We consider a compactification with a six-dimensional torus in the type II brane gas models. We show that the dilaton and the scale of each cycle of the internal space are fixed in the presence of NS5-branes and Kaluza-Klein monopoles as well as D-branes with the gauge fields. We can construct various models that lead to fixed moduli by using T-duality transformations.

DOI: 10.1103/PhysRevD.78.064045  PACS numbers: 04.50.–h, 11.25.–w, 11.25.Mj

I. INTRODUCTION

Brane gas models have been investigated to understand the origin of the hierarchy of dimensions. Since superstring theories predict a ten-dimensional universe, we need to know the reason why four-dimensional space-time expands whereas the other spaces are small if the theories are the correct description of our space-time. This problem is related to the origin of our universe and it is natural to consider this problem in the framework of the cosmology. Brandenberger and Vafa [1] considered momentum and winding modes of strings which wrap around a 9-dimensional space. It was argued that the (3 + 1)-dimensional space-time could expand so that such a space-time loses its winding of strings. This idea was polished up by Tseytlin and Vafa [2] in the context of dilaton gravity. An extension including the effect of a gas of Dp-branes has been analyzed by Alexander, Brandenberger, and Easson [3]. The T duality in the brane gas model was investigated in Ref. [4]. The cosmological evolution of string/brane gas models was considered in [5–8].

One of the problems of the brane gas models is the stability of the moduli parameters. A lot of investigations concerning this problem have been performed [9–22]. It was expected that winding modes of Dp-branes prevent the internal space from expanding, because Dp-branes give negative pressure in the internal space [3]. If the dilaton field is fixed, the internal space becomes stable, using the effect of winding modes of Dp-branes [10]. On the other hand, we have to consider the dilaton field at the early stage of the universe. However a simple brane gas model with the dilaton field cannot stabilize both the internal scale factor and the dilaton simultaneously, although the linear combination of them can be stabilized [21]. In Ref. [23], the issue of stabilizing the dilaton was considered, assuming the existence of tensionless membranes which require the quantum analysis of the supermembrane. Recently, Danos, Frey, and Brandenberger suggested a moduli fixing procedure in the heterotic strings on an orbifold [24]. In their models, the gaugino condensation has a crucial role for the moduli fixing. Although many works have been done, it still remains to be an unavoidable problem how the dilaton and the scale of the internal space are fixed simultaneously for the description of a realistic cosmology in the brane gas model.

In this paper, we will concentrate on the problem of the moduli fixing in the type II brane gas model, using Dirac-Born-Infeld action. We will show that the NS5-branes or the Kaluza-Klein monopoles (KK5 monopoles) play a crucial role for the stability of the dilaton field. It is known that the world volume action of the KK5 monopoles and the NS5-branes is derived by the dimensional reduction of the 11-dimensional KK monopole [25] and the M5-brane [26], respectively. We will analyze the role of the gauge fields on the Dp-brane and the world volume action of the NS5-brane and the KK5-monopole in addition to the Dp-branes. We will take the six-dimensional torus T 6 as the internal space for simplicity. If those objects wrap over the cycles of T 6, we will show that the dilaton and the scale of T 6 are fixed. We can find various D-brane configurations which lead to the fixed moduli spaces and dilaton by using T dualities. Namely, the T duality allows us to obtain dual type II brane gas models once the moduli fields are fixed.

This paper is organized as follows. In the next section, we will prepare the bulk action and the metric defined by the four-dimensional Einstein frame. In Sec. III, we discuss the world volume action of the Dp-branes. In Sec. IV, we will show the construction of the world volume action of the NS5-branes and the KK5-monopoles. In Sec. V, we will investigate the problems of the moduli fixing, using the world volume action of the Dp-branes, NS5-branes, and KK5 monopoles. Section VI will be devoted to the conclusions and some discussions.

II. BULK ACTION AND FOUR-DIMENSIONAL EINSTEIN FRAME

In this section, we would like to provide the setup of the bulk action and clarify the relation between the string frame and the four-dimensional Einstein frame, because the moduli fixing should be considered by the four-dimensional Einstein frame.

