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Numerical simulation of Rhone's glacier from 1874 to 2100

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Abstract

The numerical simulation of the motion of Rhone's glacier in the Swiss Alps is performed from 1874 to 2007, and then from 2007 to 2100. Given the shape of the glacier, the velocity of ice \mathbf{u} is obtained by solving a 3D nonlinear Stokes problem. Then, the shape of the glacier is updated by computing the volume fraction of ice φ , which satisfies the transport equation

$$\frac{\partial \varphi}{\partial t} + \mathbf{u} \cdot \nabla \varphi = b \delta_{\Gamma_A}.$$

Here $b \delta_{\Gamma_A}$ is a source term acting only on the ice-air interface Γ_A which accounts for the accumulation or ablation of ice due to snow falls or melting.

A decoupling algorithm allows the two above problems to be solved using different numerical techniques. The nonlinear Stokes problem is solved on a fixed, unstructured finite element mesh made of tetrahedrons. The transport equation is solved using a fixed, structured grid made of smaller cells.

The numerical simulation is performed between 1874 and 2007. Then, a median climatic scenario is considered in order to predict the shape of the glacier from 2007 until 2100.

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1 Introduction

Since 1850, glaciologists have documented the retreat of the glaciers in the Swiss Alps. The motion of glaciers is relevant not only for the tourism industry but also for the future management of natural risks, hydroelectric plants and water supply for agriculture.

The dynamics of a glacier is the result of different phenomena. Due to snow-falls, ice is accumulating on the upper part of the glacier whereas it is melting on the lower part because of higher temperatures. Moreover, due to gravity, ice is flowing down to the valley. When studying ice flows during years or centuries, ice can be considered as an incompressible non-Newtonian fluid and the convected derivative can be disregarded. Therefore, given a glacier shape, the mass and momentum equations reduce to a nonlinear stationary Stokes problem. Mass conservation along the ice-air interface yields to a transport equation which can be used to determine the new glacier shape. A source term - the so-called mass balance - must be added to the right hand side of this transport equation in order to take into account ice accumulation or ablation due to snow falls or melting. This term contains the climatic input of the model and several scenario can be explored in order to predict the future retreat of Alpine glaciers.

Most of the numerical simulations presented so far in glaciology have been performed using Lagrangian or Arbitrary Lagrangian Eulerian methods. When considering ice flows during centuries, topological changes may appear so that Eulerian methods such as Level Set or Volume of Fluid (VOF) seem to be more appropriate. Level set methods in glaciology have been considered in [PF04] to compute the onset of crevasse formation in 2D. The VOF formulation has been used in [JPRB08] to recover stationary 3D glacier shapes.

In this paper, the dynamics of Rhone's glacier is computed in 3D for 227 years using the method presented in [JPRB08]. The determination of the mass balance is the result of 150 years of measurements performed by glaciologists; it corresponds to a parameter identification procedure that involves weather reports, climate prediction, measurements of ice flow, ice depth, snow falls and ice melting [HBF08]. Using this mass balance, the simulated glacier shape can then be compared to the measured one from 1874 to 2007. Then, a median climatic scenario is considered in order to predict the future shape of the glacier from 2007 to 2100.

The paper is organised as follows. The mathematical model is presented in the next Section and corresponds to the model already presented in [JPRB08]. The numerical procedure also corresponds to [JPRB08]. At each time step, the velocity computation is decoupled from the glacier shape computation, which

allows different numerical techniques to be used for solving each of these two subproblems, as in [MPR03,BPL06]. The transport equation is solved on a fixed structured grid of small cells and SLIC post-processing is used to reduce numerical diffusion, see for instance [SZ99]. The nonlinear Stokes problem is solved using a fixed unstructured finite element mesh of tetrahedrons. The computation of the mass balance is presented in Section 4; it has been already published in [HBF08]. Numerical results are presented in Section 5. Computed shapes are compared to measured ones from 1874 to 2007. Finally, the predicted glacier shape in 2100 is reported when using a median climatic scenario taken from [HFBF08].

