Discussion Paper, Series A, No. 2008-201

# Partial Tax Coordination in a Repeated Game Setting

Jun-ichi Itaya, Makoto Okamura, Chikara Yamaguchi

October, 2008

Graduate School of Economics & Business Administration
Hokkaido University
Kita 9 Nishi 7, Kita-Ku, Sapporo 060-0809, JAPAN

# Partial Tax Coordination in a Repeated Game Setting*

Jun-ichi Itaya†, Makoto Okamura‡, Chikara Yamaguchi§

October 19, 2008

**Abstract**

This paper addresses the problem of partial tax coordination among regional or national sovereign governments in a repeated game setting. We show that partial tax coordination is more likely to prevail if the number of regions in a coalition subgroup is smaller and the number of existing regions in the entire economy is larger. We also show that under linear utility, partial tax coordination is more likely to prevail if the preference for a local public good is stronger. The main driving force for these results is the response of the intensity of tax competition. The increased (decreased) intensity of tax competition makes partial tax coordination more (less) sustainable.

*JEL classification:* H71; H77

*Keywords:* Partial tax coordination; Repeated game; Tax competition

*The first (Itaya), second (Okamura) and third (Yamaguchi) authors gratefully acknowledge the financial support provided by the Grant-in-Aid for Scientific Research from the Japan Society for the Promotion of Science (#19530145), (#20330052), and (#20730226), respectively.

†Corresponding author. Graduate School of Economics and Business Administration, Hokkaido University, Sapporo 060-0809, Japan. Tel:#+81-11-706-2858; Fax:#+81-11-706-4947; E-mail: itaya@econ.hokudai.ac.jp

‡Economics Department, Hiroshima University, 1-2-1 Kagamiyama, Higashihiroshima, Hiroshima 739-8526, Japan. Tel:#+81-82-424-7275; E-mail: okamuram@hiroshima-u.ac.jp

§Faculty of Economic Sciences, Hiroshima Shudo University, 1-1-1, Ozukahigashi, Asaminami-ku, Hiroshima 731-3195, Japan. Tel:#+81-82-830-1238; Fax:#+81-82-830-1313; E-mail: chikara@shudo-u.ac.jp

# 1 Introduction

This paper addresses the question of how and under what conditions partial tax coordination is sustained in a repeated interactions model. The coordination of tax policies among sovereign jurisdictions has often been considered as a remedy against inefficiently low taxes on mobile tax bases induced by tax competition. Although tax coordination among all the regions in the whole economy is desirable, generally, it is difficult to achieve full tax coordination because some regions may prefer a lower tax status for commercial reasons (i.e., the so-called tax heaven) or because the differences in social, cultural, and historical factors or economic fundamentals such as endowments and technologies may prevent the regions from accepting a common tax rate. Therefore, partial tax coordination, rather than global or full tax harmonization, is politically more acceptable. As a result, one could be compelled to resort to partial tax coordination. Indeed, the scenario with partial tax coordination is of particular importance since it is more likely to occur within a subgroup of countries like the European Union (EU) member states with close economic and political links.

The academic concern has been fuelled by the increasing public debate on partial tax coordination such as EU corporate tax coordination, which has let to produce several literature on partial tax coordination. Konrad and Schjelderup (1999) demonstrate that in the standard tax competition framework with identical countries, partial tax coordination among some regions can improve not only the welfare of the cooperating regions but also of the noncooperating ones. Rasmussen (2001) points out that by using a numerical analysis, the critical mass of countries needed for partial coordination to matter significantly is likely to be a very large percentage of the economies of the world, with the main benefit accrued to the nonparticipating countries. More recently, Sugahara et al. (2007) extended Konrad and Schjelderup's model by introducing regional asymmetries and showed that Konrad and Schjelderup's conclusion remains valid even in a multilateral asymmetric tax competition model. Marchand et al. (2003) addresses capital and labor taxes in a partial tax coordination model and show that when the taxes are used for redistributive purposes, the redistribution from capital owners to workers enhances welfare.

These papers provide valuable insights into partial tax cooperation; however, they employ a static framework despite the obvious fact that the interaction between regional or state governments is not with once. Apart from the reality of a repeated interactions setting, it is well known that repeated interactions facilitate cooperation, and hence, the use of a repeated interactions model would provide a better explanation of sustained fiscal cooperation among regional governments. More importantly, because of the limitation of a static analysis, the analytical focus of the abovementioned papers lies mainly on whether or not there is a welfare-enhancing tax coordination as compared to a fully noncooperative Nash equilibrium, which does not allow for any coalition among regions. However, it does not suffice to guarantee the sustainability of such a coalition of a subgroup. This is because in the context of static (one-shot) tax competition, the structure of payoffs accrued to regions displays characteristics of "Prisoner's dilemma", which is mainly caused by a positive fiscal externality associated with regional tax policies (see Wildasin, 1989). In this case, the coordinating regions are unable to reach a Pareto superior (or efficient) outcome even if it exists and to sustain it as a self-enforced equilibrium, because there is a strong incentive for them to deviate from an agreed coordinated tax rate in order to reap gains.

Furthermore, most of the abovementioned papers take a group of countries that may form a tax-harmonized area as given and focus on whether or not tax harmonization is beneficial to this given group. In other words, the question of how such a partial tax coordination among sovereign jurisdictions arises is abstracted. Thus, the given number of regions in the coalition subgroup may be too large to deter some regions to deviate from this coalition. Therefore, it is natural to explore under what conditions a coalition of a subgroup of regions is sustainable as a next step for the analysis. Burbidege et al. (1997), within the context of static tax competition games, have explored whether a subgroup of regions satisfies "the stability of coalition" suggested by d'Aspremont et al. (1983). According to this concept, the current participating and nonparticipating regions should not have any incentive to change their positions; in other words, a coalition is internally (externally) stable if there is no incentive for a member (non-member) region to withdraw from (join) the coalition. However, partial cooperation in their models need to implicitly or explicitly assume the existence of an enforcement mechanism for

such collusive behavior. Indeed, Martin (2002, pp.297) criticizes this concept of cartel stability by stating in the context of cartels among firms restricting output that "Static models omit an essential element of the cost of defecting from an output-restricting equilibrium-the profit that is lost once rivals realize that the agreement is being violated....Whether or not output restriction is stable in a dynamic sense depends on whether or not gains from output expansion. But this trade-off cannot, by its inherently intertemporal nature, be analyzed in a static model." In contrast, the repeated game setting, by comparing such losses and gains, would induce the participating regions to implement partial tax coordination as an equilibrium outcome or their self-enforced behavior but also explicitly provide an enforcement mechanism by fully utilizing punishment schemes to deter deviation, as long as the participating regions are sufficiently patient.

There are several papers that investigate tax coordination in a repeated game setting. Cardarelli et al. (2002) and Contenaro and Vidal (2006) utilize a repeated interactions model to demonstrate that coordinated fiscal policies or tax harmonization is sustainable. More recently, Itaya et al. (2007) showed that as the regional asymmetries in capital net exporting positions, which is caused by regional differences in endowments and/or production technologies, increase, regions are more likely to cooperate on capital taxes. Nevertheless, all these papers deal only with global tax coordination among all regions.

This paper discusses partial tax coordination among regions in repeated interactions models with two types of governments: one that behaves as tax-revenue maximizers and the other that behaves as utility maximizers. In either setting, we not only show that partial tax coordination is possible if competing governments are sufficiently patient but also that it is more likely to prevail if the number of cooperating regions is smaller and the total number of existing regions in the economy is larger. Further, these findings not only reveal that tax competition potentially enhances the incentive of the regions to sustain partial tax coordination but also that both regions within and outside the coalition end up being better off.

The remainder of the paper is organized as follows. Section 2 presents the basic model structure and characterizes its one-shot, noncooperative solution. Section 3 constructs a repeated interactions model of partial tax coordination in which some regions cooperate with

regard to their tax policies, while the other regions do not. Section 4 investigates the likelihood of partial coordination in a repeated interactions setting. Section 5 conducts the same analysis in a model with linear utility. Section 6 concludes the paper with a discussion on extending our model. Some mathematical derivations are relegated to appendices.

## 2 The Model of Tax Competition

Consider an economy composed of $N$ identical regions. The regions are indexed by the subscript $i \in \mathbf{N} = \{1, \cdots, N\}$. In each region, there exist a regional government, households, and firms; households are immobile across the regions, while capital is perfectly mobile. These factors are used in the production of a single homogenous good. Following Bucovetsky (1991) and Haufler (1997), we assume the constant-returns-to-scale production function: $f(k_i) \equiv (A - k_i)k_i$, $i \in \mathbf{N}$, where the parameter $A > 0$ represents the level of productivity, and $k_i$ is the per capita amount of capital employed in region $i$. We further assume that $A > 2k_i$ in order to ensure a positive but diminishing marginal productivity of capital.

