Motion of Aquatic Plants Interacting with Water Flows

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1 Introduction

In this study, an interaction between motions of sea grasses and sea water flows is studied numerically in order to optimize the way of transplanting sea grasses. Transplantation of sea grasses is widely tried in many places in Japan because sea grass field is playing a very important role in coastal ecosystems: it yields oxygen by photosynthesis, provides egg-laying site for fishes, and so on. However, it has been widely lost by shore protection works, reclamation works, and etc. The objectives of this study are to simulate motions of sea grasses by wave actions and to provide useful information to transplantation of sea grasses.

2 Governing Equations

2.1 Motion of a sea grass

A Motion of a sea grass is assumed to be governed by the following balance equation;

$$\rho_a \frac{\partial^2 \boldsymbol{x}}{\partial t^2} + c_a \frac{\partial \boldsymbol{x}}{\partial t} + EI \frac{\partial^4 \boldsymbol{x}}{\partial s^4} - \frac{\partial}{\partial s}\left(T \frac{\partial \boldsymbol{x}}{\partial s} \right) = \boldsymbol{f}, \tag{1}$$

where $\boldsymbol{x}$ is a position vector at s, c_a is a dumping coefficient, EI is a bending stiffness and T is a tension along the plant body. A sea grass is assumed to be incompressible;

$$\left| \frac{\partial \boldsymbol{x}}{\partial s} \right|^2 = 1 \tag{2}$$

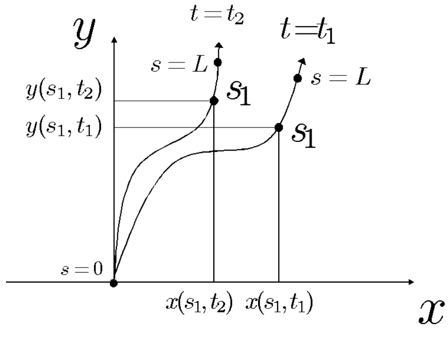

Figure 1: Geometry

Equations (1) and (2) are discretized for time as follows;

$$\rho_a \frac{\boldsymbol{x}^{n+1} - 2\boldsymbol{x}^n + \boldsymbol{x}^{n-1}}{\Delta t^2} + c_a \frac{\boldsymbol{x}^{n+1} - \boldsymbol{x}^n}{\Delta t} + EI \frac{\partial^4 \boldsymbol{x}^{n+1}}{\partial s^4} - \frac{\partial}{\partial s}\left(T^{n+1} \frac{\partial \boldsymbol{x}^{n+1}}{\partial s}\right) = \boldsymbol{f}^n, \tag{3}$$

$$\left|\frac{\partial \boldsymbol{x}^{n+1}}{\partial s}\right|^2 = 1 \tag{4}$$

Then, by taking the inner product between (3) and $\frac{\partial \boldsymbol{x}^{n+1}}{\partial s}$, differentiating it with respect to s, and substituting (4) into it, an equation for the tension T^{n+1} is obtained;

$$\frac{\partial^2 T^{n+1}}{\partial s^2} - T^{n+1}\left(\frac{\partial^2 \boldsymbol{x}^{n+1}}{\partial s^2}\right)^2 = \frac{\rho_a}{\Delta t^2}\left(1 - 2\frac{\partial \boldsymbol{x}^{n+1}}{\partial s} \cdot \frac{\partial \boldsymbol{x}^n}{\partial s} + \frac{\partial \boldsymbol{x}^{n+1}}{\partial s} \cdot \frac{\partial \boldsymbol{x}^{n-1}}{\partial s}\right)$$
$$+ \frac{c_a}{\Delta t}\left(1 - \frac{\partial \boldsymbol{x}^{n+1}}{\partial s} \cdot \frac{\partial \boldsymbol{x}^n}{\partial s}\right) + EI \frac{\partial^4 \boldsymbol{x}^{n+1}}{\partial s^4} \cdot \frac{\partial \boldsymbol{x}^{n+1}}{\partial s} - \frac{\partial \boldsymbol{f}^n}{\partial s} \cdot \frac{\partial \boldsymbol{x}^{n+1}}{\partial s} \tag{5}$$