2 The mathematical model

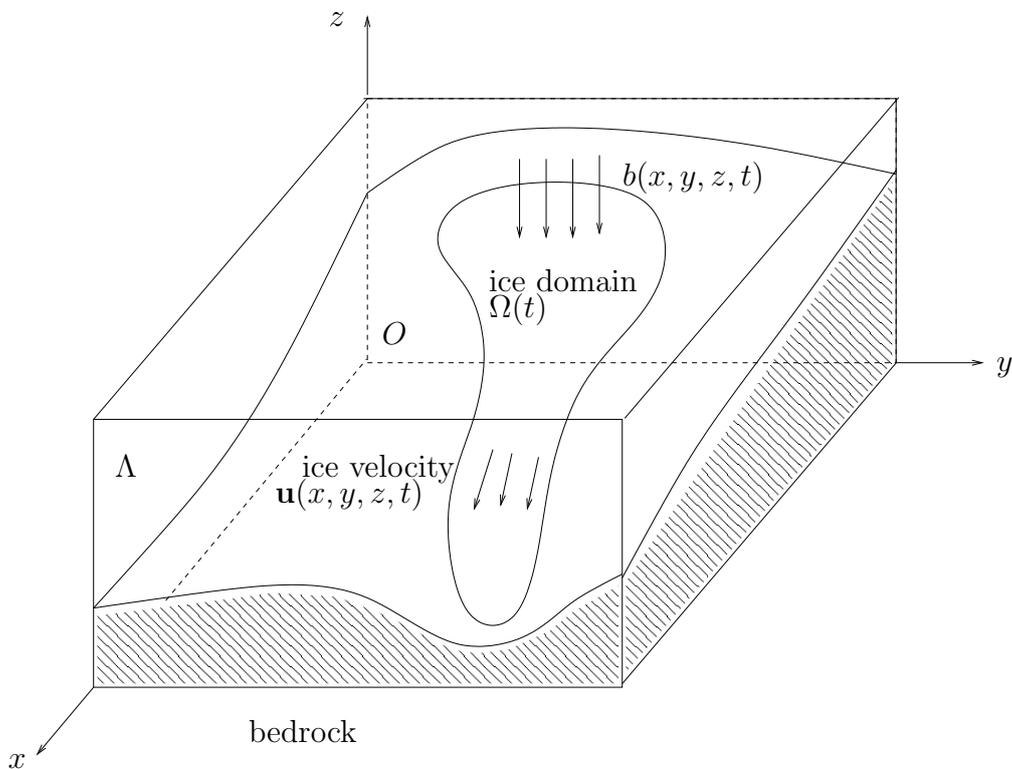


Fig. 1. Notations.

We are interested in computing the shape of a glacier between time $t = 0$ and $t = T$. Let Λ be a cavity of \mathbb{R}^3 in which the ice domain is contained. At time t the ice domain is denoted by $\Omega(t)$, the bedrock-ice interface is $\Gamma_B(t)$ and the ice-air interface is $\Gamma_A(t)$. Let Q_T be the ice region in the space-time domain :

$$Q_T = \{(x, y, z, t) \in \Lambda \times (0, T); (x, y, z) \in \Omega(t); 0 < t < T\},$$

and let $\mathbf{u} = (u, v, w) : Q_T \rightarrow \mathbb{R}^3$ and $p : Q_T \rightarrow \mathbb{R}$ be the ice velocity and

pressure, respectively. When considering the motion of a glacier during years or centuries, ice can be considered as an incompressible non-Newtonian fluid. Moreover, a dimensionless scaling shows that inertial terms can be disregarded. Therefore, the mass and momentum equations reduce at time t to a stationary nonlinear Stokes problem in the ice domain $\Omega(t)$:

$$-2\operatorname{div}(\mu\varepsilon(\mathbf{u})) + \nabla p = \rho\mathbf{g}, \quad (1)$$

$$\operatorname{div} \mathbf{u} = 0. \quad (2)$$

Hereabove $\varepsilon(\mathbf{u}) = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ denotes the rate of strain tensor and Glen's flow law [Gle58,Hut83] holds for the viscosity $\mu = \mu(\mathbf{u})$. More precisely, for a given velocity field \mathbf{u} , the viscosity μ satisfies the following nonlinear equation:

$$\frac{1}{2\mu} = A \left(\sigma_0^{n-1} + \left(2\mu\sqrt{\frac{1}{2}\operatorname{tr}(\varepsilon(\mathbf{u})\varepsilon(\mathbf{u}))} \right)^{n-1} \right), \quad (3)$$

where A is a positive number known as the rate factor, n is Glen's exponent and σ_0 is a regularisation parameter which prevents infinite viscosity for zero strain. It should be noted that A depends on ice temperature but, since temperature varies very little in most Alpine glaciers, A can be taken as a constant for Rhone's glacier.