In the present paper, we assume that the winding modes of large directions have annihilated and the three-dimensional space is the Euclid space. We also assume that D p-branes, NS5-branes, and KK5 monopoles distribute...
homogeneously in the internal space. Taking into account those assumptions, in the string frame, we consider a ten-dimensional metric with a $T^6$ compactification as

$$ds_{10}^2 = -e^{2\lambda_0(t)} dt^2 + e^{2\lambda_3(t)} dx^2 + \sum_{m=4}^{9} e^{2\lambda_m(t)} (dy^m)^2,$$

where $dx^2 \equiv \sum_{i=1}^{3} (dx^i)^2$ represents a three-dimensional flat space. The scale factor $\exp[\lambda_m(t)]$ is the moduli of $T^6$ which represents the size of the torus. Using the degree of freedom of the scale factor $\exp[\lambda_m(t)]$, we can define $T^6$ by $y^m \sim y^m + 2\pi \sqrt{\alpha'}/C_{21}$. This construction of $T^6$ indicates that the radius of the cycle of $T^6$ is given by $2\pi R_m = \int dy^m \exp[\lambda_m(t)] = 2\pi \sqrt{\alpha'} \exp[\lambda_m(t)]$ for each cycle.

In the brane gas model, we will focus on the dilaton gravity sector of the bulk action of the closed string:

$$S_0 = \frac{1}{16\pi G_{10}} \int d^{11}X \sqrt{-G(X)} e^{-2\phi} [R + 4(\nabla \phi)^2].$$

We assume that the dilaton field depends only on time, namely $\phi = \phi(t)$. It is known that this action is invariant under the $T$-duality transformation [2,27] given by

$$\lambda_m(t) \rightarrow -\lambda_m(t), \quad \phi(t) \rightarrow \phi(t) - \lambda_m(t).$$

In order to obtain the four-dimensional Einstein-Hilbert term, we will consider the following transformation:

$$\lambda_0(t) = n(t) + \phi(t) - \frac{1}{2} \tilde{\lambda}(t), \quad \lambda_i(t) = A_i(t) + \phi(t) - \frac{1}{2} \tilde{\lambda}(t),$$

where $n(t)$ and $A_i(t)$ are functions of time.

In these variables, $\exp[n(t)]$ and $\exp[A_i(t)]$ represent the lapse function and the three-dimensional scale factor in the four-dimensional Einstein frame. Indeed, the metric (1) implies that the coefficient of $R$ is proportional to $\exp[\lambda_0(t) + 3\lambda_3(t)] \times \exp[\sum_{m=4}^{9} \lambda_m(t) - 2\phi(t)] R$ which can be normalized as $\exp[n(t) + 3\lambda_3(t)] R_a + \cdots = \sqrt{-g_a(x)} R_a + \cdots$ under the transformation (4).

### III. WORLD VOLUME ACTION OF D-BRANE

In order to construct the brane gas model, we have to include the world volume action of the D$p$-brane. The role of the winding mode of the D-brane has been investigated, however the role of the gauge fields on the D-brane is still unclear in the brane gas model. In this section, we would like to derive the potential of the world volume action of the D-brane with the gauge fields, because the explicit form of the potential is required to discuss the moduli fixing.

We consider the D$p$-brane wrapping over the cycles of the six-dimensional torus $T^6$. If the D$p$-brane wraps over the $(m_1, \ldots, m_p)$ cycle $(0 \leq p \leq 6)$, the gauge fields on the D-brane exist in the $(m_1, \ldots, m_p)$ directions. We assume that the gauge field is Abelian and that the gauge potential and the transverse coordinates depend only on the time variable, i.e. $A_{m_i}(t)$ and $X^{m_a}(t)$ which means the homogeneous distributions of the branes. We adopt the coordinate system as $\xi^0 = t$, $\xi^{m_a} = y^{m_a}$ $(a = 1, 2, \ldots, p)$. Then, the Dirac-Born-Infeld action of D$p$-brane wrapping over the $(m_1, \ldots, m_p)$ cycle is given by

$$S_{Dp}^{(m_1, \ldots, m_p)} = -T_p \int_{R \times \Sigma_p} d^{p+1} \xi e^{-\phi(t)} \sqrt{-\det(\gamma_{ab} + 2\pi \alpha' F_{ab})}$$

$$= -2(2\pi \sqrt{\alpha'})^p T_p \int d(t) e^{-\phi(t)} \lambda_0(t) + \sum_{m=1}^{p} \lambda_m(t) \left[1 - (2\pi \alpha')^2 \sum_{a=1}^{p} e^{-2\lambda_0(t) - 2\lambda_a(t)} (\dot{\lambda}_a(t))^2 \right]^{1/2}$$

$$- \frac{6}{b+p+1} e^{-2\lambda_0(t)} (X_{m_b}(t))^2 \right]^{1/2},$$

where $T_p = (2\pi)^{-p} (\alpha')^{-(p+1)/2}$, $\alpha'$ is related with the string length as $L_s = \alpha'^{1/2}$ [28] and $\int_{\Sigma_p} d^p \xi = (2\pi \sqrt{\alpha'})^p$.

