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Employment and Hours of Work

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Abstract

This paper develops a dynamic model of the labor market in which the degree of substitution between employment and hours of work is determined as part of a search equilibrium. Each firm chooses its demand for working hours and number of vacancies, and the earnings profile is determined by Nash bargaining. The earnings profile is generally nonlinear in hours of work, and defines the trade-off between employment and hours of work. Concave production technology induces firms to overemploy and, as a result, hours of work are below their optimal level. The Hosios condition is not sufficient for efficiency. When there are two industries, workers employed by firms with higher recruitment costs work longer and earn more. That is, "good jobs" require longer hours of work. Interestingly, technology differentials cannot account for working hours differentials.

JEL classification: J21, J23, J31, J64.

Keywords: employment, hours of work, search frictions.

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1 Introduction

There are extensive and intensive margins for adjusting labor input: the number of workers and hours of work per employee. Understanding how firms utilize these two margins is crucial for understanding the long-run trend in hours of work (Prescott, 2004; Rogerson, 2006; Pissarides, 2007), the movements of employment and hours over the business cycle (Burnside et al., 1993), the likely effect of regulation of hours of work (Hoel, 1986; Booth and Schiantarelli, 1987; Hunt, 1998; Marimon and Zilibotti, 2000), and cross-sectional differences in hours of work (Hamermesh, 1993), to name a few.

Recent decades have witnessed declines in hours of work in major developed economies (Rogerson, 2006; Pissarides, 2007). Despite a decline in average working hours, disparities in working hours among workers have gradually increased. This disparity is particularly apparent for workers in their thirties. The Japanese Labour Force Survey (2006) documented that the proportion of male employees aged between 35 and 39 years who work more than 60 hours per week rose from 18.9% in 1993 to 23.5% in 2003.[1] Similarly, the proportion of those who work less than 35 hours per week also rose from 6.4% to 7.1%. An important question is, what is the major cause of the dispersion in hours of work?

In this paper, we construct a dynamic equilibrium model of labor demand under *search frictions* and focus on how firms utilize employment and hours of work. The idea is that hours of work can be chosen instantly whereas employment adjustment is frictional. A novel feature of this paper is that the source of labor adjustment costs is search frictions, and that the cost of adjustment is influenced by labor market tightness. Thus, the degree of substitution between employment and hours of work is determined as part of the search equilibrium. This sharply contrasts with traditional models of working hours (Hoel, 1986; Booth and Schiantarelli, 1987; Calmfors and Hoel, 1988; Cahuc and Zylberberg, 2004) and adjustment cost models (Sargent, 1978; Hamermesh, 1993), in which the degree of substitution between working hours and employment is structurally

[1] For the US economy, Kuhn and Lozano (2005) investigated this issue and documented that "highly educated, high-wage, salaried men" had the greatest increase in long working hours. See also Hamermesh (1993).

given by the production function or by the adjustment cost function.

To understand the role of search frictions, consider the standard neoclassical model (Lucas and Rapping, 1969; Prescott, 2004), in which workers face a labor–leisure choice and choose hours of work optimally, taking the market wage rate as given. In the neoclassical framework, *firms can employ any quantity of labor at the market wage rate.* Thus, from the firm's viewpoint, it does not matter whether workers work longer or shorter hours because the competitive market ensures that the quantity of labor demanded equals the quantity of labor supplied. On the other hand, with search frictions and bilateral trading, if employees work shorter hours, then the firm must pay extra search costs to maintain its labor input. Thus, firms do care about working hours.

The basic model is an extension of Smith (1999), which is particularly useful for our purpose because the number of workers in each firm is determined endogenously, although modeling firm size in a search equilibrium is generally a formidable task.[2] We incorporate the choice of working hours into Smith's (1999) framework. Each firm chooses the number of vacancies and hours of work per employee to maximize the value of the firm.

A novel feature of the model of this paper is that Nash bargaining determines the earnings profile, which relates earnings and hours of work. In particular, the earnings profile is shown to be a nonlinear function of hours of work, which reflects the concave production technology and workers' convex utility costs of longer hours of work. Interestingly, the earnings profile is consistent with the standard upward-sloping labor supply curve, and it defines the trade-off the firm faces when choosing employment and hours of work.

Smith's (1999) main result is that firms overemploy. The mechanism is intuitive. The social planner does not take into account the wage rate when choosing the optimal employment level because wages are simply transfers among the members of society. However, firms do care about the wage rate. When the production technology is concave, an increase in employment results in a reduction in the wage rate. Firms have incentives to exploit this opportunity, resulting in overemployment. This mechanism also operates in our framework.

[2] Bertola and Caballero (1994), Bertola and Garibaldi (2001), Cahuc and Wasmer (2001), and Cahuc et al. (2008) also developed search models incorporating large firms. See also Kudoh and Sasaki (forthcoming).

The overemployment effect has an important implication for the efficiency of hours of work. Since the level of employment is too high, the marginal product of an additional hour of work is inefficiently low. This leads to shorter hours of work. We show that in the absence of the overemployment effect, employment and hours of work are efficient. An important finding is that, because of the overemployment effect, the Hosios (1990) condition is not sufficient for efficiency. To restore efficiency under concave production technology, we need to impose in addition that the firm appropriates all the bargaining surplus. In this case, the overemployment effect is internalized because the firm faces all the costs and benefits of choosing employment and hours of work.

It has become increasingly important to ask whether regulating hours of work to implement some form of "work sharing" increases employment (Hoel, 1986; Booth and Schiantarelli, 1987; Hunt, 1998; Marimon and Zilibotti, 2000). According to our model, under the regulation of working hours, a reduction in hours of work generally increases employment *at each firm*. Whether it expands aggregate employment depends on whether the marginal product of labor exceeds the marginal disutility of work. However, the laissez-faire level of hours of work is below its optimum, suggesting that implementing work sharing may expand employment at the cost of efficiency.

We also study two other forms of regulation. One is wage regulation. We consider a scenario in which the earnings profile is perfectly regulated, and find that an earnings profile that mimics the worker's disutility function induces the efficient levels of employment and hours of work. The key requirement is to match the marginal hourly wage rate to the marginal disutility of longer hours of work. The other regulation we consider is entry regulation. Recently, this issue has become increasingly important (Bertrand and Kramarz, 2002; Blanchard and Giavazzi, 2003; Fang and Rogerson, 2007). We consider a scenario in which the number of firms is regulated. We find that regulation that reduces the number of firms lowers hours of work and increases the number of employees at each firm. However, it results in higher unemployment because there are fewer firms. Interestingly, this result is overturned if a regulation takes the form of imposing a fixed cost of entry. This suggests that the details of implementing entry regulation matter.

Hamermesh (1993) documented that there are sizable differences in working hours across industries. For example, in 1990 in the US, weekly hours of work averaged 44 in mining, 40.8 in manufacturing, 32.6 in services, and 28.8 in retail trade. This finding is interesting because it suggests that there are large differences in working hours among industries, each of which provides a variety of jobs. We extend the basic model to investigate why some individuals work longer than others. In doing so, we focus on job characteristics rather than worker characteristics as determinants of differences in hours of work.