Public expenditures are financed by a source-based tax on capital. Firms behave competitively, and thus, production factors are priced at their marginal productivity:

$$r = f'(k_i) - \tau_i = A - 2k_i - \tau_i, \tag{1}$$

$$w_i = f(k_i) - k_i f'(k_i) = k_i^2, \tag{2}$$

where $\tau_i$ is the capital tax rate imposed by the government in region $i$, $r$ is the net return on capital, and $w_i$ is the region-specific wage rate. The entire supply of capital in the economy is $\overline{K}$. Each household inelastically supplies one unit of labor to regional firms so that the households in each region own $\overline{k} \equiv \overline{K}/N$ units of capital. Capital is allocated across the regions until the net return on capital is equalized. As a result, the arbitrage condition, $f'(k_i) - \tau_i = r = f'(k_j) - \tau_j$ for all $i$, $j$ but $i \neq j$, must hold in equilibrium. By inverting (1), the demand for capital in each region can be expressed by $k_i(r + \tau_i) = (1/2)(A - r - \tau_i)$.

After substituting all of the demand functions for capital, $k_i(r+\tau_i)$, $\forall i \in \mathbf{N}$, into $k_i$ in the capital market clearing condition, $\sum_{h=1}^{N} k_h = N\overline{k}$, we can derive the equilibrium interest rate $r^*$:

$$r^* = A - 2\overline{k} - \overline{\tau}, \tag{3}$$

where $\overline{\tau} \equiv (\sum_{h=1}^{N} \tau_h)/N$ is the average capital tax rate over all regions. By substituting (3) back into $k_i(r+\tau_i)$, the equilibrium amount of capital demanded in region $i$ can be expressed as

$$k_i^* = \overline{k} + \frac{1}{2}\left(\overline{\tau} - \tau_i\right), \ \ \forall i \in \mathbf{N}. \tag{4}$$

Differentiating (3) and (4) with respect to the capital income tax rate $\tau_i$ yields

$$\frac{\partial r^*}{\partial \tau_i} = -\frac{1}{N} < 0, \ \frac{\partial k_i^*}{\partial \tau_i} = -\frac{N-1}{2N} < 0, \text{ and } \frac{\partial k_j^*}{\partial \tau_i} = \frac{1}{2N} > 0, \ \ \forall i, \ j \in \mathbf{N} \text{ but } i \neq j. \tag{5}$$

The objective of region $i$'s government is to maximize its tax revenue, denoted by $R_i$. In the fully noncooperative symmetric Nash equilibrium, taking all of the other regional choices as given, the government in region $i$ independently chooses $\tau_i$ to maximize its tax revenue: $R_i = \tau_i k_i^*$.[1] Assuming an interior solution and taking (5) into account, we can compute the symmetric Nash equilibrium tax rate:[2]

$$\tau_{\mathbf{N}}^{NE} = \frac{2N\overline{k}}{N-1}. \tag{6}$$

Taking into account that $\tau_i = \tau_{\mathbf{N}}^{NE} = \overline{\tau}$ and using (3), the corresponding net return is given by $r^{NE} = A - 2\overline{k} - \tau_{\mathbf{N}}^{NE} > 0$. Moreover, it follows from (4) that $k_i^*$ ends up being equal to $\overline{k}$; that is, there is no capital trade in equilibrium. Combining this nontrade equilibrium condition and (6), the tax revenue, $R_{\mathbf{N}}^{NE} = \tau_{\mathbf{N}}^{NE} k_i^*$, can be rewritten as follows:

$$R_{\mathbf{N}}^{NE} = \frac{2N\overline{k}^2}{N-1}. \tag{7}$$

[1] This simplest objective function enables us to explicitly obtain the minimum discount factor for the repeated game setting described later and to carry out a comparative statics analysis with respect to some principle parameters. We will later conduct the same analysis under linear utility.

[2] We always focus on an interior solution when solving an optimization problem, and so we omit this qualification in what follows.

# 3 Partial Tax Coordination in a One-period Game

Let us suppose now that some regions coordinate their tax policies. More precisely, the subset of regions, denoted by $\mathbf{S} = \{1, \cdots, S\} \subset \mathbf{N}$, forms a coalition to coordinate their capital tax rates at some prescribed level, while the rest of the regions belonging to the complementary set $\mathbf{N} - \mathbf{S} = \{S+1, \cdots, N\}$ act fully noncooperatively. A coalition is defined as any (proper) subset of regions that contains at least two regions, and thus, the size of the coalition, $S$, is a positive integer between 2 and $N-1$. More precisely, all the participating regions cooperatively choose a capital tax rate in order to maximize the sum of the members' regional tax revenues, $R(\mathbf{S}) \equiv \sum_{h=1}^{S} R_h$, while each of the nonparticipating regions, which belongs to the set $\mathbf{N} - \mathbf{S}$, unilaterally maximizes its own regional tax revenue. By symmetry, every participating region willingly agrees to choose a common (or harmonized) capital tax rate. Taking as given the choices of tax rates by the participating, except for $i$, and the nonparticipating regions, the first-order condition for each coalition member is given by

$$\frac{\partial R(\mathbf{S})}{\partial \tau_i} = k_i^* + \sum_{h=1}^{S} \tau_h \frac{\partial k_h^*}{\partial \tau_i} = 0, \quad i \in \mathbf{S}. \tag{8}$$

Since all the regions simultaneously choose their capital tax rates, taking as given the choices of the tax rates by the coalition group and other nonparticipating regions, each nonparticipating region unilaterally chooses a capital tax rate so as to maximize its own tax revenue $R_j$. As a result, the first-order condition is

$$\frac{\partial R_j}{\partial \tau_j} = k_j^* + \tau_j \frac{\partial k_j^*}{\partial \tau_j} = 0, \quad j \in \mathbf{N} - \mathbf{S}. \tag{9}$$

Substituting (4) and (5) into (8) and (9) and utilizing symmetry, we first obtain the best-response functions of the participating and nonparticipating regions, respectively:

$$\tau_{\mathbf{S}} = \frac{1}{2}\tau_{\mathbf{N}-\mathbf{S}} + \frac{N}{N-S}\overline{k}, \tag{10}$$

$$\tau_{\mathbf{N}-\mathbf{S}} = \frac{1}{N+S-1}\left(S\tau_{\mathbf{S}} + 2N\overline{k}\right), \tag{11}$$

where $\tau_{\mathbf{S}}$ and $\tau_{\mathbf{N-S}}$ represent the capital tax rates for regions within and outside the coalition group, respectively. Note that the chosen regional tax rates given by (10) and (11) display strategic complements, thus making these reaction functions upward sloping. Hence, this property, together with the observation that $\partial\tau_{\mathbf{S}}/\partial\tau_{\mathbf{N-S}} < 1$ and $\partial\tau_{\mathbf{N-S}}/\partial\tau_{\mathbf{S}} < 1$, ensures the uniqueness of the resulting Nash equilibrium, which we call "a Nash subgroup equilibrium" in order to distinguish it from the fully noncooperative Nash equilibrium analyzed in the previous section (see Konrad and Schjelderup, 1999).

By solving the simultaneous system of equations (10) and (11) for $\tau_{\mathbf{S}}$ and $\tau_{\mathbf{N-S}}$, respectively, we can compute the Nash subgroup equilibrium tax rates for the coalition group and noncoalition regions, respectively:

$$\tau_{\mathbf{S}}^{C} = \frac{2N(2N-1)\overline{k}}{(N-S)(2N+S-2)}, \tag{12}$$

$$\tau_{\mathbf{N-S}}^{C} = \frac{2N(2N-S)\overline{k}}{(N-S)(2N+S-2)}. \tag{13}$$

The comparison of these tax rates with the fully noncooperative Nash equilibrium tax rate in (6) results in

$$\tau_{\mathbf{N}}^{NE} < \tau_{\mathbf{N-S}}^{C} < \tau_{\mathbf{S}}^{C}. \tag{14}$$

We now explore whether the subgroup of the cooperating regions can improve their tax revenues by implementing partial tax coordination. For this, we first substitute (12) and (13) into (4) to obtain the demand functions for the capital of the coalition and noncoalition regions, respectively:

$$k_{\mathbf{S}}^{C} = \overline{k} + \frac{N-S}{2N}\left(\tau_{\mathbf{N-S}}^{C} - \tau_{\mathbf{S}}^{C}\right), \tag{15}$$

$$k_{\mathbf{N-S}}^{C} = \overline{k} + \frac{S}{2N}\left(\tau_{\mathbf{S}}^{C} - \tau_{\mathbf{N-S}}^{C}\right). \tag{16}$$

By multiplying (12) and (13) by (15) and (16), respectively, we finally obtain the tax revenues

of the participating and nonparticipating regions:

$$R_{\mathbf{S}}^{C} = \frac{2N\left(2N-1\right)^{2}\overline{k}^{2}}{\left(N-S\right)\left(2N+S-2\right)^{2}}, \tag{17}$$

$$R_{\mathbf{N-S}}^{C} = \frac{2N\left(N-1\right)\left(2N-S\right)^{2}\overline{k}^{2}}{\left(N-S\right)^{2}\left(2N+S-2\right)^{2}}. \tag{18}$$

By comparing among (7), (17), and (18), we can show that

$$R_{\mathbf{N}}^{NE} < R_{\mathbf{S}}^{C} < R_{\mathbf{N-S}}^{C}. \tag{19}$$

It should be emphasized that both the regions within and outside the coalition clearly benefit from the creation of a subgroup coalition of cooperating regions. In other words, the creation of a subgroup coalition generates higher tax revenues as compared to the fully noncooperative symmetric Nash equilibrium. The intuition behind the result is as follows. A coordinated increase in the capital tax rate chosen by the coalition group tends to relax the intensity of tax competition between the coalition group and nonparticipating regions, thereby inducing the former to raise their tax rates as well (called the tax-rate effect), as indicated in (14). Moreover, since the tax rate set by the nonparticipating regions, $\tau_{\mathbf{N-S}}^{C}$, is less than that set by the coalition group, $\tau_{\mathbf{S}}^{C}$, according to (14), the participating and nonparticipating regions, respectively, become capital exporters and importers, which is implied by (15) and (16) (called the tax-base effect).