The $T$ duality of the Dirac-Born-Infeld action of the D$p$-brane was shown in [29]. The analysis is general and applicable to our brane gas model. We will consider the $T$ dual of the $(m_p)$ cycle. This $T$ dual requires that the dilaton $\phi(t)$ and the scale factor $\exp[\lambda_{m_p}(t)]$ transform as (3). We also impose the following dual transformations between the gauge fields and the transverse coordinates:

$$2\pi \alpha' \lambda_{m_p}(t) \rightarrow X^{m_p}(t).$$

Using those $T$-dual transformations (3) and (6) as well as the relation of the tension $(2\pi \sqrt{\alpha'})^p T_p = (2\pi \sqrt{\alpha'})^{p-1} T_{p-1}$, we can see that the D$p$-brane action

$$S_{Dp}^{(m_1, \ldots, m_p)}$$

is mapped to the D$(p-1)$-brane action $S_{D(p-1)}^{(m_1, \ldots, m_{p-1})}$ which wraps over the $(m_1, \ldots, m_{p-1})$ cycle. We can also consider the $T$ dual of a transverse direction, i.e. a $(m_{p+1})$ cycle. Applying the $T$-dual rule (3) for $\lambda_{m_{p+1}}(t)$ and

$$X^{m_{p+1}}(t) \rightarrow 2\pi \alpha' \lambda_{m_{p+1}}(t),$$

we find that the D$p$-brane action $S_{Dp}^{(m_1, \ldots, m_p)}$ is mapped to the D$(p+1)$-brane action $S_{D(p+1)}^{(m_1, \ldots, m_{p+1})}$.

We need to know the explicit form of the potential term in the four-dimensional Einstein frame for investigating the moduli fixing problems. Substituting (4) for Eq. (5), we
obtain the world volume action in the four-dimensional Einstein frame:

\[
S_{Dp}^{(m_1 \ldots m_p)} = -(2\pi \sqrt{\alpha'})^p T_p \times \int dt e^{\phi(t) - (1/2)\dot{\lambda}(t) + \sum_{a=1}^{p} \lambda_{a}(t) \{1 - \mathcal{A}(t)\}^{1/2},}
\]  

(8)

where \(\mathcal{A}(t)\) is

\[
\mathcal{A}(t) \equiv (2\pi \alpha')^2 \sum_{a=1}^{p} e^{-2\phi(t) - 2\lambda_a(t)} \{1 - \mathcal{A}(t)\}^{1/2},
\]

(10)

\[
e^{n(t) - 2\sqrt{\alpha'} \lambda_a(t)} \sum_{a=1}^{p} e^{-2\phi(t) - 2\lambda_a(t)} \dot{X}_{\mu a}(t) \mathcal{A}(t)
\]

(11)

where \(f_m\) and \(v_m\) are constants of integration. We can solve Eqs. (10) and (11) on \(\mathcal{A}(t)\):

\[
\mathcal{A}(t) = \frac{e^{-2\sum_{a=1}^{p} \lambda_{a}(t)} \tilde{\mathcal{A}}(t)}{1 + e^{-2\sum_{a=1}^{p} \lambda_{a}(t)} \tilde{\mathcal{A}}(t)}
\]

(12)

\[
\tilde{\mathcal{A}}(t) \equiv \sum_{a=1}^{p} e^{-2\phi(t) + 2\lambda_a(t)} \left| f_m \right|_{\mu^{m_1 \ldots m_p}}^{\mu^{m_1 \ldots m_p}} \frac{2}{2\pi \alpha'}
\]

\[
+ \sum_{b=p+1}^{6} e^{-2\phi(t) - 2\lambda_{a}(t)} \left| v_m \right|_{\mu^{m_1 \ldots m_p}}^{\mu^{m_1 \ldots m_p}}
\]

(13)

In the four-dimensional Einstein frame, the potential term is derived by

\[
u_0 = -T_0 = \frac{2g^{00}}{\sqrt{g_4(t)}} \frac{\delta S_{Dp}^{(m_1 \ldots m_p)}}{\delta g^{00}}
\]

\[
= - e^{-n(t) - 3\dot{\lambda}(t)} \frac{\delta \mathcal{L}_{Dp}^{(m_1 \ldots m_p)}}{\delta n(t)}
\]