We first address *within-industry* differentials in hours of work, such as full-time–part-time differentials. To explain these differentials, we develop a model of a single industry in which the labor market is pooled for two types of jobs. The key determinant of differentials in working hours is differences in recruitment costs. For example, Hamermesh (1993) documented that firms incur much higher costs in recruiting workers with skills and education than in filling jobs that require little skill or training. Our result implies that jobs with high recruitment costs are characterized as "good jobs" (Acemoglu, 2001), and that workers with such jobs work longer hours than so workers with "bad jobs".

We also address *interindustry* differentials in hours of work. We develop a model that incorporates two industries in which labor markets are segmented. Because workers apply for work in one of the industries, there is arbitrage between the two markets. We show that the industry with higher recruitment costs pays higher wages and requires longer hours of work. The jobs in this industry are considered as "good jobs", and the labor market tightness in this industry is lower because more workers apply for jobs in this industry. The key finding is that recruitment costs account for interindustry dispersion in hours of work. Interestingly, we find that differences in technology cannot explain working hours differentials.

2 The Model

2.1 Environment

This section presents a dynamic model of the demand for hours and employment with search frictions. The model developed in this section is an extension of Smith (1999). Consider an economy consisting of workers and firms. The measure of workers is normalized to unity. There is a large number of firms, with the exact number to be determined by free entry. Time is discrete and all agents discount the future at the common discount rate r.

The number of matches M is determined by the standard constant returns to scale matching technology, $M = m(U, V)$, where U is the total number of job seekers and V is the number of aggregate job vacancies. A vacancy is matched to a worker during a period with probability q, where

$$q \equiv \frac{M}{V} = m\left(\frac{U}{V}, 1\right). \tag{1}$$

It is easy to verify that an increase in labor market tightness $V/U \equiv \theta$ decreases the matching probability q. Similarly, the probability that a worker is matched with a vacancy is given by $M/U = m(1, V/U) = \theta q(\theta)$. Let λ be the exogenous rate of job destruction. Then, in any steady state, the flow into employment $\theta q(\theta)U$ must equal the flow into unemployment $\lambda(1-U)$. Thus:

$$m(U, V) = \lambda(1-U), \tag{2}$$

which defines the Beveridge curve, illustrated in Figure 1.

2.2 Firms

The production technology is given by $f(L)$, where $L \equiv hl$ is total labor input, h denotes hours of work per employee and l is the number of employees at each firm. We assume that $f'(L) > 0 > f''(L)$ and $f(0) = 0$. In particular, we assume that production technology is specified by $f(L) = AL^{\alpha}$, where $A > 0$ and $\alpha \in (0, 1)$.

The instantaneous payoff to a firm with l employees is given by $f(hl) - W(h, l)l - kv - c$, where v is the number of vacancies, $k > 0$ is the cost of each vacancy, which may include a recruitment cost and an equipment cost (Acemoglu, 2001), and $c \geq 0$ is a fixed cost. Each worker earns $W(h,l)$, where $W(h,l)$ is the *earnings function* or earnings profile to be determined as part of the equilibrium. The earnings profile $W(h,l)$ is negotiated in each period, so the profile is treated as a function of the state variable l. Note that $w(h,l) \equiv W(h,l)/h$ is the *average* hourly wage rate, which generally differs from the *marginal* hourly wage rate $W_h(h,l)$ if the earnings profile is a nonlinear function of h. For the firm's decision making, what matters is the marginal hourly wage rate; thus we focus on $W(h,l)$ rather than $w(h,l)$.

We assume that firms choose hours of work.[3] The key is that firms can change hours of work instantly whereas employment adjustment takes time because of search frictions. As a result, the number of employees l is treated as the state variable.

Let $J(l)$ be the value of an operating firm with l employees. The Bellman equation is given by

$$J(l) = \max_{h,v} \{ f(hl) - W(h,l)l - kv - c + \delta J(l_{+1}) \}, \tag{3}$$

where $l_{+1} = (1-\lambda)l + q(\theta)v$ is next period's employment level, $\delta \equiv 1/(1+r)$ is the discount factor, and λ is the exogenous rate of job destruction. The first-order conditions with respect to h and v, respectively, are:

$$\begin{aligned} f'(hl) - W_h(h,l) &= 0, && (4) \\ -k + q(\theta)\delta J'(l_{+1}) &= 0. && (5) \end{aligned}$$

The envelope condition yields

$$J'(l) = f'(hl)h - W(h,l) - W_l(h,l)l + (1-\lambda)\delta J'(l_{+1}). \tag{6}$$

In any steady state, the flow into unemployment equals the flow into employment:

$$\lambda l = q(\theta)v. \tag{7}$$

Then, (5) and (6) imply

$$\frac{(r+\lambda)k}{q(\theta)} = f'(hl)h - W(h,l) - W_l(h,l)l. \tag{8}$$

[3] In Section 3.2.1 we discuss the implication of this assumption.

2.3 Workers and Wages

The value of being employed by a firm of size l, denoted by $J^E(l)$, satisfies

$$J^E(l) = W(h,l) - e(h) + \lambda\delta J^U + (1-\lambda)\delta J^E(l_{+1}), \tag{9}$$

where $\delta \equiv 1/(1+r)$ is the discount factor, λ is the exogenous separation rate, $e(h)$ is the disutility of work, and J^U is the value of being unemployed. We assume that $e'(\cdot) > 0$, $e''(\cdot) > 0$ and $\lim_{h\to\infty} e(h) = \infty$.

We assume that workers and the firm share the total surplus. In addition, we assume that workers are not unionized. Consider a bargaining process between a firm and a group of workers of measure Δ. The threat point for the firm is $J(l-\Delta)$ because failing to agree on a contract implies losing the workers. The total match surplus is therefore $J(l) - J(l-\Delta) + \Delta(J^E(l) - J^U)$. If the firm's share of the surplus is given by $1-\beta \in [0,1]$, then we have $\beta[J(l) - J(l-\Delta)] = (1-\beta)\Delta(J^E(l) - J^U)$. In the limit as $\Delta \to 0$,

$$\beta J'(l) = (1-\beta)\left[J^E(l) - J^U\right]. \tag{10}$$

This is the key equation for rent sharing.[4] Note that this amounts to maximizing the asymmetric Nash product $[J'(l)]^{1-\beta}[J^E(l) - J^U]^{\beta}$ with respect to $W(h,l)$, taking J^U as given. In principle, the bargaining problem selects the appropriate function from a set of admissible functions to maximize the Nash product, and the first-order condition is exactly (10).

To determine the bargaining outcome, first we use (5) and (6) to obtain

$$J'(l) = \frac{1+r}{r+\lambda}\left[f'(hl)\,h - W(h,l) - W_l(h,l)l\right]. \tag{11}$$

Similarly, from (9), we obtain

$$J^E(l) - J^U = \frac{1+r}{r+\lambda}\left[W(h,l) - e(h) - \frac{r}{1+r}J^U\right], \tag{12}$$

Substituting (11) and (12) into (10) yields

$$W(h,l) = \beta\left[f'(hl)\,h - W_l(h,l)l\right] + (1-\beta)\,e(h) + \frac{(1-\beta)\,r}{1+r}J^U. \tag{13}$$

[4] Smith (1999) also considered wage determination in a model similar to ours, and derived the wage rate using the intuition from the sequential bargaining theory (Osborne and Rubinstein, 1990). By contrast, our method exploits the Nash sharing rule together with the envelope condition from the dynamic programming problem.