The boundary conditions corresponding to (1) are the following. Since no force applies on the ice-air interface $\Gamma_A(t)$:

$$2\mu\varepsilon(\mathbf{u})\mathbf{n} - p\mathbf{n} = \mathbf{0}, \quad (4)$$

where \mathbf{n} is the unit outer normal vector along the boundary of the ice domain $\Omega(t)$. Along the bedrock-ice interface $\Gamma_B(t)$, ice may slip or not, according to the roughness of the bedrock. However, for the sake of simplicity, only no-slip condition will be considered :

$$\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma_B(t). \quad (5)$$

We now turn to the model for the volume fraction of ice. As in [JPRB08], the presence of ice is defined by a characteristic function $\varphi : \Lambda \times (0, T) \rightarrow \mathbb{R}$ defined by :

$$\varphi(x, y, z, t) = \begin{cases} 1, & \text{if } (x, y, z) \in \Omega(t), \\ 0, & \text{else.} \end{cases} \quad (6)$$

In absence of snowfalls or melting, the volume fraction of ice would satisfy

$$\frac{\partial\varphi}{\partial t} + \mathbf{u} \cdot \nabla\varphi = 0, \quad (7)$$

in a weak sense in the space-time ice domain Q_T . In other words, φ is constant along the trajectories of the fluid particles which are given by

$$\begin{pmatrix} x'(t) \\ y'(t) \\ z'(t) \end{pmatrix} = \begin{pmatrix} u(x(t), y(t), z(t)) \\ v(x(t), y(t), z(t)) \\ w(x(t), y(t), z(t)) \end{pmatrix}.$$

We refer for instance to [SZ99] for a review of numerical methods for solving the above equation.

In Alpine glaciers snow accumulates on the upper part of the glacier whereas ice melts on the lower part. The horizontal line at which $b = 0$ is called the equilibrium line of accumulation ELA and is around 3000 meters above sea level in the Alps. Therefore, a source term must be added to the right hand side of (7) to account for ice accumulation or ablation. Let $b(x, y, z, t)$ be the ice height added or removed along the ice-air interface $\Gamma_A(t)$ within one year, the so-called mass balance function. In our model, this quantity is a given quantity and corresponds to the climate input. It can be measured by glaciologists using stakes and simple empirical expressions are available. Recently [HBF08], a complex formula has been proposed for b which involves snow precipitations and ice melting. The coefficients involved in these formula are then the result of a parameter identification problem which takes into account measurements that have been reported on Rhone's glacier since 1874.

Given the mass-balance function b and following [JPRB08], equation (7) must be updated as follows :

$$\frac{\partial \varphi}{\partial t} + \mathbf{u} \cdot \nabla \varphi = b \delta_{\Gamma_A}, \quad (8)$$

where δ_{Γ_A} is the 3D Dirac (or delta) function on the ice-air interface Γ_A .

Let us sum up the model. Our goal is to find the volume fraction of ice φ in the whole cavity, the velocity \mathbf{u} and pressure p in the ice domain satisfying equations (1), (2) and (8). Equation (1) must be supplemented with the boundary conditions (4) and (5). At initial time, the volume fraction of liquid $\varphi(x, y, z, 0)$, or equivalently the initial ice domain $\Omega(0)$, must be provided.

3 Numerical method

In order to decouple the computation of the volume fraction of ice φ to that of \mathbf{u} , p , a time discretization is proposed. Two different space discretizations are then used for solving the transport problem (8) and the nonlinear Stokes problem (1) (2). The transport problem is solved using a structured grid of

small cells having size h , with goal to reduce numerical diffusion as much as possible. On the other side, since the numerical resolution of the nonlinear Stokes problem is CPU time consuming, an unstructured mesh of tetrahedrons with larger size H is used. The use of two different grids has shown to be very efficient for solving Newtonian [MPR03] and viscoelastic flows [BPL06] with complex free surfaces. A good compromise between accuracy and efficiency is to choose $H \simeq 5h$ with a time step such that the maximum CFL number (velocity times the time step divided by the cells spacing) is close to 5. This numerical method has already been presented in [JPRB08] in order to compute stationary glacier shapes.