It is important to note that there is an incentive for regions to form a coalition in order to coordinate their capital tax rates, since the tax revenues of the cooperating regions are larger than those in the fully noncooperative symmetric Nash equilibrium. However, this gain does not necessarily deter the deviation of a member region from the coalition on the grounds that the regions can potentially benefit more from being noncoalition members.

# 4 A Repeated Game

In this section, we construct a simple repeated partial tax coordination game with a common discount factor denoted by $\delta \in [0, 1)$. Let us assume that in every period, each participating region agrees to coordinate its capital tax rate at the common tax rate $\tau_{\mathbf{S}}^{C}$ provided that all of the other member regions had followed the common tax rate in the previous period. If a participating region deviates from it, then their coalition collapses, triggering the punishment phase that results in a fully noncooperative Nash equilibrium, which persists forever. The condition to sustain partial tax coordination is given by

$$\frac{1}{1-\delta} R_{\mathbf{S}}^{C} \geq R_{i}^{D} + \frac{\delta}{1-\delta} R_{\mathbf{N}}^{NE},\ i \in \mathbf{S}, \tag{20}$$

where $R_i^D$ represents the tax revenue for the deviating region $i$. The left-hand side of (20) is the discounted total tax revenue of region $i$ when all coalition members belonging to the set $\mathbf{S}$ continue to maintain $\tau_{\mathbf{S}}^{C}$ infinitely. The right-hand side represents the sum of the current period's tax revenue associated with the best-deviation tax rate $\tau_i^D$ and the discounted total tax revenues associated with the fully noncooperative Nash equilibrium in all subsequent periods. Because of symmetry, the conditions in (20) reduce to a single condition.

The best-deviation tax rate $\tau_i^D$ is chosen so as to maximize the tax revenue of region $i$, given that the other $S-1$ participating regions and all $N-S$ nonparticipating regions follow $\tau_{\mathbf{S}}^{C}$ and $\tau_{\mathbf{N-S}}^{C}$, respectively. Solving the first-order condition for the deviating region $i$ for $\tau_i^D$ yields

$$\tau_{i}^{D} = \frac{N}{N-1}\left[\overline{k} + \frac{(S-1)\,\tau_{\mathbf{S}}^{C} + (N-S)\tau_{\mathbf{N-S}}^{C}}{2N}\right]. \tag{21}$$

By substituting (12) and (13) into (21), the best-deviation tax rate can be expressed as

$$\tau_{i}^{D} = \frac{N\,(2N-1)\,(2N-S-1)\overline{k}}{(N-1)\,(N-S)\,(2N+S-2)}. \tag{22}$$

Comparing (22) with (6), (12), and (13) reveals that

$$\tau_{\mathbf{N}}^{NE} < \tau_{i}^{D} < \tau_{\mathbf{N-S}}^{C} < \tau_{\mathbf{S}}^{C}. \tag{23}$$

That is, the best-deviation tax rate is the second lowest tax rate; in other words, it is still larger than the fully noncooperative Nash equilibrium tax rate.

By substituting (12), (13), and (22) into (4) and rearranging, we can compute the capital demanded in the deviating region $i$, participating regions in the set $\mathbf{S} - \{i\}$, and nonparticipating regions in the set $\mathbf{N} - \mathbf{S}$, respectively:

$$k_i^D = k_{\mathbf{N}-\mathbf{S}}^D + \frac{\tau_{\mathbf{N}-\mathbf{S}}^C - \tau_i^D}{2}, \tag{24}$$

$$k_{\mathbf{S}-i}^D = k_{\mathbf{S}}^C - \frac{\tau_{\mathbf{S}}^C - \tau_i^D}{2N}, \tag{25}$$

$$k_{\mathbf{N}-\mathbf{S}}^D = k_{\mathbf{N}-\mathbf{S}}^C - \frac{\tau_{\mathbf{S}}^C - \tau_i^D}{2N}. \tag{26}$$

By straightforward comparison among these capital demands, it is easy to show that $k_{\mathbf{S}-i}^D < k_{\mathbf{N}-\mathbf{S}}^D < k_i^D$, which implies that deviator $i$ not only changes its net capital position from an exporter to an importer but also becomes the largest capital importer by levying the lowest capital tax rate $\tau_i^D$ in the deviation phase. Moreover, although the unilateral deviation of region $i$ from the coordinated tax rate ends up lowering the capital demands of all regions except $i$, the capital demand of nonparticipating regions, $k_{\mathbf{N}-\mathbf{S}}^D$, is still larger than that at the fully noncooperative symmetric Nash equilibrium $\overline{k}$.

By utilizing (12), (13), (22), (24), (25), and (26), we obtain the tax revenues of the deviating region $i$, cooperating, and noncooperating regions, respectively:

$$\begin{aligned} R_i^D &= \frac{N(2N-1)^2(2N-S-1)^2\overline{k}^2}{2(N-1)(N-S)^2(2N+S-2)^2}, \\ R_{\mathbf{S}-i}^D &= \frac{N(2N-1)^2\left[2N(N-S-1)+S+1\right]\overline{k}^2}{(N-1)(N-S)^2(2N+S-2)^2}, \\ R_{\mathbf{N}-\mathbf{S}}^D &= \frac{N(2N-S)\left[2N^2(2N-S-4)+(2N-1)S+6N-1\right]\overline{k}^2}{(N-1)(N-S)^2(2N+S-2)^2}. \end{aligned} \tag{27}$$

By straightforward comparison, we find that $R_{\mathbf{S}-i}^D < R_{\mathbf{N}-\mathbf{S}}^D < R_i^D$. As expected, since deviator $i$ sets the least capital tax rate, it captures the largest one-period tax revenue.

Substituting (7), (17), and (27) into the equality in (20) and rearranging yields the mini-

mum discount factor of the coalition members as follows:

$$\delta\left(S,\ N\right) \equiv \frac{R_i^D - R_{\mathbf{S}}^C}{R_i^D - R_{\mathbf{N}}^{NE}} = \frac{\left(2N-1\right)^2\left(S-1\right)}{\left(2S-1\right)\left[2(2N+S)(N-S)+2N(2N-S-4)+5S+1\right]}. \quad (28)$$

Only when the actual discount factors for all the coalition members, $\delta$, which is common for all regions, are greater than or equal to $\delta\left(S,\ N\right)$, then the coordinated tax rate $\tau_{\mathbf{S}}^C$ can be sustained as a subgame perfect Nash equilibrium of the repeated game. It is also straightforward to confirm that $\delta\left(S,\ N\right) < 1$ for any integer $S$ and $N$.

Differentiating the minimum regional discount factor $\delta\left(S,\ N\right)$ with respect to the group size $S$ and the total number of regions $N$, respectively, yields

$$\frac{\partial\delta\left(S,\ N\right)}{\partial S} = \frac{4(2N-1)^2\left[N(2N-1)+2S(S-2)(N+S-1)+2S-1\right]}{(2S-1)^2\left[2(2N+S)(N-S)+2N(2N-S-4)+5S+1\right]^2} > 0, \quad (29)$$

$$\frac{\partial\delta\left(S,\ N\right)}{\partial N} = -\frac{4(2N-1)(S-1)\left[2S(N+S-2)+1\right]}{(2S-1)\left[2(2N+S)(N-S)+2N(2N-S-4)+5S+1\right]^2} < 0. \quad (30)$$

Furthermore, we have

$$\lim_{N\to\infty}\delta\left(S,\ N\right) = \frac{S-1}{2\left(2S-1\right)} < \frac{1}{4} \quad \text{for any integer } S \geq 2,$$

implying that partial tax coordination can be sustained *irrespective of the group size*, provided the actual discount factors of the coalition members are sufficiently close to 1.