This is a differential equation about the unknown earnings function for each h and θ.

Proposition 1 *The earnings function is given by*

$$W(h,l) = l^{-1/\beta} \int_0^l i^{\frac{1}{\beta}-1} \left[f'(hi)\, h \right] di + (1-\beta)\, e(h) + \frac{(1-\beta)\, r}{1+r} J^U.$$

Proof. See Appendix. ■

It is easy to verify that $\partial W(h,l)/\partial J^U > 0$. Thus, as in Pissarides (2000), the wage rate increases with the value of unemployment. Since the value of unemployment is increasing in labor market tightness, this implies that the wage rate increases with labor market tightness.

For $f(L) = AL^{\alpha}$, the earnings function reduces to

$$W(h,l) = \frac{\alpha A h^{\alpha} l^{\alpha-1}}{\alpha + \frac{1-\beta}{\beta}} + (1-\beta)\, e(h) + \frac{(1-\beta)\, r}{1+r} J^U, \tag{14}$$

from which it is easy to verify that

$$W_h(h,l) = \frac{\alpha^2 A \left[hl\right]^{\alpha-1}}{\alpha + \frac{1-\beta}{\beta}} + (1-\beta)\, e'(h) > 0, \tag{15}$$

$$W_l(h,l) = \frac{(\alpha-1)\, \alpha A h^{\alpha} l^{\alpha-2}}{\alpha + \frac{1-\beta}{\beta}} < 0. \tag{16}$$

These two results are worth emphasizing. First, the earnings profile is not a linear function of h. For α sufficiently close to unity, $W_{hh} > 0$. In other words, as long as the production technology is sufficiently less concave, the marginal hourly wage rate is increasing in hours of work. This explains the observed positive association between hours of work and the hourly wage rage. For example, Murphy and Topel (1997) interpreted their finding of a positive cross-sectional relationship between working hours and the hourly wage rate as supporting evidence of the standard labor supply curve based on workers' labor–leisure choices. Our result suggests that the positive association between hours of work and earnings represents the earnings profile resulting from bargaining; workers' labor–leisure choices are not necessary.

Equation (16) indicates that the wage rate is decreasing in the number of employees. As pointed out by Smith (1999), this effect arises because of the concavity of the production technology. We subsequently show that, because of this effect, firms have incentives to overemploy to reduce wage rates. Clearly, this effect disappears under linear production technology ($\alpha = 1$).

3 Equilibrium

3.1 Characterization

Following Smith (1999), we assume free entry of firms and that a firm entering the market creates a large number of vacancies to achieve the steady state level of employment l in the next period, so that there are no transitional dynamics or (transitory) size distribution of firms.[5] Because the rate of filling a vacancy is $q(\theta)$, in order to achieve l in the next period, the firm must create exactly $l/q(\theta)$ vacancies in the current period. Thus, the value of entry is given by

$$J(0) = -\frac{kl}{q(\theta)} - c + \delta J(l). \tag{17}$$

Equation (17) is interpreted as follows. The new firm creates $l/q(\theta)$ vacancies in order to employ l workers in the next period, and pays $kl/q(\theta)$ and c. Because the firm will employ l workers in the next period, the continuation value is exactly $J(l)$. The number of firms is determined by the free entry condition $J(0) = 0$. Thus, from (3) and (17), we obtain

$$f(hl) - W(h,l)l = \frac{r+\lambda}{q(\theta)}kl + (1+r)c. \tag{18}$$

Equation (7) can be rewritten as $\lambda l = [m(U,V)/V] \times v$, which implies that, assuming symmetric equilibrium, the number of firms is

$$N \equiv \frac{V}{v} = \frac{m(U,V)}{\lambda l} = \frac{1-U}{l}. \tag{19}$$

Consider the unemployed. The probability that an individual finds a job is $M/U = \theta q(\theta)$. Thus, the value of being unemployed can be written recursively as

$$J^U = b + \theta q(\theta)\delta J^E(l_{+1}) + (1-\theta q(\theta))\delta J^U, \tag{20}$$

where b is the (exogenous) unemployment benefit. Substituting (10) and (5) into (20) yields the equilibrium value of J^U as follows:

$$\frac{r}{1+r}J^U = b + \theta q(\theta)\delta\left[J^E(l) - J^U\right] = b + \theta q(\theta)\delta\frac{\beta}{1-\beta}J'(l) = b + \frac{\beta}{1-\beta}\theta k. \tag{21}$$

[5] For a model that generates a transitory size distribution of firms as a result of the absence of this assumption, see Kudoh and Sasaki (forthcoming), in which firms are of size 0, 1, or 2. It turned out that even the three-state distribution is far from tractable.

Definition 2 *A steady-state equilibrium under free entry is a set of variables h, l, θ, U, and V that satisfies $\theta = V/U$, (2), (4), (8), and (18), equipped with the earnings profile (14).*

Under free entry, U and V are determined by the Beveridge curve once θ has been determined. Thus, we focus on the determination of h, l, and θ. The equilibrium is then summarized by the following:

$$f'(hl) = W_h(h,l), \tag{22}$$

$$f(hl) = f'(hl)hl - W_l(h,l)l^2 + (1+r)c, \tag{23}$$

$$\frac{(r+\lambda)k}{q(\theta)} = W_h(h,l)h - W(h,l) - W_l(h,l)l, \tag{24}$$

where the earnings function is given by (14) and J^U is given by (21). Equation (22) follows from the firm's choice of hours of work (4), (23) is derived by substituting (8) into the free entry condition (18), and (24) is derived by substituting (4) into the equation for the choice of employment (8). With $f(L) = AL^{\alpha}$, these equations reduce to:

$$\frac{\alpha}{\alpha\beta + 1 - \beta} AL^{\alpha-1} = e'(h), \tag{25}$$

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta + 1 - \beta} AL^{\alpha} = (1+r)c, \tag{26}$$

$$\frac{(r+\lambda)k}{q(\theta)} + \beta\theta k = (1-\beta)\left[e'(h)h - e(h) - b\right], \tag{27}$$

where $L \equiv hl$ is total labor input. It is clear that the equilibrium is uniquely determined. First, equilibrium labor input L is determined by the free entry condition (26). Given L, (25) determines h. Finally, (27) determines θ. The determination of the equilibrium is shown in Figure 2. The upward-sloping curve is based on (27), which represents the optimal choice of employment: as labor market tightness increases, the firm substitutes away from employment. However, the choice of hours is independent of tightness because the firm's demand for hours of work is determined by comparing the marginal product and the marginal wage rate $W_h(h,l)$, both of which are independent of tightness. This is reflected by the horizontal line in this figure.

