Numerical and mathematical approaches to analyses of water-circulator-induced flow in ponds

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1. Introduction

Pollution and muddiness of natural and artificial reservoirs that are used to supply water irrigation have become important problems in recent years in Japan. One cause of such turbid waters in small reservoirs, in which water flow is less active, is that stagnation of oxygen and water decrease aerobic decomposition capacity. Insolation stratifies the water in such small lakes and ponds according to a temperature gradient: a so-called thermocline. Consequently, vertically circulating flow is strongly suppressed.

A rotating propeller operating at low speed set on a lake surface is proposed because it is expected that the device can induce vertical circulating flow. Typically, a three- or four-bladed propeller forces rotation of the water of the surface and vertical circulating flow is induced because of the centrifugal force. Although various experiments have shown clearly that the water quality in a lake is improved by operation of such equipment, the flow mechanism is not fully understood. To optimize such equipment, mathematical methods are strongly required.

2. Formulation of the problem

- **Coordinate**

To survey such a fluid motion mathematically in a simple system, cylindrical coordinates (r, θ, z) are adopted. A radius of the cylindrical coordinate is r_1, a height of that is s and aspect ratio $\Gamma = \frac{s}{r_1}$. The flow confined between the top lid inducing a horizontal rotating flow, the bottom stationary lid, and the stationary wall are investigated here. Reynolds number is defined as

$$Re = \frac{\max(V(r,s))r_1}{\nu},$$

where $V(r,s)$ is the top boundary condition and ν is viscosity.

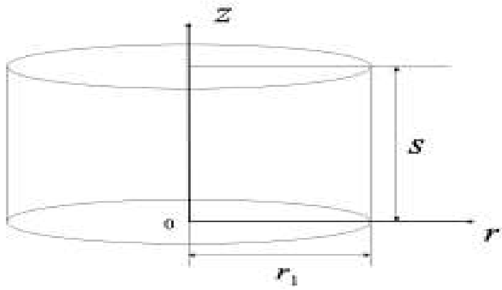

Fig. 1: Cylindrical coordinate.

The top lid

We set

$$V(r,s) = a \cdot r \cdot \exp(1 - a \cdot r) \quad (1)$$

as the top lid boundary condition, where a is selected due to $V(r_1, s) \approx 0$.

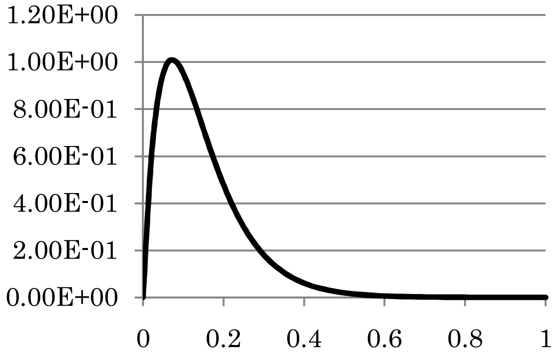

Fig. 2: A distribution of $V(r, s)$.

Governing equations

Since we hope to obtain steady-state solutions numerically for $\Gamma = 1$ and $Re = 50$ and 1000, we solve the time-dependent Navier-Stokes equation (2)-(4) and continuity equation (5) under some initial conditions and expect to obtain different steady-state solutions.

I. Momentum conversation law

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{\partial p}{\partial r} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right) \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}+w\frac{\partial v}{\partial z}+\frac{uv}{r}$$
$$=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}+\frac{\partial^2 v}{\partial z^2}+\frac{v}{r}+\frac{2}{r^2}\frac{\partial v}{\partial \theta}\right) \quad (3)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial \theta}+w\frac{\partial w}{\partial z}=-\frac{1}{r}\frac{\partial p}{\partial z}+\frac{1}{Re}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}+\frac{\partial^2 w}{\partial z^2}\right) \quad (4)$$

II. Mass conversation law

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{\partial w}{\partial z}=0 \quad (5)$$

In numerical computations, we assume that steady-state solutions have been obtained when residuals are under 10^{-4}.

- **Initial conditions**

We survey three kinds of initial conditions, Case 1, Case 2 and Case 3.

$$\text{Case 1}\begin{cases}u=0\\ v=0\\ w=0\end{cases},\ \text{Case 2}\begin{cases}u=0\\ v=V(r,s)\\ w=0\end{cases},\ \text{Case 3}\begin{cases}u=0\\ v=V(r,z)\\ w=0\end{cases}$$

In Case 3, a velocity component for the azimuthal direction, $V(r,z)$, is set as follows;

$$V(r,z)=\sum_{l=1}^{\infty}A_l J_1(k_1 r)\sinh(k_l z) \quad (6),$$

where A_l is a constant number, J_1 is Bessel function of the first kind and k_l is zero points. $V(r,z)$ is zero on the axis and nearly equals zero on the wall.

3 Results and future works

- $\boldsymbol{Re=50}$

  I. It is considered that Case 1 and Case 2 are the same distributions.
  II. It is expected that flow fields converged the same distribution assuming Case 1 or Case 2 as initial conditions.
  III. A distribution of Case 3 is different from those of Case 1 and Case 2.

- $\boldsymbol{Re=1000}$

  I. It is expected that Case 1, Case 2 and Case 3 are not the same distributions.

It is planned that future works will include numerical investigations with respect to various Reynolds numbers and aspect ratios.

References

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