(14)

Using (8), (12), and (13), we obtain the potential term of the Dp-brane wrapping over the \((m_1 \ldots \cdot m_p)\) cycle:

\[
u_0 = e^{-3\dot{\lambda}(t) (2\pi \sqrt{\alpha'})^p T_p (e^{-\dot{\lambda}(t) + 2 \sum_{a=1}^{p} \lambda_{a}(t)} + e^{-\dot{\lambda}(t)} \tilde{\mathcal{A}}(t))^{1/2}.}
\]

(15)

Taking into account the definition of \(\tilde{\mathcal{A}}(t)\), it is easily found that \(\nu_0^{(m_1 \ldots m_p)}\) cannot fix the dilaton.
\[ u_{\text{NS}5}^{(m_1 \cdots m_m)} = e^{-3A(t)}(2\pi\sqrt{\alpha'})^5 T_{\text{NS}5} e^{-\phi(t)-(1/2)\tilde{\lambda}(t) + \sum_{a=1}^{4} \lambda_a(t)}, \]  
(17)

\[ u_{\text{KK}5}^{(m_1 \cdots m_m)} = e^{-3A(t)}(2\pi\sqrt{\alpha'})^5 T_{\text{KK}5} e^{-\phi(t)-(1/2)\tilde{\lambda}(t) + 2\sum_{a} \lambda_a(t)}, \]  
(18)

V. MODULI FIXING AND T DUALITY

In this section, we are going to show that there exist models where the dilaton field and scales of \( T^6 \) are fixed simultaneously. Once moduli fields will be fixed, we are able to obtain dual type IIA/IIB brane gas models by the \( T \)-duality transformations.

All we need is the existence of \( D_p \)-branes with gauge fields and NS5-branes or KK5 monopoles. At first, we consider the type IIB theory with D1-branes and KK5 monopoles. We require that these branes and monopoles distribute homogeneously. Namely, we consider a type IIB brane gas model in which the D1-branes wrap over each 1-cycle and the KK5 monopoles wrap over the (45678) cycle and its cyclic permutations. Although we can see

\[
(2\pi\sqrt{\alpha'}) T_1 N_{\text{D}1} \sum_{a=4}^{9} e^{-(1/2)\tilde{\lambda}_a(t)+\lambda_a(t)} f^2 e^{2\phi(t)} (1 + f^2 e^{2\phi(t)})^{-(1/2)} - (2\pi\sqrt{\alpha'})^5 T_{\text{KK}5} N_{\text{KK}5} \sum_{a=4}^{9} e^{-\phi(t)+(1/2)\tilde{\lambda}_a(t)+\lambda_a(t)} = 0,
\]  
(20)

\[
(2\pi\sqrt{\alpha'}) T_1 N_{\text{D}1} \left( 2 e^{\lambda_1(t)} - \sum_{b=4}^{9} e^{\lambda_b(t)} \right) e^{\phi(t)} (1 + f^2 e^{2\phi(t)})^{1/2} + (2\pi\sqrt{\alpha'})^5 T_{\text{KK}5} N_{\text{KK}5} e^{\tilde{\lambda}(t)} \left( 2 e^{\lambda_1(t)} + \sum_{b=4}^{9} e^{\lambda_b(t)} \right) = 0.
\]  
(21)

Using (21), \((\frac{\partial}{\partial \lambda_a} - \frac{\partial}{\partial \alpha} U_{\text{IIB}}) = 0\) gives

\[ e^{\lambda_1(t)} = e^{\lambda_b(t)}, \]  
(22)

where \((a, b = 4, 5, \ldots, 9)\). We will define \( \lambda_1(t) = \lambda_2(t) = \cdots = \lambda_9(t) \). Substituting (22) for (20) and (21), the result is given by

\[ e^{2\phi_{\text{min}}} = \frac{1}{f^2}, \quad e^{2\lambda_1(t)} = \left( \frac{N_{\text{D}1}}{N_{\text{KK}5}} \right)^{1/3} \left( \frac{1}{f^2} \right)^{1/6}, \]  
(23)

where we have used \( T_{\text{KK}5} = T_{\text{NS}5} = T_5 \) and \( T_p = (2\pi)^p (\alpha')^{-(p+1)/2} \). If we choose the initial condition of the gauge fields as \( 1 \ll f^2 \), the weak string coupling is realized. \( N_{\text{D}1} \) and \( N_{\text{KK}5} \) control the scale of the six-dimensional torus.