Proposition 3 *Suppose that $e(h) = \varepsilon h^2$. (a) An increase in A increases h and θ. (b) An increase in k has no effect on h but lowers θ. (c) An increase in ε decreases h and θ.*

Proof. (a) Equation (26) implies that an increase in A reduces L. Since (25) and (26) imply $e'(h)L = (1+r)c\alpha/(1-\beta)(1-\alpha)$, it is evident that L and h are negatively related. Thus, an increase in A increases h. (b) Note that k appears only in (27). An increase in k shifts the employment curve upward in Figure 2. (c) From (26), changes in ε have no effect on L. Given this, (25) and (27) can be rewritten as $H = 2\varepsilon h$ and $(r+\lambda)k/q(\theta)+\beta\theta k = (1-\beta)[\varepsilon h^2 - b]$, where H is a constant. Eliminating h from these two expressions yields $(r+\lambda)k/q(\theta)+\beta\theta k = (1-\beta)[H^2/4\varepsilon - b]$, from which it is clear that $d\theta/d\varepsilon < 0$. ■

The results are intuitive. An increase in productivity induces potential firms to enter the market, which tightens the labor market and raises the expected cost of a vacancy. Firms respond to this by substituting employment for hours of work. An increase in the cost of a vacancy directly raises the expected cost of a vacancy. This has two effects, direct and indirect. The direct effect induces firms to respond to an increase in the vacancy cost by substituting employment for hours of work. The indirect effect is to deter entry; this reduces tightness, to which firms respond by substituting hours for employment. The overall effect of an increase in recruitment costs on hours of work is neutral. An increase in the marginal disutility of work reduces hours of work even though hours of work is the firm's choice variable. The key is that the earnings profile is determined by bargaining. Because an increase in the marginal disutility of work is reflected in the earnings profile, firms choosing longer hours of work must pay a higher marginal wage rate.

Here we present a numerical example. The parameters are $A = 0.9$, $\alpha = 0.8$, $\beta = 0.4$, $r = 0.01$, $\lambda = 0.2$, $c = 0.1$, $k = 0.1$, $b = 0.1$, $e(h) = h^2$, and $q = \theta^{-0.5}$. The equilibrium is given by $L = 0.83$, $h = 0.41$, $l = 2.04$, and $\theta = 0.58$. Figure 3 shows the equilibrium earnings profile. Since we have chosen the parameters so that the disutility function is sufficiently convex and the production function is not too concave, the earnings profile is convex as shown in the figure.

3.1.1 Does it Matter who Incurs the Cost of Longer Hours of Work?

To answer this question, we briefly discuss an alternative environment in which firms incur the cost of longer hours of work. Let $J(l)$ be the value of an operating firm with l employees. The Bellman equation is given by $J(l) = \max_{h,v}\{f(hl) - e(h)l - W(h,l)l - kv - c + \delta J(l_{+1})\}$, where

$l_{+1} = (1-\lambda)l + qv$. The first-order conditions with respect to h and v are $f'(hl) - e'(h) - W_h(h,l)h = 0$ and $-k + q\delta J'(l_{+1}) = 0$, respectively. The envelope condition implies $J'(l) = f'(hl)\,h - e(h) - W(h,l) - W_l(h,l)l + (1-\lambda)\delta J'(l_{+1})$. The wage equation becomes $W(h,l) = \beta[f'(hl)h - e(h) - W_l(h,l)l] + (1-\beta)rJ^U/(1+r)$. The corresponding earnings function is

$$W(h,l) = \frac{\alpha\beta A h^\alpha l^{\alpha-1}}{\alpha\beta + 1 - \beta} - \beta e(h) + (1-\beta)\frac{r}{1+r}J^U.$$

The equilibrium conditions are: $f'(hl) = e'(h) + W_h(h,l)$, $f(hl) = f'(hl)hl - W_l(h,l)l^2 + (1+r)c$, and $(r+\lambda)k/q = e'(h)h - e(h) + W_h(h,l)h - W(h,l) - W_l(h,l)l$. It is easy to verify that these conditions reduce to (25)–(27).

Proposition 4 *It does not matter who incurs the cost of longer hours of work.*

3.2 Welfare

In this section, we consider the efficiency of the equilibrium. Following Smith (1999), we consider the social planner's problem for $r = 0$ and focus on steady-state welfare, $N[f(hl) - e(h)l - c] + bU - kV$, where N is the number of firms in steady state. Thus,

$$\max_{h,l,U,V} \frac{1-U}{l}[f(hl) - e(h)l - c] + bU - kV \text{ subject to } m(U,V) = \lambda(1-U).$$

The efficient allocation must satisfy:

$$f'(L) = e'(h), \tag{28}$$

$$f(L) = f'(L)L + c, \tag{29}$$

$$\frac{f(L)-c}{l} - e(h) - b = \frac{m_U + \lambda}{m_V}k, \tag{30}$$

$$m(U,V) = \lambda(1-U). \tag{31}$$

The efficiency conditions (28)–(31) are comparable with the decentralized equilibrium conditions.

Proposition 5 $L \geq L^*$, $h \leq h^*$, *and* $l \geq l^*$, *where* L^*, h^*, *and* l^* *denote their efficient levels.*

Proof. See Appendix. ■

First, we compare (29) with (23). These two conditions are not identical because of the term $W_l(h,l) < 0$. Given $W_l(h,l) < 0$, the equilibrium L is larger than the efficient one. This replicates Smith's (1999) overemployment result.[6] The term $W_l(h,l) < 0$ is the source of overemployment because a firm has an incentive to overemploy in order to reduce the wage rates of all employees by exploiting the relationship between the marginal product of labor and the wage rate. The source of this externality is easily understood by observing the social planner's objective function. For the social planner, wages are transfers among the members of society. Thus, for efficiency, wages should not influence agents' decisions. However, for individual agents, wages significantly influence their decisions.

The same mechanism operates in terms of the efficiency of hours of work. When determining hours of work, the firm takes account of the earnings profile. By contrast, the social planner is concerned only with the marginal disutility of longer hours of work. The effect of overemployment is to lower the marginal product of an additional hour of work, and thereby reduce the demand for hours. It is clear that as α approaches unity, the overemployment effect tends to zero and hours of work approach the efficient level.

Proposition 6 *The equilibrium allocation is efficient if and only if $\eta(\theta) = \beta$ and $\beta = 0$, where $\eta(\theta) \equiv -q'(\theta)\theta/q(\theta)$.*

Proof. When $\beta = 0$, (25) and (26) reduce to $\alpha A L^{\alpha-1} = e'(h)$ and $(1-\alpha)AL^{\alpha} = (1+r)c$. These conditions coincide with (28) and (29) at $r = 0$. Note that $[1-\eta(\theta)]q(\theta) = m_V$ and $[1-\xi(\theta)]\theta q(\theta) = m_U$, where $\eta(\theta) \equiv -q'(\theta)\theta/q(\theta)$ is the elasticity of $q(\theta)$ with respect to θ, and $\xi(\theta) = 1-\eta(\theta)$ is the elasticity of $\theta q(\theta)$ with respect to θ. It is then easy to rewrite (30) as

$$\frac{f(L)-c}{l} - e(h) - b = \frac{[1-\xi(\theta)]\theta q(\theta) + \lambda}{[1-\eta(\theta)]q(\theta)} k. \tag{32}$$

Now consider (18). At $r = 0$, this can be rewritten as

$$\frac{f\left(hl\right)-c}{\left(1-\beta\right)l} - \frac{\alpha\beta A h^{\alpha} l^{\alpha-1}}{\left(1-\beta\right)\left[\alpha\beta+1-\beta\right]} - e\left(h\right) - b = \frac{\beta\theta q(\theta)+\lambda}{\left(1-\beta\right)q(\theta)} k.$$

[6] In Smith (1999), there is no employment–hours split. Thus, too large a level of L implies overemployment.