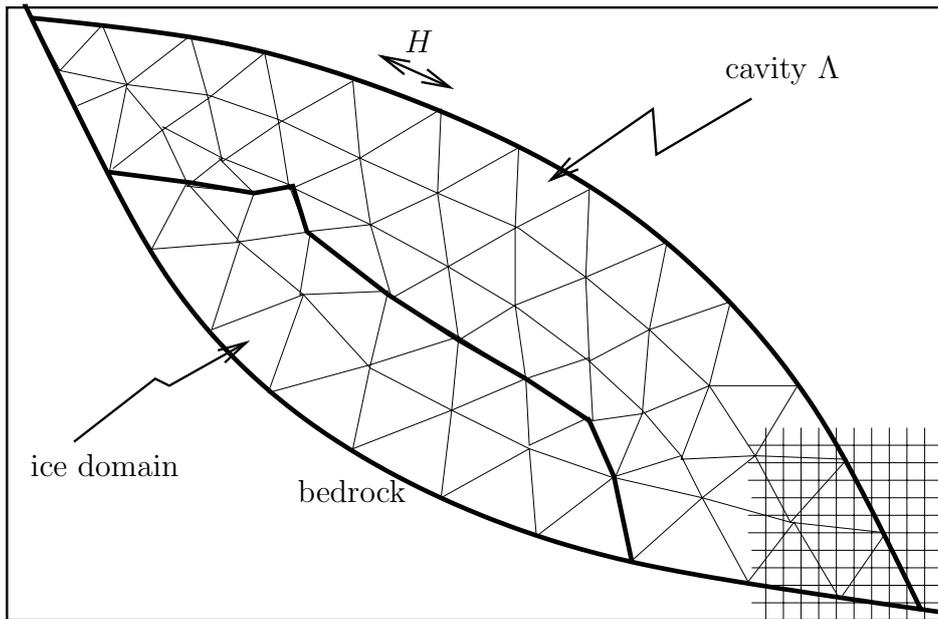


Fig. 2. The two grids (2D figure). The cavity Λ is meshed once for all with unstructured finite elements having size H . Then, at each time step, the ice region is the union of all the elements being filled with ice. The cavity is also covered with structured cells having smaller size $h \simeq H/5$.

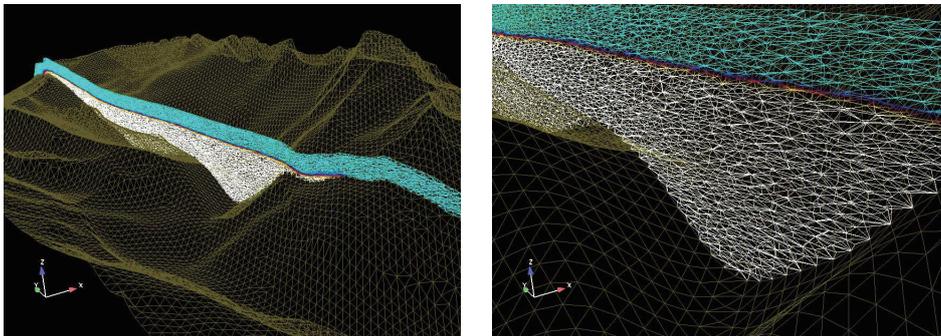


Fig. 3. Cut of the finite element mesh used for Rhone's glacier. Brown : bedrock, blue : cut of the cavity, white : cut of the ice domain at year 1874.

4 The mass balance function b

A model for the mass balance function b accounting for ice acculimation or ablation has been proposed in [HBF08]. The parameters involved in this model have been tuned in order to fit measured values from 1874 to 2007. Then a median climatic scenario [HFBF08] is considered in order to predict glacier shapes until 2100. It corresponds to an increase of $3.8^\circ C$ in temperature and a decrease of 6% in precipitations.

