SIMULTANEOUS ANALYSIS METHOD OF DEFORMATION AND INTERNAL EROSION FOR SOIL STRUCTURES

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INTRODUCTION

Recently, failures of embankments such as irrigation ponds and levees have been reported because of frequent occurrences of storm rainfall, together with significant economic and environmental damages. Piping, as the result of internal erosion of embankments, is a primary cause of embankment breaks. The inner states of embankments are needed to be estimated to take appropriate measures for the prevention of embankment failures. In this study, the equation of the conservation of soil particles inside of embankments is introduced to the conventional soil-water coupled equations, which enables internal erosion, i.e. transport of soil particles, and deformation of embankments to be calculated simultaneously. This approach offers a straightforward method to solve the complex phenomena of soil structures, such as piping.

GOVERNING EQUATIONS

Governing equations to analyze the deformation and the internal erosion of soils are given as follows:

$$\frac{\partial}{\partial x_j}\left(\dot{T}_{ij}+T_{ij}L_{kk}-T_{ik}L_{jk}\right)+\rho\dot{g}_i=0 \quad , \tag{1}$$

$$\frac{\partial q_i}{\partial x_i}+\dot{\theta}+\theta L_{ii}=0 \quad , \tag{2}$$

$$\dot{n}-(1-n)L_{ii}=E\,R_s \quad , \tag{3}$$

where t , x_i , T_{ij} , L_{ij} , ρ , g_i , θ , q_i , n and E denote time, Cartesian coordinates, Cauchy stress tensor, velocity gradient tensor, density of soil mass, gravity acceleration, volumetric water content, flux of pore water, porosity and erosion rates of soils, respectively. Erosion rates of soils are defined as the volume of eroded soil particles included in unit area within unit time. R_s in Eq. (1) denotes the internal surface area per unit volume of soils, where soil particles are eroded. A dot above the variables in Eqs. (1) to (3) means Lagrangian time differentiation. Eqs. (1) and (2) have been conventionally used to analyze the deformation of soils as the soil-water coupled problem. In this study, Eq. (3) is introduced to express the internal erosion of soils.

In order to solve the three equations above simultaneously, the change of the material parameters of soils governing the deformation has to be considered because internal erosion of soils decrease the density and change the particle size distribution. However, the studies on the alteration of the material characteristics of soils due to internal erosion have not been carried out up to now. Therefore, in this paper, the deformation of soils is neglected and the internal erosion is analyzed. In this case, Eq. (1) is not necessary, so that the governing equations are reduced as follows:

$$\frac{\partial n}{\partial t}=E\,R_s \quad , \tag{4}$$

$$\frac{\partial \theta}{\partial t}+\frac{\partial q_i}{\partial x_i}=0 \ , \quad q_i=k\,\frac{\partial h}{\partial x_i} \quad , \tag{5}$$

where h and k denote hydraulic head and permeability of saturated or unsaturated soils, respectively, and Darcy's law is applied to describe the flow in porous media. It has been empirically known that erosion rates of soils can be given as the following form as a function of applied shear stress on the pore wall τ (Reddi & Bonala, 1997):

$$E=\alpha(\tau-\tau_c) \quad , \tag{6}$$

where α is the erodibility coefficient expressing the rate of change of erosion rate; and τ_c is the critical shear stress which determines the onset of erosion. In order to obtain the applied shear stress τ, the following equation is available (Reddi, Lee & Bonala, 2000):

$$\tau=\rho_w gI\sqrt{2K/n} \quad , \tag{7}$$

$$K=\frac{k\mu}{\rho_w g} \quad , \tag{8}$$

where ρ_w , I , K and μ denote the water density, hydraulic gradient, intrinsic permeability and the viscosity of water ($1.005\text{x}10^{-3}$ kg/m/s). The porosity and the intrinsic

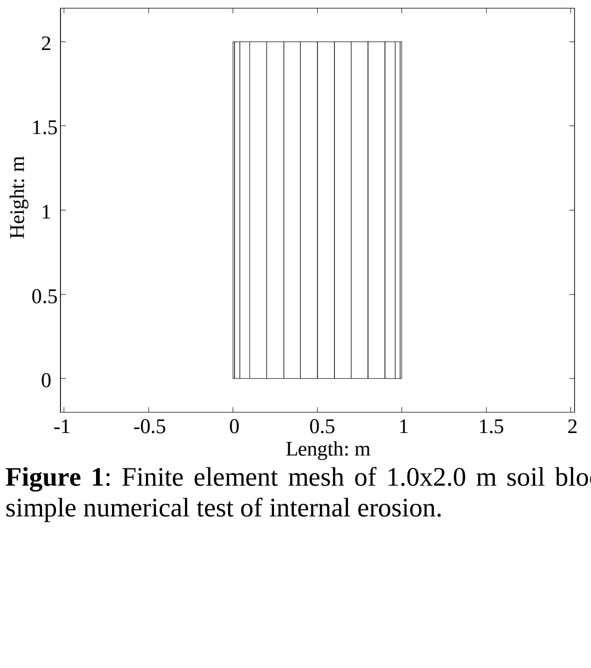

Figure 1: Finite element mesh of 1.0x2.0 m soil block for simple numerical test of internal erosion.

permeability are sufficient for computation of shear stress exerted onto pore walls. The internal surface area per unit volume R_s can be calculated from the particle size distribution curve of the soil.