It is evident that this expression is identical to (32) if and only if $\eta(\theta) = \beta$ and $\beta = 0$. ■

Interestingly, the Hosios (1990) condition ($\eta(\theta) = \beta$) is not sufficient for efficiency. This is because of the presence of the overemployment effect, which exists as long as the production function is concave ($\alpha < 1$). To eliminate this effect, one must impose $\beta = 0$, which implies that the firm completely appropriates the bargaining surplus. In this case, the firm no longer has an incentive to overemploy in order to lower the wage rate. This is because the firm derives all the costs and benefits of the employment relationship.

3.2.1 Does it Matter who Chooses Hours of Work?

To address this question, suppose that each employed individual chooses hours of work to maximize the value of employment. Then, $J^E(l) = \max_h\{W(h,l) - e(h) + \lambda\delta J^U + (1-\lambda)\delta J^E(l_{+1})\}$. The first-order condition is $W_h(h,l) = e'(h)$. This is equivalent to

$$\frac{\alpha^2 A\,[hl]^{\alpha-1}}{\alpha\beta + 1 - \beta} = e'\,(h)\,. \tag{33}$$

The other equilibrium conditions are

$$\begin{aligned} \frac{(1-\beta)\,(1-\alpha)}{\alpha\beta+1-\beta} AL^{\alpha} &= (1+r)\,c, \\ \frac{r+\lambda}{q(\theta)}k + \beta\theta k &= (1-\beta)\left[e'\,(h)\,h - e\,(h) - b\right] + f'\,(hl)\,h - e'\,(h)\,h. \end{aligned}$$

It is now evident that in terms of L, it does not matter who chooses hours of work. Comparing (33) with (25) indicates that $e'(h^w)/e'(h^f) = \alpha < 1$, where h^w denotes hours of work chosen by workers and h^f denotes hours of work chosen by the firm. We can conclude that $h^w < h^f$.

Consider the scenario in which hours of work are determined during the bargaining stage. In particular, consider the case in which the wage rate and hours of work are determined to maximize the generalized Nash product. The Nash bargaining problem is given by $\max_{W,h}[J'(l)]^{1-\beta}[J^E(l) - J^U]^{\beta}$ subject to (6) and (9), taking J^U as given. The first-order conditions with respect to W and h are (10) and

$$(1-\beta)\left[J^E - J^U\right]\frac{\partial\,[J'\,(l)]}{\partial h} + \beta J'\,(l)\,\frac{\partial\left[J^E(l) - J^U\right]}{\partial h} = 0. \tag{34}$$

These expressions reduce to

$$\frac{\partial\left[J'\left(l\right)\right]}{\partial h}+\frac{\partial\left[J^{E}(l)-J^{U}\right]}{\partial h}=0, \tag{35}$$

where $\partial[J'(l)]/\partial h$ and $\partial[J^E(l)-J^U]/\partial h$ are computed from (11) and (12). The earnings function is the same as before and is given by (14). Thus, (35) becomes $f'(hl)+f''(hl)hl-W_{lh}(h,l)l-e'(h)=0$, which is equivalent to

$$f'\left(hl\right)+f''\left(hl\right)hl=\frac{\left(\alpha-1\right)\alpha^{2}A\left[hl\right]^{\alpha-1}}{\alpha+\frac{1-\beta}{\beta}}+e'\left(h\right).$$

With $f(L)=AL^{\alpha}$, we obtain $[\alpha\beta+1-\beta]^{-1}\alpha^{2}AL^{\alpha-1}=e'(h)$, which is identical to (33). This and the other equilibrium conditions imply that employment and hours of work in this economy coincide with employment and hours of work when individual workers choose their own hours.

Let h^b denote the hours of work that result from bargaining. We can summarize the results as follows.

Proposition 7 *$h^w=h^b<h^f<h^*$.*

Thus, it *does* matter who chooses hours of work. Moreover, as long as the choice is decentralized, hours of work are generally below the optimal level because of the presence of the overemployment effect. In fact, under linear technology ($\alpha=1$), it follows that $h^w=h^b=h^f=h^*$.

Intuition suggests that it should be irrelevant who chooses hours of work. Thus, Proposition 7 seems counterintuitive. The key is that the asymmetric Nash product is given by $[J'(l)]^{1-\beta}[J^E(l)-J^U]^{\beta}$. This means that the bargaining solution maximizes the weighted product of the *marginal increase* in the firm's value and the level of the worker's value. By contrast, the firm chooses hours of work to maximize the *level* of the firm's value. This is a common feature of large firm models with search and bargaining as long as all workers are treated as the marginal worker in the bargaining game.

3.3 Implications for Regulation

3.3.1 The Employment Effect of Restricting Working Hours

There is a policy debate about whether regulating hours of work can increase aggregate employment (Booth and Schiantarelli, 1987, Marimon and Zilibotti, 2000). Suppose that hours of work can be perfectly controlled by regulation. Does restricting working hours stimulate employment?[7] The purpose of this section is to address this issue.

The Bellman equation is given by $J(l) = \max_v\{f(hl) - W(l)l - kv - c + \delta J(l_{+1})\}$, where $l_{+1} = (1-\lambda)l + q(\theta)v$. Here, hours of work h is a policy parameter (Marimon and Zilibotti, 2000). The first-order condition with respect to v is $-k + q(\theta)\delta J'(l_{+1}) = 0$, and the envelope condition is given by (6). From these equations, $(r+\lambda)k/q = f'(hl)h - W(l) - W'(l)l$. With free entry, we have $f(hl) = W(l)l + (r+\lambda)kl/q + (1+r)c = f'(hl)hl - W'(l)l^2 + (1+r)c$. The earnings function is given by (14) and J^U is given by (21). After some algebra, it is easy to show that the equilibrium is a pair of l and θ that satisfy

$$\begin{aligned}
\frac{(1-\beta)(1-\alpha)A[hl]^{\alpha}}{\alpha\beta+1-\beta} &= (1+r)c, \\
\frac{r+\lambda}{q(\theta)}k + \beta\theta k &= \frac{(1-\beta)\alpha A[hl]^{\alpha-1}}{\alpha\beta+1-\beta}h - (1-\beta)[e(h)+b].
\end{aligned}$$

Proposition 8 *Suppose that working hours h are perfectly regulated. A reduction in hours of work increases the number of employees at each firm. It increases aggregate vacancies and decreases unemployment if and only if $\alpha A[hl]^{\alpha-1} > [\alpha\beta+1-\beta]e'(h)$.*

A change in regulated hours of work influences the marginal product of labor and the worker's disutility, and through these channels, the earnings function. This proposition states that if a decrease in hours of work increases the marginal product of labor by more than it reduces worker disutility, then the firm expands employment and entry is induced.