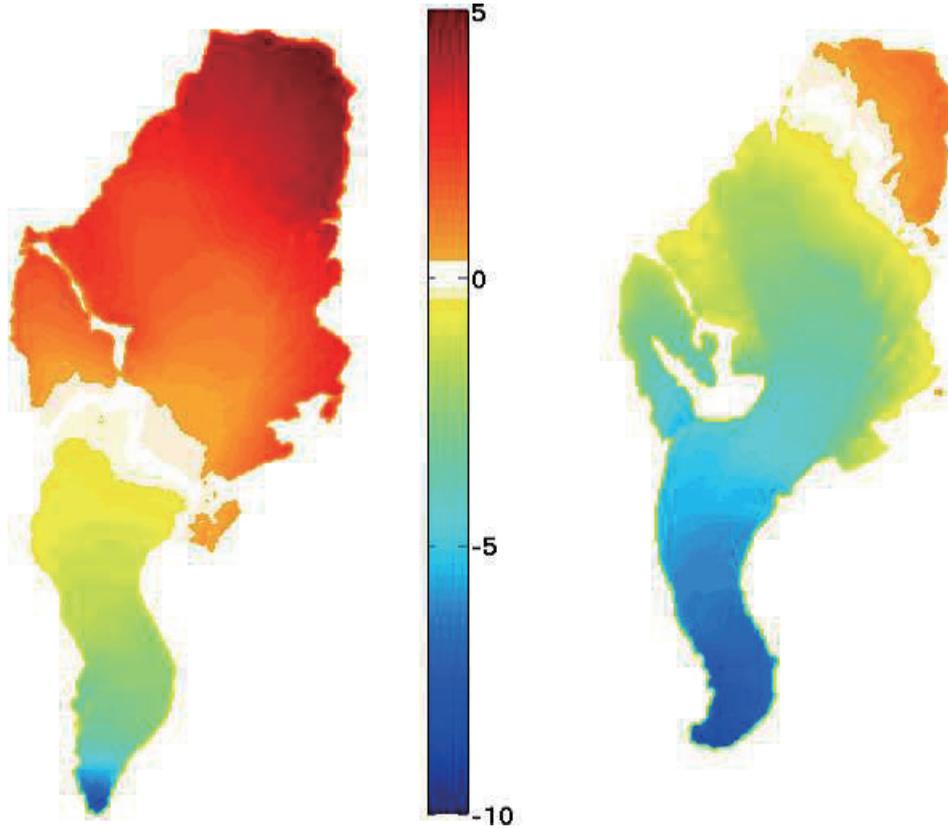


Fig. 4. The mass balance function b at year 1913 (left, cold year) and 2003 (right, hot year).

5 Numerical results

The cavity Λ is contained in the block $(0, 4000) \times (0, 10000) \times (1700 \times 3600)$. At each vertex (x_i, y_j) of a structured grid in the Oxy plane, the bedrock elevation $B(x_i, y_j)$ and the initial ice thickness $T(x_i, y_j)$ are provided, $i = 1, 80$, $j = 1, 200$. The mesh stepping in the x, y directions is $50 m$. A triangular finite element mesh of the bedrock is then generated. A triangular finite element mesh of the top surface of the cavity Λ is also generated by adding $150 m$ to the initial ice thickness. Then, a Delaunay unstructured mesh of tetrahedrons

is generated between the bedrock and the top surfaces using TetMesh-GHS3D [FG08], thus filling the cavity Λ with tetrahedrons of typical size 50 m . The MeshAdapt remesher [Dis03] is used in order to refine the mesh in the Oz direction only (mesh size 10 m). The final mesh of the cavity has 240147 vertices. The number of vertices of the cavity contained in Ω^0 is 84161. The block $(0, 4000) \times (0, 10000) \times (1700 \times 3600)$ containing the cavity Λ is cut into $400 \times 1000 \times 200$ structured cells. As in [MPR03] a hierarchic data structure is used in order to activate the cells and decrease the required memory. The time step is half a year. All the computations have been performed on an AMD Opteron 242 CPU with less than 8Mb memory. The simulation is performed with $A = 0.08$ from 1874 to 2007 and requires one week of CPU time. The initial and final glacier shapes are reported in Fig. 5.

Starting from the measured shape of the glacier in 2007, the numerical simulation of the motion of Rhone’s glacier is then performed until 2100 using the median climatic scenario discussed in Section 4. The results are displayed in Fig. 6. Clearly, when using such a scenario, the glacier would almost disappear in 2100.

