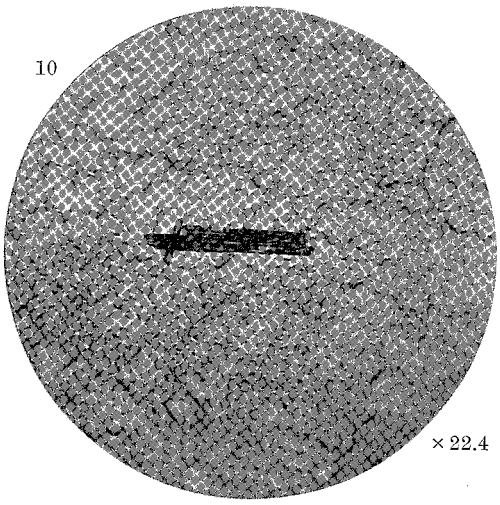
×13.2

9b



$m = 0.506$ mg
$v = 108$ cm/sec

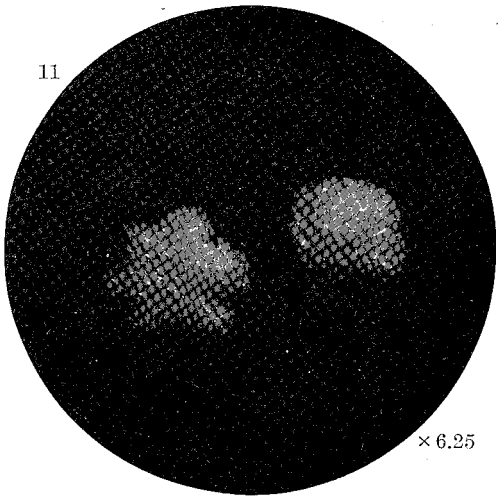
10



×22.4

$v = 46$ cm/sec

11



×6.25

left, $m = 1.56$ mg
right, $m = 1.35$ mg