[7] Proposition 7 suggests that restricting hours of work cannot be welfare improving.

3.3.2 Regulated Wages

Suppose that wage rates are regulated. In particular, we assume that the earnings profile $W(h)$ is determined by the regulator and is exogenous. To simplify the analysis, we assume that the earnings function is continuous and differentiable with respect to h. In addition, $W'(h) > 0$.

The Bellman equation is given by $J(l) = \max_{h,v}\{f(hl) - W(h)l - kv - c + \delta J(l_{+1})\}$, where $l_{+1} = (1-\lambda)l + q(\theta)v$. The first-order conditions are $f'(hl) - W'(h) = 0$ and $-k + q(\theta)\delta J'(l_{+1}) = 0$, respectively. The envelope condition is $J'(l) = f'(hl)h - W(h) + (1-\lambda)\delta J'(l_{+1})$. Importantly, in the absence of bargaining, we have $W_l(h) = 0$. This rules out the overemployment effect because firms have no incentive to overemploy in order to reduce wage rates. With free entry, the equilibrium is given by $f'(hl) = W'(h)$, $f(hl) = f'(hl)hl + (1+r)c$, and $(r+\lambda)k/q(\theta) = W'(h)h - W(h)$.

Proposition 9 *The following earnings function implements the efficient levels of employment and hours of work: $W(h) = e(h) + \bar{W}$, where $\bar{W}$ is constant.*

Proof. It is evident that total employment is efficient at $r = 0$. The level of hours of work is efficient if h satisfies $f'(hl) = e'(h)$. Thus, the regulator must make sure that $W'(h) = e'(h)$. Taking integrals of both sides yields $W(h) = e(h) + \bar{W}$, where $\bar{W}$ is an arbitrary constant. ■

This proposition suggests that, for efficiency, the regulator must match the marginal increase in the wage rate to the marginal disutility of longer hours of work. Although regulators are usually concerned with the minimum level of earnings, $\bar{W}$, this does not matter for efficiency of employment and hours of work. Given $(r+\lambda)k/q(\theta) = W'(h)h - W(h)$, an increase in $\bar{W}$ decreases θ, and thereby raises the unemployment rate. In other words, the policymaker faces a trade-off between $\bar{W}$ and unemployment.

3.3.3 Entry Regulation

Some critics argue that entry regulation can serve as a useful labor market policy by reducing product market competition and thereby increasing employment. By using a model of monopolistic competition and rent sharing, Blanchard and Giavazzi (2003) showed that tougher entry regulation leads to lower employment. Bertrand and Kramarz (2002) argued that stronger entry regulation slowed down employment growth in France. Does entry regulation increase employment? To address this important issue, we modify the model so that it can be used to examine the effect of regulating the number of firms.

Suppose that the number of firms N is constant, and is sufficiently large. We treat N as a continuous variable for ease of exposition. In this environment, equation (19) must be replaced with

$$\sum_{i=1}^{N} l_i = 1 - U(\theta), \tag{36}$$

where $U'(\theta) < 0$ from the Beveridge curve. Thus, for each firm $i = 1, ..., N$,

$$\frac{\alpha}{\alpha\beta + 1 - \beta} A l_i^{\alpha - 1} = h_i^{1-\alpha} e'(h_i), \tag{37}$$

$$\frac{(r+\lambda)k}{q(\theta)} + \beta\theta k = (1-\beta)\left[e'(h_i)h_i - e(h_i) - b\right]. \tag{38}$$

In addition, the free entry condition is replaced with (36). We look for a Nash equilibrium in which individual l_i is determined as the solution to (36)–(38), taking as given all other firms' employment levels. It is easy to verify that (37) and (38) define $l_i = \Phi(\theta)$ for all i in the symmetric equilibrium, where $\Phi'(\theta) < 0$. Thus, (36) reduces to $N\Phi(\theta) = 1 - U(\theta)$, which determines equilibrium labor market tightness.

Proposition 10 *A decrease in N decreases h and θ, and increases l and U.*

Proof. The symmetric equilibrium must satisfy $N\Phi(\theta) = 1 - U(\theta)$, where the left-hand side is decreasing and the right-hand side is increasing. Thus, a decrease in N decreases θ. Expression (38) implies that a decrease in θ is associated with a decrease in h for all firms. Since $l = \Phi(\theta)$ is decreasing, a decrease in θ must be associated with an increase in l for all firms. U increases because it is decreasing in θ. ■

Although entry regulation expands employment in each individual firm, it increases aggregate unemployment because the number of firms is limited.

An alternative way to model entry regulation is to introduce a fixed cost of entry while maintaining the assumption of free entry. In this case, the free entry condition is replaced by $J(0) = F$, where $F > 0$ is the cost of entry, which is the policy parameter. The equilibrium is characterized by (25)–(27) and (26) is replaced with

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta} AL^{\alpha} = (1+r)\,c + rF.$$

It follows that an increase in F increases L and reduces h and θ. In other words, if entry regulation is implemented by introducing a fixed entry fee, either pecuniary or nonpecuniary, then stronger regulation reduces hours of work and unemployment. This suggests that the form of entry regulation matters: direct control of the number of firms reduces employment whereas imposing an entry cost expands employment.

3.4 Endogenous Entry

In the basic model, we assumed that $J(0) = 0$. This implies that there is entry of firms as long as $J(0) > 0$. However, from a potential entrant's point of view, the opportunity cost of starting up a company must be the value of being a worker (Fonseca et al., 2001). Thus, we assume that an entrepreneur starts up a company if and only if $J(0) \geq J^U$. We maintain the assumption of a unit measure of workers. Thus, if $J(0) < J^U$, then firms exit without increasing the total measure of workers.[8]

Under endogenous entry, we replace $J(0) = 0$ with $J(0) = J^U$. Thus, $J^U = -kl/q - c + \delta J(l)$, so (23) is replaced with $f(hl) = f'(hl)hl - W_l(h,l)l^2 + (1+r)c + rJ^U$, where J^U is given by (21). The equilibrium under endogenous entry is characterized by (25), (27), and the following:

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta} AL^{\alpha} = (1+r)\left[c + b + \frac{\beta}{1-\beta}\theta k\right]. \tag{39}$$

From this we define $L = L(\theta)$ with $L' > 0$. Thus, the equilibrium is characterized by a pair of h

[8] Alghough there is a continuum of workers, the number of entrepreneurs equals the number of firms, which is finite. Thus, because it requires a mass of entrepreneurs to change the total measure of workers, we can safely assume that the measure of workers is invariant to entrepreneurs' decisions.

and θ that satisfy

$$\frac{\alpha}{\alpha\beta + 1 - \beta} A[L(\theta)]^{\alpha-1} = e'(h), \tag{40}$$

$$\frac{(r+\lambda)k}{q(\theta)} + \beta\theta k = (1-\beta)\left[e'(h)h - e(h) - b\right]. \tag{41}$$

Figure 4 illustrates the determination of equilibrium. Equation (41) is depicted as the upward-sloping curve, as in Figure 2. Equation (40) produces the downward-sloping curve. The basic intuition is as follows. An increase in labor market tightness induces firms to substitute employment for longer hours of work, as represented by the upward-sloping curve. An increase in labor market tightness makes entrepreneurship less attractive, which lowers the number of firms. Incumbent firms react by increasing total labor input, which reduces the marginal product of labor. Since firms want to match the marginal product with the marginal disutility of longer hours, the demand for hours decreases.