These observations lead to the following proposition:

**Proposition 1** *(i) If all the participating regions are sufficiently patient, partial tax coordination can be sustained as a subgame perfect Nash equilibrium of the repeated game irrespective of the size of the coalition;*
*(ii) the larger (smaller) the number of participating regions, the more difficult (easier) it is for partial tax coordination to prevail; and*
*(iii) the larger (smaller) the total number of regions in the economy, the easier (more difficult) it is for partial tax coordination to prevail.*

To gain the insight underlying Proposition 1, we need to know how an increase in the

coalition size $S$ (or the total number of regions, $N$) affects the tax revenues of the respective regions at all phases of the present repeated game. To this end, we first differentiate $R_{\mathbf{S}}^C$ with respect to $S$ to get

$$\frac{\partial R_{\mathbf{S}}^C}{\partial S} = \underbrace{k_{\mathbf{S}}^C \frac{\partial \tau_{\mathbf{S}}^C}{\partial S}}_{\text{tax-rate effect } (+)} + \underbrace{\tau_{\mathbf{S}}^C \frac{\partial k_{\mathbf{S}}^C}{\partial S}}_{\text{tax-base effect } (-)} = \frac{2N(2N-1)^2(3S-2)\overline{k}^2}{(N-S)^2(2N+S-2)^3} > 0, \tag{31}$$

which reveals that increasing the group size $S$ has two opposite effects on the tax revenues of the participating regions; that is, the first and second terms in the middle expression of (31) stand for the *positive* tax-rate and *negative* tax-base (i.e., fiscal externality) effects, respectively. Since an increase in $S$ mitigates the intensity of tax competition, the tax rates imposed by the participating and nonparticipating regions both rise (recall that the choice variables are strategic complements). Moreover, since it is confirmed by straightforward computation that the tax rate set by the coalition subgroup rises more than that set by the noncoalition regions, i.e., $\partial \tau_{\mathbf{S}}^C / \partial S > \partial \tau_{\mathbf{N-S}}^C / \partial S > 0$, it further increases the tax differential, $\tau_{\mathbf{S}}^C - \tau_{\mathbf{N-S}}^C > 0$. This impact in turn shrinks the tax bases of the coalition members, as indicated in (15), simply because the relatively higher tax rate $\tau_{\mathbf{S}}^C$ causes a capital flight from the coalition group to the noncoalition regions. As shown in the last expression in (31), however, the *positive* tax-rate effect dominates the *negative* tax-base effect in absolute value, thus resulting in larger tax revenues accrued to the coalition members.

Similarly, the effect of an increase in $S$ on the tax revenue of the deviating region $i$ can be decomposed into the tax-rate and tax-base effects as stated above:

$$\frac{\partial R_i^D}{\partial S} = \underbrace{k_i^D \frac{\partial \tau_i^D}{\partial S}}_{(+)} + \underbrace{\tau_i^D \frac{\partial k_i^D}{\partial S}}_{(+)} > 0. \tag{32}$$

Although an increase in $S$ unambiguously increases $\tau_i^D$ as a result of the mitigated pressure of tax competition as before, in order to precisely identify the effect on $k_i^D$, we need to know the effect of increasing $S$ on the tax rates set by the respective regions (recall (24)). It is straightforward to show by verifying (12) and (13) that $\partial \tau_{\mathbf{S}}^C / \partial S > \partial \tau_{\mathbf{N-S}}^C / \partial S > \partial \tau_i^D / \partial S > 0$. Hence, an increase in $S$ enlarges the gap between the taxes set by the deviating region $i$ and

the coalition group, as well as that set by the deviating region $i$ and the nonparticipating regions. As a result, since the deviating region can attract more capital from the participating and nonparticipating regions, the tax revenue accrued to the deviating region unambiguously increases due to the resulting larger tax base multiplied by the higher tax rate.

With these results, we can shed some light on how changes in the group size affect the likelihood of cooperation. Each participating region has to compare the gain from its unilateral deviation with the opportunity cost when reverting to the fully noncooperative Nash equilibrium in all the subsequent periods in order to decide on whether or not to remain in the coalition. To this end, replacing the actual discount factor $\delta$ with the minimum discount factor $\delta(S,\ N)$ in (20) and subtracting $R_{\mathbf{S}}^{C}$ from the resulting equality yields the following expression:

$$\frac{\delta(S,\ N)}{1-\delta(S,\ N)}\left(R_{\mathbf{S}}^{C}-R_{\mathbf{N}}^{NE}\right)=R_{i}^{D}-R_{\mathbf{S}}^{C}. \tag{33}$$

The left-hand side of (33) represents the discounted future (opportunity) costs from region $i$'s unilateral deviation if this region has the discount factor $\delta(S,\ N)$, while its right-hand side is the current gain from deviating. For ease of exposition, we further decompose the discounted future costs into two components: the discount factor component $\delta(S,\ N)/(1-\delta(S,\ N))$ and the opportunity cost incurred by the deviator, $R_{\mathbf{S}}^{C}-R_{\mathbf{N}}^{NE}$. It follows from (7) ($R_{\mathbf{N}}^{NE}$ is independent of $S$), (31), and (32) that the future loss, $R_{\mathbf{S}}^{C}-R_{\mathbf{N}}^{NE}$, and the current gain, $R_{i}^{D}-R_{\mathbf{S}}^{C}$, are both increasing in $S$. Since (29) indicates that the gain is larger than the loss, this region has an incentive to deviate. This stems from the fact that the effect of increasing $S$ on $R_{i}^{D}$ should be much larger than that on $R_{\mathbf{S}}^{C}$, since the tax-rate and tax-base effects on $\partial R_{\mathbf{S}}^{C}/\partial S$ operate in opposite directions, whereas these two effects on $\partial R_{i}^{D}/\partial S$ do in the same direction. Hence, the minimum discount factor $\delta(S,\ N)$ should be higher so as to satisfy the equality in (33).

Next, we can investigate how increasing the total number of regions, $N$, affects the incentive of the participating regions to maintain partial tax coordination in an analogous manner.

Differentiating $R_{\mathbf{S}}^C$ with respect to $N$ yields

$$\frac{\partial R_{\mathbf{S}}^C}{\partial N} = \underbrace{k_{\mathbf{S}}^C \frac{\partial \tau_{\mathbf{S}}^C}{\partial N}}_{(-)} + \underbrace{\tau_{\mathbf{S}}^C \frac{\partial k_{\mathbf{S}}^C}{\partial N}}_{(+)} < 0.$$

Although we can identify the tax-rate and tax-base effects of the increase in $N$ on the tax revenues accrued to the participating regions as before, the signs of these two effects are reversed to those resulting from increasing $S$. An increase in $N$ intensifies tax competition, which in turn depresses the tax rates set by the participating and nonparticipating regions. Since it can be further verified that $\partial \tau_{\mathbf{S}}^C / \partial N < \partial \tau_{\mathbf{N-S}}^C / \partial N < 0$ – that is, the tax rate set by the participating regions falls more than that set by the nonparticipating regions – their tax differential will shrink, thus reducing the amount of capital flight from the coalition to the nonparticipating regions and increasing the tax bases of the participating regions. Nevertheless, since the negative tax-rate effect dominates such a positive tax-base effect, the increase in $N$ reduces the tax revenues accrued to the participating regions $R_{\mathbf{S}}^C$.

In the deviation phase, by differentiating (22) and (24) with respect to $N$, we can confirm that the tax-rate and tax-base effects are both negative; hence, the effect on the tax revenue of the deviating region is as follows:

$$\frac{\partial R_i^D}{\partial N} = \underbrace{k_i^D \frac{\partial \tau_i^D}{\partial N}}_{(-)} + \underbrace{\tau_i^D \frac{\partial k_i^D}{\partial N}}_{(-)} < 0.$$

Since it is straightforward to check that $\partial \tau_{\mathbf{S}}^C / \partial N < \partial \tau_{\mathbf{N-S}}^C / \partial N < \partial \tau_i^D / \partial N < 0$, the increase in $N$ reduces all taxes as a result of the intensified tax competition. Since the tax rates chosen by the participating and nonparticipating regions fall more than that chosen by the deviating region, the decreased tax differential between the deviating and other regions reduces the tax base of deviator $i$, $k_i^D$. This impact, together with the negative tax-rate effect, reduces its tax revenue as well.

Finally, in the symmetric Nash equilibrium phase, an increase in $N$ creates the only negative

tax-rate effect via the intensified tax competition (recall $k_i^* = \overline{k}$):

$$\frac{\partial R_{\mathbf{N}}^{NE}}{\partial N} = \overline{k}\frac{\partial \tau_{\mathbf{N}}^{NE}}{\partial N} < 0.$$

To sum up, an increase in the total number of regions, $N$, reduces both the current gain from deviating, $R_i^D - R_{\mathbf{S}}^C$, and the future opportunity cost incurred by the deviator, $R_{\mathbf{S}}^C - R_{\mathbf{N}}^{NE}$, in every period, and (30) indicates that the reduction in $R_i^D$ is in absolute value much larger than that in $R_{\mathbf{S}}^C$. As a result, the only lower minimum discount factor $\delta(S,\ N)$ can satisfy the equality of (33) at the resulting equilibrium.

Although the general message of the literature on tax competition is that there are various potential inefficiencies associated with tax competition, our analysis based on the repeated interactions model indicates that the intensified tax competition (associated with the larger $N$) makes the coalition members more cooperative to sustain partial tax coordination, *which ends up benefiting all the regions.* In the context of partial tax coordination, therefore, tax competition is beneficial. Since it is well documented that tax competition may have efficiency- or welfare-enhancing effects, such as the benefits of restraining Leviathan tendencies for over-expansion of the public sector (Edwards and Keen, 1996), or of limiting the incentive for time-inconsistent governments to increase capital income taxes once an investment location decision has been made (Conconi et al. 2007), our finding, which has not been addressed in the literature that focuses on full-tax coordination would also provide one of such possibilities.