EXAMPLES OF NUNERICAL ANALYSIS

To solve Eqs. (4) and (5), finite element method (FEM) was applied and the simple numerical test was conducted first. Fig. 1 shows the used finite element mesh. The number of the element was 14. The initial porosity was given as 0.31. The value of the critical shear stress was given as 0.8x10^{-5} kPa and the erodibility coefficient 0.51x10^{-6} m/kPa/s. The water pressure of 9.8 kPa was applied on the left side and 0 kPa on the right side. The undrained condition was imposed on the top and the bottom. When the gradient of the water pressure is applied as the boundary conditions, the pore water flowed rightward in Fig.1 and the internal erosion was induced. However, the state of internal erosion became steady, where the internal erosion did not happen, after the fine particles of the soil were fully eroded and erodible soil particles vanished. Fig. 2 shows three particle distribution curves, one of which presents the initial particle distribution. Other two curves show the steady particle distributions of the soils with the initial permeability of 0.5x10^{-6} and 1.0x10^{-6}. It is found that the soil with the initial permeability of 0.5x10^{-6} keeps more of fine particles than the other with the initial permeability of 1.0x10^{-6}. This result was obtained since the soil with smaller permeability is denser and the soil particles were restrained from moving.

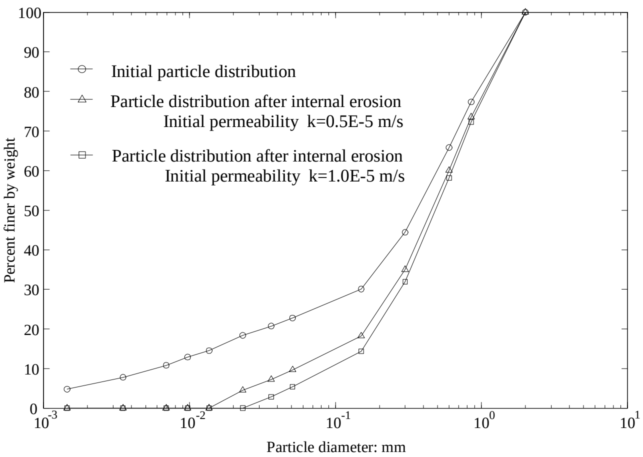

Figure 2: Alteration of particle distribution curve due to internal erosion from the initial state (○) to the steady state (△ and □), where internal erosion stops.

Next, the effect of initial imperfection was investigated, giving a scar at the center of the right side of a 1.0x2.0m soil block. Figs. 3 and 4 shows the finite element mesh and the numerical result of the pore water flux and the porosity distribution in a soil block which has been subjected to internal erosion for 166 hours. The values of the material parameters used in this analysis are the initial porosity of 0.31, the initial permeability of 1.0x10^{-6}, the critical shear stress of 0.8x10^{-5} kPa and the erodibility coefficient of 0.51x10^{-6} m/kPa/s. These values are similar to the previous analysis presented above, but the value critical shear stress is different. The water pressure of 9.8 kPa was applied on the left side and 0 kPa on the right side. The undrained condition was imposed on the top and the bottom. The pore water flowed rightward. As shown in Fig. 3, the internal erosion concentrated to the scar, the porosity just upstream of it increased rapidly, and the internal erosion of the soil block developed backward. These results were obtained when the shear stress exerted by the pore water flow onto the pore wall is slightly greater than the imposed critical shear stress.

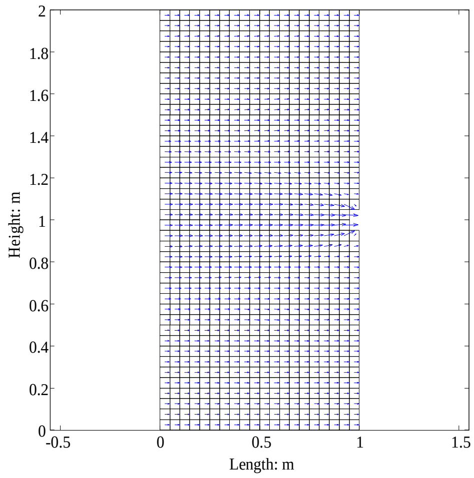

Figure 3: Finite element mesh and pore water flux in a 1.0x2.0m soil block which has a scar at the center of its right side (166 hours after internal erosion started).

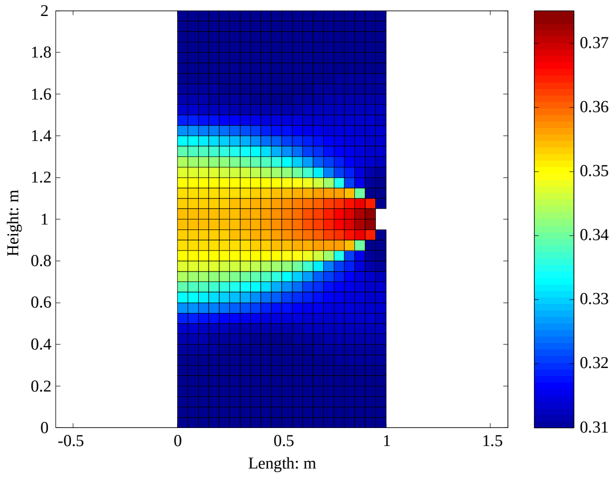

Figure 4: Finite element mesh and porosity distribution in a 1.0x2.0m soil block which has a scar at the center of its right side (166 hours after internal erosion started).

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