2.2 Motion of sea water

A motion of sea water around sea grasses are governed by incompressible Navier-Stokes equations;

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla) = \frac{1}{\rho_w} \nabla p + \nu \nabla^2 \boldsymbol{u} - c\chi(\boldsymbol{u} - \boldsymbol{u}_a), \tag{6}$$

$$\nabla \cdot \boldsymbol{u} = 0, \tag{7}$$

where the existence of sea grasses is represented by fictitious domain method. χ is a characteristic function of sea grasses and $\boldsymbol{u}_a$ is the moving velocity of sea grasses. External force f in the motion equation for the sea grass is computed by an integration of pressure p around the plant body and a gravitational force.

3 Numerical Results

Figures (2), (3) and (4) show the shapes of sea grasses and velocity vectors of sea water.

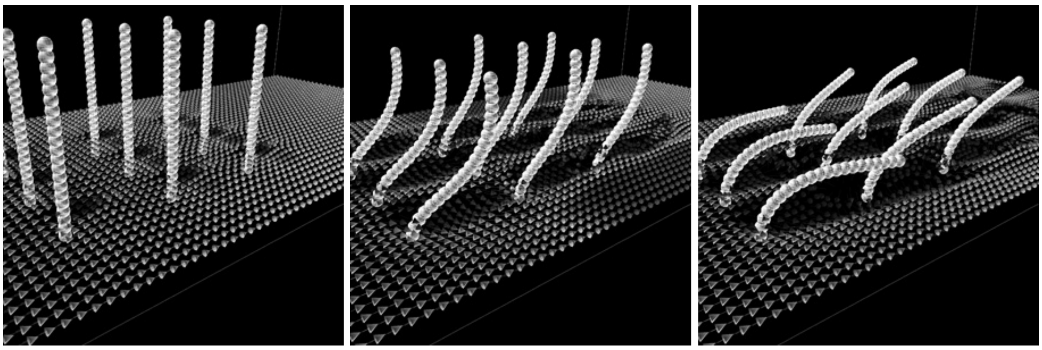

Figure 2: Shapes of sea grasses and a flow field (horizontal cross sections)

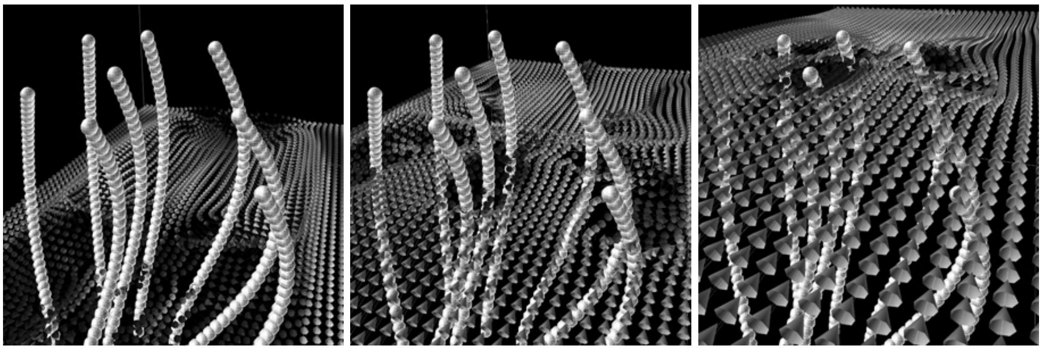

Figure 3: Shapes of sea grasses and a flow field (horizontal cross sections)

Figure 4: Shapes of sea grasses and a flow field (vertical cross sections)

4 Conclusions

In this study, a numerical model for computing the motions of sea grasses interacting with water flow is presented. Although further investigation for more accurate and realistic numerical model is required, this kind of simulation is expected to be helpful for understanding the mechanisms of its motion and to provide useful information to transplantation of sea grasses.

5 Acknowledgement

The authors would like to thank Prof. H. Tamaki at Ishinomaki-Senshu University for providing real-site data for sea grass fields.

References

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