Therefore, we see that the dilaton and the scale of \( T^6 \) can be fixed in the type IIB brane gas model. If \( U_{\text{IIB}} \) does not include the gauge fields or the KK5 monopoles, we cannot fix the dilaton field and the scale of \( T^6 \) simultaneously. It is stressed that the scale of \( T^6 \) becomes isotropic because of the isotropic initial conditions for gauge fields on D1-branes. This property can be seen from the invariance of \( U_{\text{IIB}} \) under \( \lambda_a \rightarrow \lambda_a \). We have argued with one type of D-branes. Moreover, we have considered \( u_{m_1 \cdots m_m} = 0 \).
where $N_{DS}^{\text{dual}} = N_{D1}$ and $N_{NS5}^{\text{dual}} = N_{KK5}$. The first term indicates the D5-brane action moving to the transverse direction. The second term describes the world volume action of the NS5-branes. $e^{2\phi_{\text{min}}}$ and $e^{2\phi_{\text{max}}}$ are mapped to the dual values which are not self-dual by (3).

Note that we can obtain possible dual type IIA or IIB models by the $T$ duality.

VI. CONCLUSION AND DISCUSSIONS

In this paper, we have considered the moduli fixing problem in the type II brane gas model. It is stressed that this moduli fixing including dilaton field is realized in the presence of NS5-branes or KK5 monopoles as well as the winding modes of the D-brane and the effects of the gauge fields on the D-brane. NS5, KK5, and gauge fields on D-brane are new ingredients because those objects have not been considered in [9–24]. The reason why the moduli fixing has been difficult in the brane gas model so far is that we do not deal with the effect of the gauge fields on the D-branes, the world volume action of NS5-branes, and KK5 monopoles.

The key idea of introducing NS5-branes and KK5 monopoles is the difference of the dilaton dependence between the D-brane and NS5-brane/KK5 monopole. In the string frame, the dilaton dependence of the D-brane is $e^{-\phi}$, on the other hand, NS5-branes and KK5 monopoles have the dependence of $e^{-2\phi}$. This fact implies that NS5-branes and KK5 monopoles generate a stronger pressure for the dilaton than the D-branes.

A brane gas model whose moduli fields are fixed can be mapped to a dual brane gas model with fixed moduli fields by the $T$-duality transformations. Once moduli fields are fixed, we are able to obtain dual type IIA or IIB brane gas models by $T$ duality. Then, the fixed values of the moduli fields are not necessarily self-dual. This fact is different from the string gas model [1] where moduli fields are fixed at the self-dual radius of the $T$ dual without the dilaton field.

In our model, the problem on the selection of vacua remains. At this stage, the brane configurations, the number of branes, and the initial values of gauge fields are given by hand. A lot of initial conditions give the various minimum of the potential. However, once the initial condition is given, the moduli fields can be fixed.

It is interesting to classify the cases where the moduli fields are fixed in our model. This analysis may lead to the moduli fixing which includes nondiagonal components of the metric of $T^6$.

The toroidal compactification with fixed moduli fields is a first and significant step to obtaining the realistic compactification with $K3 \times T^2$ or the Calabi-Yau threefold [31]. Our analysis of this paper includes the configuration where D2-branes wrap around two-cycles in type IIA theory and D3-branes wrap around three cycles in type IIB theories. Therefore, it is expected that a similar analysis can be made even for the models with Calabi-Yau compactifications. The moduli fixing procedure discussed in our paper may also be applicable to a heterotic brane gas model with $T^6$ compactification which is expected to be dual to a type IIA with $K3 \times T^2$ compactification [32–34].

An application of our model to the accelerating universe [35] is also interesting. The method used in [36] may be useful for the analytic investigation.

ACKNOWLEDGMENTS

We would like to thank the Yukawa Institute for Theoretical Physics at Kyoto University. Discussions during the YITP-W-07-05 on “String theory and field theory—frontier of quantum and space-time” and during the YITP-W-07-10 on “KIAS-YITP Joint Workshop String Phenomenology and Cosmology” were useful to complete this work. We are grateful to thank Kenji Hotta, Katsumi Itoh, Testuji Kimura, Nobuyoshi Ohta, Yuji Satoh, Makoto Tanabe, Seiji Terashima, and Kunihito Uzawa for valuable comments.