# 5 The Model under Linear Preferences

In this section, we assume that the objective of regional governments is to maximize the representative resident's utility rather than the tax revenues. Each household residing in region $i$ derives utility from the consumption of a single homogenous good $x_i$ and a local public good (or redistributed income to households) $G_i$. By making use of (2), (3), and (4), the budget constraint of a representative inhabitant in the region $i$, $x_i = w_i + r^*\overline{k}$, can be rewritten as $x_i = f(k_i^*) + r^*\left(\overline{k} - k_i^*\right) - \tau_i k_i^*$. Given the budget constraints for households and the region $i$'s government, $G_i = \tau_i k_i^*$, the government selects $\tau_i$ so as to maximize its resident's

utility $U_i$ defined below. To facilitate explicit analytical solutions to our repeated interactions model, following Cardarelli et al. (2002) and Itaya et al. (2007), we assume a linear utility function such as

$$\begin{aligned} U_i &\equiv U(x_i, G_i) \equiv x_i + \gamma G_i, \\ &= (A - k_i^*)\, k_i^* + r^* \left(\overline{k} - k_i^*\right) + (\gamma - 1)\, \tau_i k_i^*, \end{aligned} \tag{34}$$

where $\gamma > 1$ denotes a preference parameter toward the local public good $G_i$.[3] Taking (5) into account, we can obtain the fully noncooperative symmetric Nash equilibrium tax rate:

$$\tau_{\mathbf{N}}^{NE} = \frac{2N(\gamma - 1)\overline{k}}{(N-1)\gamma}. \tag{35}$$

As in the previous model, taking as given the choices of tax rates by all the other regions, all coalition members cooperatively choose a common capital tax rate so as to maximize the welfare function: $W(\mathbf{S}) \equiv \sum_{h=1}^{S} U_h$. The resulting first-order condition is

$$\frac{\partial W(\mathbf{S})}{\partial \tau_i} = (\gamma - 1)k_i^* + \gamma \sum_{h=1}^{S} \tau_h \frac{\partial k_h^*}{\partial \tau_i} + \sum_{h=1}^{S} \frac{\partial r^*}{\partial \tau_i}(\overline{k} - k_h) = 0,\ i \in \mathbf{S}, \tag{36}$$

which, by symmetry, is reduced to a single equation. As before, each nonparticipating region simultaneously and independently chooses its capital tax rate in order to maximize its own utility $U_j$. The first-order condition leads to

$$\frac{\partial U_j}{\partial \tau_j} = (\gamma - 1)k_j^* + \gamma \tau_j \frac{\partial k_j^*}{\partial \tau_j} + \frac{\partial r^*}{\partial \tau_j}(\overline{k} - k_j^*) = 0,\ j \in \mathbf{N} - \mathbf{S}, \tag{37}$$

which also is reduced to a single equation. Solving (36) and (37) simultaneously for $\tau_i$ and $\tau_j$, together with (4) and (5), yields the Nash subgroup equilibrium tax rates (see Appendix

[3] Since the marginal rate of substitution between private consumption $x_i$ and a local public good $G_i$ is equal to 1, the condition $\gamma > 1$ is necessary to ensure an interior solution for $G_i$.

A for derivations):

$$\tau_{\mathbf{S}}^{C} = \frac{2(\gamma-1)\left[N(N(\gamma-1)+\gamma(N-1)+S)-S(S-1)\right]\overline{k}}{\gamma(N-S)\left[(2\gamma-1)(N-1)+\gamma S\right]}, \tag{38}$$

$$\tau_{\mathbf{N-S}}^{C} = \frac{2(\gamma-1)\left[N(N(2\gamma-1)-S(\gamma-1))-S(S-1)\right]\overline{k}}{\gamma(N-S)\left[(2\gamma-1)(N-1)+\gamma S\right]}. \tag{39}$$

The first-order condition, after substituting (38) and (39), yields the best-deviation tax rate (see Appendix A for derivations):

$$\tau_{i}^{D} = \frac{2(\gamma-1)\overline{k}\Lambda}{\gamma(N-1)(N-S)\left[N(2\gamma-1)+1\right]\left[(2\gamma-1)(N-1)+\gamma S\right]}, \tag{40}$$

where $\Lambda \equiv N(N-1)[(2\gamma-1)N+S][\gamma(N-S)+N(\gamma-1)+1]-(S-1)[N(\gamma-1)+1][S(N-1)+\gamma N] > 0$. Comparing the taxes given by (35), (38), (39), and (40), we obtain precisely the same ranking regarding the tax rates as (23) (see Appendix B), so does the net capital-exporting position of regions associated with the respective phases.

With these results, the condition to sustain partial tax coordination for each coalition member region is expressed as

$$\frac{1}{1-\delta}U_{\mathbf{S}}^{C} \geq U_{i}^{D} + \frac{\delta}{1-\delta}U_{\mathbf{N}}^{NE},\ i \in \mathbf{S}, \tag{41}$$

where $U_{\mathbf{S}}^{C}$, $U_{i}^{D}$, and $U_{\mathbf{N}}^{NE}$ represent the utility levels associated with the cooperative (i.e., partial coordination), deviation, and punishment (i.e., the fully noncooperative symmetric Nash equilibrium) phases, respectively. Utilizing (3), (4), (34), (35), (38), (39), and (40) and rearranging, we can compute the minimum discount factor (see Appendix C for derivations):

$$\delta(S,\ N,\ \gamma) \equiv \frac{U_{i}^{D}-U_{\mathbf{S}}^{C}}{U_{i}^{D}-U_{\mathbf{N}}^{NE}} = \frac{\gamma N^{2}(S-1)[N(2\gamma-1)-(\gamma-1)]^{2}}{[NS(2\gamma-1)+S-\gamma N]\Omega} < 1, \tag{42}$$

where $\Omega \equiv [(N-1)(\gamma-1)+\gamma N]\left[2(N-S)[N(2\gamma-1)+1]+\gamma N(S-1)\right]+\gamma(N-S)(S-1)\left[N(2\gamma-1)+1\right] > 0$. Differentiating the minimum discount factor in (42) with respect to the group size $S$, the total number of regions $N$, and the preference parameter $\gamma$, respectively,

yields the following results (see Appendix D):

$$\frac{\partial \delta\left(S,\ N,\ \gamma\right)}{\partial S} > 0, \tag{43}$$

$$\left.\frac{\partial \delta\left(S,\ N,\ \gamma\right)}{\partial N}\right|_{\gamma \in \left(1,\ \frac{7+\sqrt{13}}{6}\right)} \gtreqless 0 \ \text{ and } \ \left.\frac{\partial \delta\left(S,\ N,\ \gamma\right)}{\partial N}\right|_{\gamma \in \left[\frac{7+\sqrt{13}}{6},\ \infty\right)} < 0, \tag{44}$$

$$\frac{\partial \delta\left(S,\ N,\ \gamma\right)}{\partial \gamma} < 0. \tag{45}$$

Furthermore, it follows from (42) that if the number of regions, $N$, goes to infinity, then partial tax coordination can be sustained *irrespective of the coalition size*, provided the actual discount factor of the coalition members is sufficiently close to 1:

$$\lim_{N,\ S \longrightarrow \infty} \delta\left(S,\ N,\ \gamma\right) = \frac{\gamma}{4\gamma - 2} < \frac{1}{2} \ \text{ for any number of } \gamma > 1.$$

These observations lead to the following proposition:

**Proposition 2** *For the linear utility function given by (34), we have the following:*
*(i) If all the cooperating regions are sufficiently patient, partial tax coordination can be sustained as a subgame perfect equilibrium of the repeated game irrespective of the coalition size and the strength of preference toward a local public good;*
*(ii) partial tax coordination is more likely to prevail if the coalition size of tax coordination is smaller and/or the preference toward a local public good is stronger; and*
*(iii) partial tax coordination is more likely to prevail if the total number of regions in the economy is larger, provided the preference toward a local public good is strong enough.*

In order to understand how the parameters $S$, $N$, and $\gamma$ affect the behavior of the respective regions, we first differentiate the welfare levels of the regions at the respective phases of the

repeated game with respect to $S$, as follows:

$$\frac{\partial U_{\mathbf{N}}^{NE}}{\partial S} = 0, \tag{46}$$

$$\frac{\partial U_{\mathbf{S}}^{C}}{\partial S} = \underbrace{(\gamma-1)\,k_{\mathbf{S}}^{C}\frac{\partial \tau_{\mathbf{S}}^{C}}{\partial S}}_{\text{tax-rate effect }(+)} + \underbrace{\gamma\tau_{\mathbf{S}}^{C}\frac{\partial k_{\mathbf{S}}^{C}}{\partial S}}_{\text{tax-base effect }(-)} + \underbrace{\left(\overline{k}-k_{\mathbf{S}}^{C}\right)\frac{\partial r^{C}}{\partial S}}_{\text{terms-of-trade effect }(-)} > 0, \tag{47}$$

$$\frac{\partial U_{i}^{D}}{\partial S} = \underbrace{(\gamma-1)\,k_{i}^{D}\frac{\partial \tau_{i}^{D}}{\partial S}}_{(+)} + \underbrace{\gamma\tau_{i}^{D}\frac{\partial k_{i}^{D}}{\partial S}}_{(+)} + \underbrace{\left(\overline{k}-k_{i}^{D}\right)\frac{\partial r^{D}}{\partial S}}_{(+)} > 0, \tag{48}$$