Comparative statics results are summarized below.

Proposition 11 *(a) An increase in A increases h and θ. (b) An increase in k has an ambiguous effect on h and decreases θ. (c) An increase in λ increases h and decreases θ.*

4 Dispersion in Hours of Work

4.1 Preliminaries

According to the Groningen Growth & Development Centre (GGDC) 60-Industry Database,[9] in 2003 in the US, annual hours of work per employee were 2,306 in mining, 1,615 in retail trade, and 1,350 in hotels and catering. In 2003 in the European Union, annual hours worked per employee were 1,738 in mining, 1,515 in retail trade, and 1,503 in hotels and catering. In 2002 in Japan, annual hours of work per employee were 2,055 in mining, 1,661 in retail trade, and 1,661 in hotels and catering.

[9] The 60-Industry Database is available at http://www.ggdc.net/index-dseries.html. For a comprehensive account of this database, see Rogerson (2006), who focuses on (the evolution of) cross-country differentials in hours of work.

Using the model developed in the preceding sections, we investigate possible determinants of the dispersion in working hours. There are many potential explanations of differences in working hours, and we do not intend to give a comprehensive list of reasons. Instead, we focus on job characteristics (rather than worker characteristics such as differences in preferences) as determinants of dispersion in hours of work. In particular, we present a simple model with two types of firms and explore the possibility of differentials in working hours. The key question in this section is: which structural parameters are responsible for generating differentials in working hours, and which parameters are not?

In this section and the next, we consider two models of hours dispersion. First, we develop a model to explain within-industry hours dispersion. In this model, we assume that the labor market is pooled for all jobs; thus, a job seeker can receive a job offer from a firm in either industry. Then, we develop a model of interindustry hours dispersion. In this model, we assume that labor markets are segmented; thus, a job seeker in one industry cannot receive a job offer from a firm in the other industry.

4.2 Within-industry Differentials

Suppose that there is a single industry, but there are two types of firms, type 1 and type 2. As before, the matching technology is $m(U, V)$, and we define labor market tightness by $\theta \equiv V/U$. The probability that a vacancy is filled is $q(\theta) \equiv m(U/V, 1)$, and the probability that a worker finds a job is $\theta q(\theta)$. We assume a single labor market. Thus, all firms and workers face the same labor market tightness θ.

The Bellman equation for a firm of type $j = 1, 2$ is given by $J_j(l_j) = \max_{h_j, v_j}\{A_j[h_j l_j]^\alpha - W^j(h_j, l_j)l_j - k_j v_j - c_j + \delta J_j(l^j_{+1})\}$, where $l^j_{+1} = (1-\lambda)l_j + q(\theta)v_j$. Note that both types of firms face the same labor market tightness. This reflects the assumption that the two types of firms are in the same industry (markets are not segmented), and the assumption that workers will accept all types of jobs.[10] We assume that the bargaining weight is the same for all firms.[11] In addition, we assume that all firms face the same separation rate.[12] The first-order conditions with respect to h_j and v_j, and the envelope condition are $\alpha A_j[h_j l_j]^{\alpha-1} = W_h^j(h_j, l_j)$, $q(\theta)\delta J_j'(l_j) = k_j$, and $(r+\lambda)\delta J_j'(l_j) = \alpha A_j[h_j l_j]^{\alpha-1}h_j - W^j(h_j, l_j) - W_l^j(h_j, l_j)l_j$, respectively.

As before, we assume that workers and the firm share the rent. The firm's share of rent is denoted by $1-\beta$, where $\beta \in [0,1]$. The Nash bargaining solution requires $\beta J_j'(l_j) = (1-\beta)(J_j^E - J^U)$ for $j = 1, 2$, where the value of being employed by a firm of type j is $J_j^E = P_j + \lambda\delta J^U + (1-\lambda)\delta J_j^E$, where $P_j \equiv W^j(h_j, l_j) - e(h_j)$. Thus, the earnings function is given by

$$W^j(h_j, l_j) = \frac{\alpha A_j h_j^\alpha l_j^{\alpha-1}}{\alpha + \frac{1-\beta}{\beta}} + (1-\beta)\,e\,(h_j) + \frac{(1-\beta)\,r}{1+r} J^U. \tag{42}$$

Let ϕ denote the equilibrium proportion of type 1 vacancies. The probability that a job seeker finds a position is $\theta q(\theta)$. Given that a job seeker finds a position, he or she is employed by a firm of type 1 with probability ϕ. The value of being unemployed is $J^U = b + \theta q(\theta)[\phi\delta J_1^E + (1-\phi)\delta J_2^E] + (1-\theta q(\theta))\delta J^U$, or equivalently:

$$\frac{r}{1+r} J^U = b + \theta q\,(\theta)\left[\phi\delta\left(J_1^E - J^U\right) + (1-\phi)\,\delta\left(J_2^E - J^U\right)\right]. \tag{43}$$

Noting $J_j^E - J^U = (1-\beta)^{-1}\beta(1+r)k_j/q(\theta)$, we have

$$\frac{r}{1+r} J^U = b + \frac{\beta\theta\left[\phi k_1 + (1-\phi)\,k_2\right]}{1-\beta}. \tag{44}$$

Lemma 12 *$P_1 > P_2$ if and only if $k_1 > k_2$.*

Proof. From the employee's value functions, it is easy to verify that $J_1^E - J_2^E = (r+\lambda)^{-1}(1+r)[P_1 - P_2]$. Similarly, the bargaining outcome and the vacancy choice imply $J_j^E - J^U = (1-$

[10] For this, we must focus on equilibria in which there is a nonnegative surplus to shared. This requires that the fixed cost c_i is not too high.

[11] We do not focus on the bargaining weight because the foundation of this parameter is the relative frequency with which a party makes an offer in the context of strategic bargaining (Osborne and Rubinstein, 1990).

[12] It is evident that firms facing greater separation rates would choose longer working hours. A model with on-the-job search (Burdett and Mortensen, 1998) is required to generate a search equilibrium with endogenously dispersed separation rates.

$\beta)^{-1}\beta(1+r)k_j/q(\theta)$ for $i = 1, 2$. From these equations, we obtain

$$P_1 - P_2 = \frac{\beta}{1-\beta}\frac{r+\lambda}{q(\theta)}(k_1 - k_2).$$

Thus, $P_1 > P_2$ if and only if $k_1 > k_2$. ■

In other words, jobs with higher recruitment costs are "good jobs" (Acemoglu, 2001).