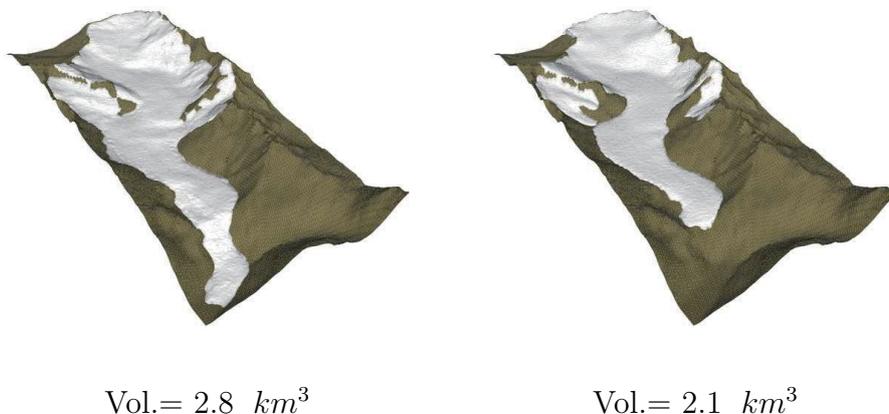


Fig. 5. Simulation over the period 1874-2007.

References

- [BPL06] A. Bonito, M. Picasso, and M. Laso. Numerical simulation of 3D viscoelastic flows with free surfaces. *J. Comput. Phys.*, 215(2):691–716, 2006.

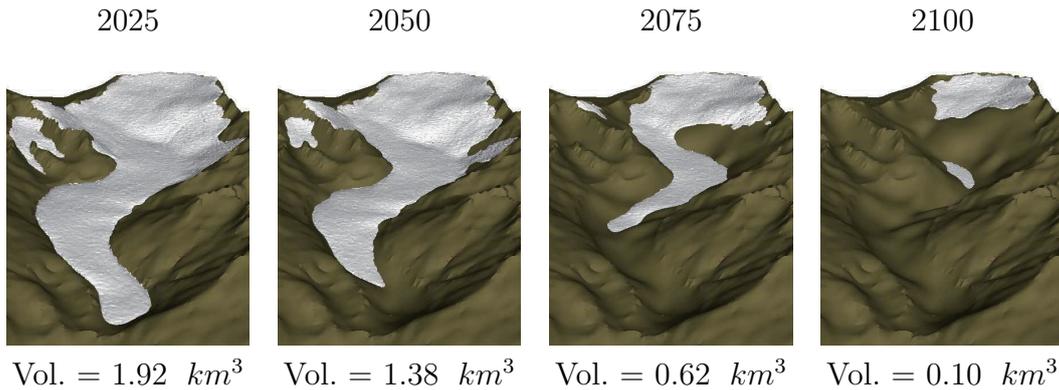


Fig. 6. Simulation over the period 2007-2100.

- [Dis03] Distene S.A.S., Pôle Teratec - BARD-1, Domaine du Grand Rué, 91680 Bruyères-le-Chatel, France. *MeshAdapt : A mesh adaptation tool, User's manual Version 3.0*, 2003.
- [FG08] P. Frey and P.-L. George. *Mesh generation*. ISTE, London, second edition, 2008. Application to finite elements.
- [Gle58] J.W. Glen. The flow law of ice. *IUGG/IAHS Symposium of Chamonix IAHS Publication*, 47:171183, 1958.
- [HBF08] M. Huss, A. Bauder, and M. Funk. Determination of the seasonal mass balance of four alpine glaciers since 1865. *Journal of Geophysical research*, 2008.
- [HFBF08] M. Huss, D. Farinotti, A. Bauder, and M. Funk. Modelling runoff from highly glacierized alpine drainage basins in a changing climate. *Hydrological Processes*, 2008.
- [Hut83] K. Hutter. *Theoretical Glaciology*. Reidel, 1983.
- [JPRB08] G. Jovet, M. Picasso, J. Rappaz, and H. Blatter. A new algorithm to simulate the dynamics of a glacier: theory and applications. *Journal of Glaciology*, 2008.
- [MPR03] V. Maronnier, M. Picasso, and J. Rappaz. Numerical simulation of three-dimensional free surface flows. *Internat. J. Numer. Methods Fluids*, 42(7):697–716, 2003.
- [PF04] A. Pralong and M. Funk. A level-set method for modeling the evolution of glacier geometry. *J. Glaciology*, 50(171):485–491, 2004.
- [SZ99] R. Scardovelli and S. Zaleski. Direct numerical simulation of free-surface and interfacial flow. *Ann. Rev. Fluid Mech.*, 31(7):567–603, 1999.