where $r^{C} \equiv A-2\overline{k}-[S\tau_{\mathbf{S}}^{C}+(N-S)\tau_{\mathbf{N-S}}^{C}]/N$ and $r^{D} \equiv A-2\overline{k}-[(S-1)\tau_{\mathbf{S}}^{C}+(N-S)\tau_{\mathbf{N-S}}^{C}+\tau_{i}^{D}]/N$ represent the corresponding net returns in the cooperation and deviation phases, respectively. Since the tax rates chosen by the participating and nonparticipating regions both rise in response to the weakened intensity of tax competition, so does the average tax rate $\overline{\tau}$. The increase in $S$, therefore, reduces the equilibrium net return: $\partial r^{C}/\partial S < 0$ and $\partial r^{D}/\partial S < 0$. Moreover, (47) and (48), together with (4), (38), (39), and (40), imply that although the signs of the tax-rate and tax-base effects are the same as those in the previous model, the terms-of-trade effect emerges as an additional term. This terms-of-trade effect tends to reduce the welfare of the coalition members (i.e., capital exporters), whereas it tends to enhance that of the deviating region (i.e., capital importers), thereby unambiguously strengthening the incentive of deviation. Since the capital importers (i.e., the nonparticipating regions) benefit from the higher tax rate due to lower capital payments resulting from a lower interest rate (i.e., the terms-of-trade effect), they enjoy more tax revenues than the capital exporters (i.e., the participating regions) do. This result is essentially the same as that in an asymmetric two-country model of Wilson (1991), which demonstrates that a small country is always better off than a large country as a result of tax competition. Since we can view the coalition subgroup of cooperating regions and each noncooperating region outside the coalition as "a large country" and "a small country", respectively, each noncooperating region will be more better off than any of the coalition members.[4]

Replacing (41) with the minimum discount factor $\delta\,(S,\ N,\ \gamma)$ and then subtracting $U_{\mathbf{S}}^{C}$

[4] Also note that this result is consistent with Proposition 1 in Konrad and Schjelderup (1999) since tax rates are strategic complements.

from both sides of the resulting equality, we can obtain the following expression:

$$\frac{\delta(S,\ N,\ \gamma)}{1-\delta(S,\ N,\ \gamma)}\left(U_{\mathbf{S}}^{C}-U_{\mathbf{N}}^{NE}\right)=U_{i}^{D}-U_{\mathbf{S}}^{C}. \tag{49}$$

A straightforward calculation shows that the one-period loss, $U_{\mathbf{S}}^{C}-U_{\mathbf{N}}^{NE}$, and the current gain, $U_{i}^{D}-U_{\mathbf{S}}^{C}$, are both increasing in the group size $S$. Moreover, it can be verified that the gain from deviation, $U_{i}^{D}$, is larger than the opportunity cost from punishment, $U_{\mathbf{S}}^{C}$, thereby increasing the minimum discount factor $\delta(S,\ N,\ \gamma)$.

Similarly, we can compute the effect of the larger $N$ on the utility levels of the respective regions in all phases of the repeated game as follows:

$$\frac{\partial U_{\mathbf{N}}^{NE}}{\partial N} = (\gamma-1)\,\overline{k}\frac{\partial\tau_{\mathbf{N}}^{NE}}{\partial N}<0, \tag{50}$$

$$\frac{\partial U_{\mathbf{S}}^{C}}{\partial N} = \underbrace{(\gamma-1)\,k_{\mathbf{S}}^{C}\frac{\partial\tau_{\mathbf{S}}^{C}}{\partial N}}_{(-)}+\underbrace{\gamma\tau_{\mathbf{S}}^{C}\frac{\partial k_{\mathbf{S}}^{C}}{\partial N}}_{(+)}+\underbrace{\left(\overline{k}-k_{\mathbf{S}}^{C}\right)\frac{\partial r^{C}}{\partial N}}_{(+)}<0, \tag{51}$$

$$\frac{\partial U_{i}^{D}}{\partial N} = \underbrace{(\gamma-1)\,k_{i}^{D}\frac{\partial\tau_{i}^{D}}{\partial N}}_{(-)}+\underbrace{\gamma\tau_{i}^{D}\frac{\partial k_{i}^{D}}{\partial N}}_{(-)}+\underbrace{\left(\overline{k}-k_{i}^{D}\right)\frac{\partial r^{D}}{\partial N}}_{(-)}<0. \tag{52}$$

The intuition is as follows. An increase in $N$ strengthens the intensity of tax competition, which decreases the average tax rate. It follows that $\partial r^{C}/\partial N>0$ and $\partial r^{D}/\partial N>0$. On the other hand, it is seen from (50), (51), and (52) that at all phases, the signs of the tax-rate and tax-base effects are reversed to those of the effects of changes in $S$, because of the opposite effects on the intensity of tax competition to those in changes in $S$. Moreover, the terms-of-trade effect also shows an opposite sign to that of changes in $S$. This terms-of-trade effect tends to increase the welfare of the coalition members (i.e., capital exporters), while it tends to reduce the welfare of the nonparticipating regions (i.e., capital importers). In addition, the level of welfare at the fully noncooperative Nash equilibrium falls due to the negative tax-rate effect (recall that there is no capital trade in this equilibrium). Although these results reveal that the increase in $N$ tends to deter deviation, the precise effect on the minimum discount factor depends on the size of the preference parameter $\gamma$. As shown in Appendix D, if $\gamma\geq(7+\sqrt{13})/6\simeq1.767$, the minimum discount factor $\delta(S,\ N,\ \gamma)$ unambiguously

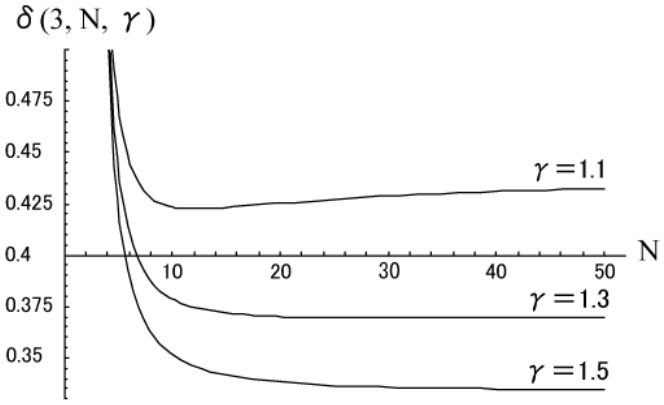


Figure 1: The minimum discount factor $\delta(S,\ N,\ \gamma)$ for $S = 3$.

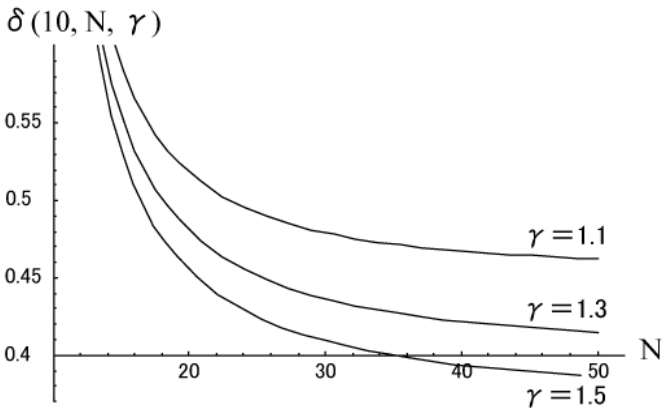


Figure 2: The minimum discount factor $\delta(S,\ N,\ \gamma)$ for $S = 10$.

falls in $N$, while it may or may not fall in $N$ if $1 < \gamma < (7 + \sqrt{13})/6$. To further identify the latter case, we employ a numerical analysis, which shows that when $\gamma$ and $S$ are both small, the minimum discount factor rises in $N$. Figure 1 illustrates that both $\delta\,(3,\ N,\ 1.1)$ and $\delta\,(3,\ N,\ 1.3)$ decline in $N$ for lower values of $N$ and then rise slightly in $N$. When $S$ is relatively large (i.e., $S = 10$ and $S = 25$), $\delta\,(S,\ N,\ \gamma)$ monotonically falls in $N$ irrespective of the size of $\gamma$, as shown in Figures 2 and 3. In sum, it may be concluded that even in the model with linear utility, increasing $N$ in general strengthens the incentive of cooperation, except when the group size $S$ is extremely small.

Finally, differentiating the welfare levels of the respective regions at all phases with respect

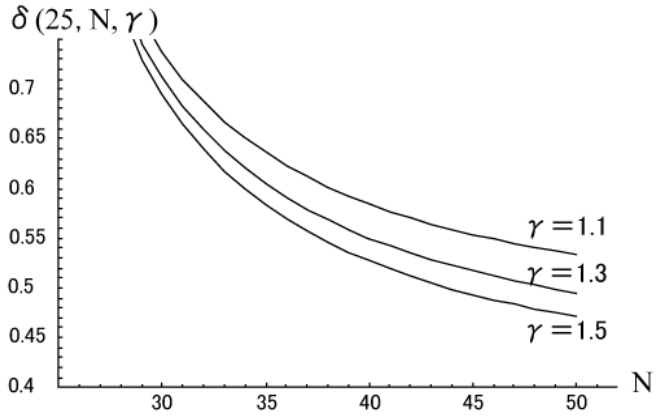


Figure 3: The minimum discount factor $\delta(S,\ N,\ \gamma)$ for $S = 25$.

to $\gamma$ yields the following:

$$\frac{\partial U_{\mathbf{N}}^{NE}}{\partial \gamma} = \underbrace{(\gamma - 1)\,\overline{k}\frac{\partial \tau_{\mathbf{N}}^{NE}}{\partial \gamma}}_{(+)} + \underbrace{\tau_{\mathbf{N}}^{NE}\overline{k}}_{\text{strengthened preference effect } (+)} > 0, \tag{53}$$

$$\frac{\partial U_{\mathbf{S}}^{C}}{\partial \gamma} = \underbrace{(\gamma - 1)\,k_{\mathbf{S}}^{C}\frac{\partial \tau_{\mathbf{S}}^{C}}{\partial \gamma}}_{(+)} + \underbrace{\gamma\tau_{\mathbf{S}}^{C}\frac{\partial k_{\mathbf{S}}^{C}}{\partial \gamma}}_{(-)} + \underbrace{\left(\overline{k} - k_{\mathbf{S}}^{C}\right)\frac{\partial r^{C}}{\partial \gamma}}_{(-)} + \underbrace{\tau_{\mathbf{S}}^{C}k_{\mathbf{S}}^{C}}_{(+)} > 0, \tag{54}$$

$$\frac{\partial U_{i}^{D}}{\partial \gamma} = \underbrace{(\gamma - 1)\,k_{i}^{D}\frac{\partial \tau_{i}^{D}}{\partial \gamma}}_{(+)} + \underbrace{\gamma\tau_{i}^{D}\frac{\partial k_{i}^{D}}{\partial \gamma}}_{(+)} + \underbrace{\left(\overline{k} - k_{i}^{D}\right)\frac{\partial r^{D}}{\partial \gamma}}_{(+)} + \underbrace{\tau_{i}^{D}k_{i}^{D}}_{(+)} > 0. \tag{55}$$

A higher $\gamma$ stimulates the demand for a local public good in all phases, so that the inhabitants in the regions prefer higher capital tax rates. This in turn weakens the intensity of tax competition, thereby raising the average tax rate and thus lowering the net return, i.e., $\partial r^{C}/\partial\gamma < 0$ and $\partial r^{D}/\partial\gamma < 0$. Hence, the resulting signs of the tax-rate, tax-base, and terms-of-trade effects are the same as those of changes in $S$. There is an additional direct effect on welfare, which is caused by the stronger preference toward a local public good (which we call the preference effect). This preference effect enhances the welfare level of all the regions including that of a deviator. Although the increase in $\gamma$ weakens the pressure of tax competition like the increase in $S$, they have opposite effects on the minimum discount factor, which is implied by (43) and (45). This stems from the fact that the direct preference effect exerts a dominant

impact on the incentive of cooperation.[5]

# 6 Concluding Remarks

In this paper, we constructed a repeated interactions model where some regions collude to coordinate their capital tax rates and the other regions do not and found that partial tax coordination can sustain as an equilibrium outcome if the regions are sufficiently patient. We have also shown that partial tax coordination is more likely to prevail if the number of cooperating regions is smaller and the total number of regions in the whole economy is larger. In light of our results, global tax coordination is the most vulnerable outcome, simply because the incentive to deviate becomes the strongest. Hence, our repeated interactions model further suggests that the size of a coalition, which implements tax coordination, should be set equal to the maximum sustainable number of members in a coalition, since the welfare of cooperating and noncooperating regions both increase in a coalition size. In other words, there is a desirable intermediate coalition size in order to implement tax coordination because there is a trade-off between the sustainability of a coalition and the welfare level of cooperating regions; that is, although a much more encompassing tax coordination leads to a first-best solution, sustainability becomes more difficult.

These results would also provide a useful lesson for the intense discussion on corporate tax coordination, including tax-rate harmonization, in the EU for many years. Partial tax coordination within EU member nations would be desirable rather than world-wide organizations such as a "World Tax Organization" suggested by Tanzi (1998) or multilateral agreements such as a "GATT for Taxes" to achieve global tax coordination for the following reasons: first, partial tax coordination would be more sustainable because of the existence of the significant fringe of competing countries in the tax competition; second, it is beneficial not only for EU member nations, which maintain partial tax cooperation, but also for the other nations outside the EU, since a collapse of the coalition would lead to a harmful "race to the bottom" with

[5] It is interesting to note that $\lim_{\gamma \to \infty} \delta(S, N, \gamma) = \delta(S, N)$. That is, as $\gamma$ becomes infinitely larger, the present model becomes essentially the same as the one in the previous section. This occurs mainly because in the limit, the governments care only for the provision of the local public good, which is identical with tax revenues.

all the nations in the world.

There is another policy lesson that can be deduced from the present analysis. If the stream of future tax revenues which accrues to jurisdictional governments is discounted by an interest rate, a lower interest rate policy would facilitate the cooperation of tax coordination and allow for a larger size of coalition, which entails the larger tax revenues or higher welfare of each coalition member. The lower interest rate policy thus serves in promoting the sustainability of partial tax coordination.

The results obtained in this analysis critically rely on the restrictive structure of the present model, such as linear utility and a quadratic production function. To make the model more realistic and the results more robust, it is certainly desirable that the analysis should be conducted under more general forms of those functions. For this purpose, we need to resort to a numerical analysis. The more important extension is to include the introduction of regional asymmetries in terms of capital endowments and/or production technologies and explore how the likelihood of partial tax coordination is affected by changes in the degree of the asymmetries.

# Appendix A

Making use of (4), (5), and (36) for all $i \in \mathbf{S}$ yields the reaction function as follows:

$$\tau_{\mathbf{S}} = \frac{(N-S)\left[N(\gamma-1)+S\right]\tau_{\mathbf{N-S}} + 2N^2(\gamma-1)\overline{k}}{(N-S)\left[N(2\gamma-1)+S\right]}, \tag{A.1}$$

where $\tau_{\mathbf{S}}$ and $\tau_{\mathbf{N-S}}$ denote the capital tax rates for cooperators and noncooperators, respectively. Similarly, from (4), (5), and (37) for all $j \in \mathbf{N}-\mathbf{S}$, we have the following best-response function:

$$\tau_{\mathbf{N-S}} = \frac{S\left[N(\gamma-1)+1\right]\tau_{\mathbf{S}} + 2N^2(\gamma-1)\overline{k}}{N\left[\gamma(N-1)+S(\gamma-1)\right]+S}. \tag{A.2}$$

Solving (A.1) and (A.2) for $\tau_{\mathbf{S}}$ and $\tau_{\mathbf{N-S}}$, respectively, gives the tax rates (38) and (39) in the Nash equilibrium with a coalition subgroup. Further, using (4) and (5), the first-order

condition for the deviating region $i$ can be rewritten as

$$\tau_i^D = \frac{[N(\gamma-1)+1]\left[(S-1)\tau_{\mathbf{S}} + (N-S)\tau_{\mathbf{N-S}}\right] + 2N^2(\gamma-1)\overline{k}}{(N-1)\left[N(2\gamma-1)+1\right]}. \tag{A.3}$$

Substituting (38) and (39) into (A.3) and manipulating yields the best-deviation tax rate (40).

# Appendix B

From (35), (38), (39), and (40), we obtain the following:

$$\tau_{\mathbf{S}}^C - \tau_{\mathbf{N-S}}^C = \frac{2N\left(\gamma-1\right)\left(S-1\right)\overline{k}}{\left(N-S\right)\left[\left(2\gamma-1\right)\left(N-1\right)+\gamma S\right]} > 0, \tag{B.1}$$

$$\tau_{\mathbf{N-S}}^C - \tau_{\mathbf{N}}^{NE} = \frac{2S\left(\gamma-1\right)\left(S-1\right)\left[N(\gamma-1)+1\right]\overline{k}}{\gamma(N-1)\left(N-S\right)\left[\left(2\gamma-1\right)\left(N-1\right)+\gamma S\right]} > 0, \tag{B.2}$$

$$\tau_{\mathbf{N-S}}^C - \tau_i^D = \frac{2N\left(\gamma-1\right)\left(S-1\right)[N(\gamma-1)+1]\overline{k}}{\left(N-1\right)\left(N-S\right)\left[N(2\gamma-1)+1\right]\left[\left(2\gamma-1\right)\left(N-1\right)+\gamma S\right]} > 0, \tag{B.3}$$

$$\tau_{\mathbf{S}}^C - \tau_i^D = \frac{2N^2\left(\gamma-1\right)\left(S-1\right)[N(\gamma-1)+\gamma(N-1)+1]\overline{k}}{\left(N-1\right)\left(N-S\right)\left[N(2\gamma-1)+1\right]\left[\left(2\gamma-1\right)\left(N-1\right)+\gamma S\right]} > 0, \tag{B.4}$$

$$\tau_i^D - \tau_{\mathbf{N}}^{NE} = \frac{2\left(\gamma-1\right)\left(S-1\right)[N(\gamma-1)+1][N(\gamma(S-1)+S(\gamma-1))+S]\overline{k}}{\gamma(N-1)\left(N-S\right)\left[N(2\gamma-1)+1\right]\left[\left(2\gamma-1\right)\left(N-1\right)+\gamma S\right]} > 0, \tag{B.5}$$

which together produce the following ranking: $\tau_{\mathbf{N}}^{NE} < \tau_i^D < \tau_{\mathbf{N-S}}^C < \tau_{\mathbf{S}}^C$.

# Appendix C

From (3), (4), (34), (35), (38), (39), and (40), we have

$$U_i^D - U_{\mathbf{S}}^C = k_i^D\left(k_i^D + \gamma\tau_i^D\right) - k_{\mathbf{S}}^C\left(k_{\mathbf{S}}^C + \gamma\tau_{\mathbf{S}}^C\right) + (r^D - r^C)\overline{k} = \frac{\left(\tau_{\mathbf{S}}^C - \tau_i^D\right)\Delta_1}{4N^2}, \tag{C.1}$$

where $\Delta_1 \equiv 2(N-S)[N(\gamma-1)+1](\tau_{\mathbf{S}}^C - \tau_{\mathbf{N-S}}^C) + (N-1)^2(\tau_{\mathbf{S}}^C - \tau_i^D) - 2N[2N(\gamma-1)\overline{k} - \gamma(N-1)\tau_i^D]$, and $r^C = A - 2\overline{k} - [S\tau_{\mathbf{S}}^C + (N-S)\tau_{\mathbf{N-S}}^C]/N$ and $r^D = A - 2\overline{k} - [(S-1)\tau_{\mathbf{S}}^C + (N-S)\tau_{\mathbf{N-S}}^C + \tau_i^D]/N$ denote the equilibrium net returns at the cooperation and deviation

phases, respectively. Substituting (B.1) and (B.4) into (C.1) and rearranging yields

$$U_i^D - U_{\mathbf{S}}^C = \frac{N^2 (\gamma-1)^2 (S-1)^2 [N(2\gamma-1)-(\gamma-1)]^2 \overline{k}^2}{(N-1)(N-S)^2 [N(2\gamma-1)+1][(2\gamma-1)(N-1)+\gamma S]^2}. \tag{C.2}$$

Similarly, we have

$$\begin{aligned} U_i^D - U_{\mathbf{N}}^{NE} &= k_i^D \left(k_i^D + \gamma \tau_i^D\right) - \overline{k}\left(\overline{k} + \gamma \tau_{\mathbf{N}}^{NE}\right) + (r^D - r^{NE})\overline{k} \\ &= (\gamma-1)\left(\tau_i^D - \tau_{\mathbf{N}}^{NE}\right)\overline{k} + \frac{\Delta_2}{4N^2}, \end{aligned} \tag{C.3}$$

where $\Delta_2 \equiv [(N-S)(\tau_{\mathbf{S}}^C - \tau_{\mathbf{N-S}}^C) - (N-1)(\tau_{\mathbf{S}}^C - \tau_i^D)][(N-S)(\tau_{\mathbf{S}}^C - \tau_{\mathbf{N-S}}^C) - (N-1)(\tau_{\mathbf{S}}^C - \tau_i^D) - 2N\gamma\tau_i^D]$, and $r^{NE} = A - 2\overline{k} - \tau_{\mathbf{N}}^{NE}$ stands for the net return at the fully noncooperative symmetric Nash equilibrium. Inserting (B.1), (B.4), and (B.5) into (C.3) and rearranging results in

$$U_i^D - U_{\mathbf{N}}^{NE} = \frac{(\gamma-1)^2 (S-1)[NS(2\gamma-1)+S-\gamma N]\overline{k}^2 \Omega}{\gamma(N-1)(N-S)^2 [N(2\gamma-1)+1][(2\gamma-1)(N-1)+\gamma S]^2}. \tag{C.4}$$

Substituting (C.2) and (C.4) into the formula $\delta(S,\ N,\ \gamma) \equiv (U_i^D - U_{\mathbf{S}}^C)/(U_i^D - U_{\mathbf{N}}^{NE})$ yields the minimum discount factor (42).

# Appendix D

Differentiating the minimum discount factor (42) with respect to $S$, $N$, and $\gamma$, respectively, yields

$$\begin{aligned} \frac{\partial \delta(S,\ N,\ \gamma)}{\partial S} &= \frac{\gamma N^2 [N(2\gamma-1)-(\gamma-1)]^2}{[NS(2\gamma-1)+S-\gamma N]^2 \Omega^2} \times \\ &\quad \left[\{N(\gamma-1)+1\}\Omega - (S-1)\{N(\gamma(S-1)+S(\gamma-1))+S\}\frac{\partial \Omega}{\partial S}\right] > 0, \\ \frac{\partial \delta(S,\ N,\ \gamma)}{\partial N} &= \frac{-2\gamma N (S-1)[N(2\gamma-1)-(\gamma-1)]\Phi}{[NS(2\gamma-1)+S-\gamma N]^2 \Omega^2} \gtreqless 0 \text{ if and only if } \Phi \lesseqgtr 0, \\ \frac{\partial \delta(S,\ N,\ \gamma)}{\partial \gamma} &= \frac{-2N^2 (N-1)(N-S)(S-1)[N(2\gamma-1)-(\gamma-1)]\Psi}{[NS(2\gamma-1)+S-\gamma N]^2 \Omega^2} < 0, \end{aligned}$$

where $\partial\Omega/\partial S \equiv -[N(\gamma-1)+1][N(2\gamma-1)+\gamma(2S-3)+2]-2\gamma^2N(S-1) < 0$, $\Phi \equiv S(N-1)^3[N+S(N-2)]+2\gamma^4N^3(2S-1)[2S(N+S-2)+1]-N\gamma^3[2S^3[6N(N-1)+1]+2S^2[2N(N-1)(5N-9)-3]+2NS(5N-3)+S-N^2]+S\gamma^2(N-1)[N^3(18S+7)+N^2[S(6S-43)-3]+NS(23-6S)+S(S-3)]-S\gamma(N-1)^2[N^2(7S+5)+N[S(S-15)-2]-S(S-5)]$, and $\Psi \equiv S[N(2\gamma-1)+1][N^2(2\gamma-1)^2-\gamma[\gamma(S-3)+3]+N[\gamma(7-5\gamma)-2]+1]-N\gamma^3 > 0$. Further computation reveals that $\Phi > 0$ if $\gamma \geq (7+\sqrt{13})/6$, while the sign of $\Phi$ is ambiguous if $1 < \gamma < (7+\sqrt{13})/6$.

# References


Bucovetsky, S., Asymmetric tax competition, *Journal of Urban Economics* **30** (1991) 167-181.

Burbidge, J. B., Depater, J. A., Myers, G. M., and A. Sengupta, A coalition-formation approach to equilibrium federations and trading blocs, *American Economic Review* **87** (1997) 940-956.

Cardarelli, R., Taugourdeau, E., and J-P. Vidal, A repeated interactions model of tax competition, *Journal of Public Economic Theory* **4** (2002) 19-38.

Catenaro, M., and J-P. Vidal, Implicit tax co-ordination under repeated policy interactions, *Recherches Economiques de Louvain* **72** (2006) 1-17.

Conconi, P., Perroni, C., and R. Riezman, Is partial tax harmonizaiton desirable? *Journal of Public Economics* **92** (2008) 254-267.

D'Aspremont, C., Jacquenmin, A., Gabszwicz, J.J., and J.A. Weymark, On the stability of collusive price leadership, *Canadian Journal of Economics* **16** (1983) 17-25.

Edwards, J., and M. Keen, Tax competition and Leviathan, *European Economic Reveiw* **40** (1996) 113-134.

Haufler, A., Factor taxation, income distribution, and capital market integration, *Scandinavian Journal of Economics* **99** (1997) 425-446.

Itaya, J., Okamura, M., and C. Yamaguchi, Are regional asymmetries detrimental to tax coordination in a repeated game setting? *Journal of Public Economics* (2007) (forthcoming).

Marchand, M., Pestieau, P., and M. Sato, Can partial fiscal coordination be welfare worsening? A model of tax competition, *Journal of Urban Economics* **54** (2003) 451-458.

Martin, S., *Advanced Industrial Economics, 2nd edn.*, Blackwell Publishers (2002).

Sugahara, K., Kunizaki, M., and K. Oshima, Partial tax coordination in multilateral asymmetric tax competition, **63**rd Congress of the International Institute of Public Finance (2007).

Konrad, K. A., and G. Schjelderup, Fortress building in global tax competition, *Journal of Urban Economics* **46** (1999) 156-167.

Rasmussen, S., Partial vs. global coordination of capital income tax policies, *Working Paper No.* 2001-3, University of Aarhus (2001).

Tanzi, V., *Taxation in a Integrating World*, Washington, DC; Brookings Institution (1995).

Wildasin, D. E., Interjurisdictional capital mobility: Fiscal externality and a corrective subsidy, *Journal of Urban Economics* **25** (1989) 193-212 .

Wilson, J. D., Tax competition with interregional differences in factor endowments, *Regional Science and Urban Economics* **21** (1991) 423-451.