Assuming endogenous entry ($J(0) = J^U$), the equilibrium conditions are:[13]

$$\frac{\alpha}{\alpha\beta + 1 - \beta} A_j L_j^{\alpha-1} = e'(h_j), \tag{45}$$

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta + 1 - \beta} A_j L_j^{\alpha} = (1+r)\left[c_j + b + \frac{\beta\theta\left[\phi k_1 + (1-\phi) k_2\right]}{1-\beta}\right], \tag{46}$$

$$\frac{(r+\lambda) k_j}{q(\theta)} + \beta\theta k_j = (1-\beta)\left[e'(h_j) h_j - e(h_j) - b\right], \tag{47}$$

for $j = 1, 2$. The equilibrium is characterized by 6 equations in 6 unknowns, L_1, L_2, h_1, h_2, θ, and ϕ. Equation (47) suggests that, given the equilibrium labor market tightness θ, there is a one-to-one relationship between k_j and h_j. This suggests that within-industry differentials in hours of work can only arise when $k_1 \neq k_2$. The parameter k_j captures the cost of creating a vacancy, thus playing the same role as does the cost of creating a job in Acemoglu's (2001) model, in which differences in the costs of equipment for each job create differences in job characteristics, such as wage and productivity differentials. The following proposition emphasizes this point.

Proposition 13 *Within-industry differentials in working hours arise if and only if $k_1 \neq k_2$. In particular, $k_1 > k_2$ implies that $h_1 > h_2$. Technology differentials cannot account for differentials in hours of work.*

Jobs with higher recruitment costs are "good jobs", as shown by Lemma 12. Further, Proposition 13 implies that good jobs are associated with longer hours of work. Since there are two types of jobs in a single industry, a possible interpretation of this result is that it describes differentials in

[13] Unfortunately, the basic model cannot be used to study heterogeneous firms because it cannot support an equilibrium with heterogeneous firms. Thus, the model we employ in what follows is the one with endogenous entry presented in Section 3.4. The consequence of adopting free entry is presented in Appendix C.

hours of work between full-time and part-time jobs, or differentials between jobs that require training and those that do not. For evidence of differences in recruitment costs, we quote Hamermesh (1993, p. 208):[14]

> A survey of employers in the Rochester, New York, area in 1965–66 found an average hiring cost for all occupations of $910, but an average for professional and managerial workers of $4,600. A survey in Los Angeles in 1980 found hiring and training costs of $13,790 for salaried workers, and $5,110 for production workers. [...] In a nationwide survey of large employers in 1979 the cost of hiring a secretary was $680, but for a college graduate was $2,200.

Thus, there are substantial differences in recruitment costs, and jobs that require more skills are more costly to fill. Our model suggests that such jobs pay more, and require longer hours of work.

4.3 Interindustry Differentials

To account for interindustry hours dispersion, in this section we analyze a directed search version of the model in the spirit of Acemoglu (2001, Section III.B) and Moen (1997). The key feature is that there are two industries and their labor markets are segmented: once entered, a worker will never receive a job offer from a firm in the other industry. However, each unemployed individual freely chooses to look for work in any one of the industries at the beginning of each period. Arbitrage dictates and the values of seeking a job in the two industries must be balanced in equilibrium.

Suppose that the markets for the two types of jobs are segmented, and each worker can apply to either industry 1 or industry 2. We assume that the two industries face the same matching technology. Let U_j denote the number of unemployed workers applying to a firm in industry j. Similarly, V_j denotes the number of vacancies in industry j. Thus, the probability that a worker applying to industry j finds a job in that industry is $\theta_j q(\theta_j)$, where $\theta_j \equiv V_j/U_j$. Let μ be

[14] All values are in 1990 US dollars.

the proportion of workers in industry 1. Then, (2) is replaced with $m\,(U_1, V_1) = \lambda(1 - U_1)\mu$ and $m\,(U_2, V_2) = \lambda(1-U_2)(1-\mu)$, which define the Beveridge curves for industries 1 and 2, respectively.

Following Acemoglu (2001), we assume that each industry produces a distinct commodity. Let $p_1(\mu)$ and $p_2(\mu)$ be the prices of the products in industries 1 and 2, respectively. We assume that $p_1(\mu)$ is decreasing and $p_2(\mu)$ is increasing.[15]

As before, we assume that the bargaining weight is the same for all industries. In addition, we assume that all firms face the same separation rate. The Bellman equation for a firm in industry $j = 1, 2$ is given by $J_j(l_j) = \max_{h_j,v_j}\{p_j(\mu)A_j[h_j l_j]^\alpha - W^j(h_j, l_j)l_j - k_j v_j - c_j + \delta J_j(l^j_{+1})\}$, where $l^j_{+1} = (1 - \lambda)l_j + q(\theta_j)v_j$. The first-order conditions with respect to h_j and v_j, and the envelope condition are $\alpha p_j(\mu)A_j[h_j l_j]^{\alpha-1} = W^j_h(h_j, l_j)$, $q(\theta_j)\delta J'_j(l_j) = k_j$, and $(r + \lambda)\delta J'_j(l_j) = \alpha p_j(\mu)A_j[h_j l_j]^{\alpha-1}h_j - W^j(h_j, l_j) - W^j_l(h_j, l_j)l_j$, respectively.

The Nash bargaining solution requires $\beta J'_j(l_j) = (1 - \beta)(J^E_j - J^U_j)$ for $j = 1, 2$. The value of applying to industry $j = 1, 2$ is $J^U_j = b + \theta_j q(\theta_j)\delta J^E_j + (1 - \theta_j q(\theta_j))\delta J^U_j$. Similarly, the value of being employed is $J^E_j = P_j + \lambda\delta J^U_j + (1 - \lambda)\delta J^E_j$ for $j = 1, 2$, where $P_j = W_j(h_j, l_j) - e(h_j)$. From these expressions, we obtain

$$\frac{r}{1 + r}J^U_j = b + \frac{\beta}{1 - \beta}\theta_j k_j$$

(see also (21)). For the two sectors to coexist, we must have $J^U_1 = J^U_2$, which requires that in any equilibrium,

$$k_1\theta_1 = k_2\theta_2. \tag{48}$$

Lemma 14 *$P_1 > P_2$ if and only if $k_1 > k_2$.*

Proof. The employee's value functions imply that $J^E_1 - J^E_2 = (r+\lambda)^{-1}(1+r)[P_1 - P_2]$. Similarly, the bargaining outcome and the vacancy choice imply $J^E_j - J^U = (1 - \beta)^{-1}\beta(1 + r)k_j\theta_j/\theta_j q(\theta_j)$ for $i = 1, 2$. From these equations, we obtain

$$P_1 - P_2 = \frac{\beta}{1 - \beta}(r + \lambda)k_1\theta_1\left[\frac{1}{\theta_1 q(\theta_1)} - \frac{1}{\theta_2 q(\theta_2)}\right].$$

[15] In general, the price function is more complex. However, we simply postulate the price function in order to avoid further complication of the analysis. For a derivation of the price function, see Acemoglu (2001), for example.

Thus, $P_1 > P_2$ if and only if $\theta_2 > \theta_1$. It is then easy to verify that (48) implies $k_1 > k_2$. ■

As stated in Lemma 12, jobs with higher recruitment costs are "good jobs". Industries with higher recruitment costs face a slacker labor market because they pay higher wages and attract more workers.

With endogenous entry, the equilibrium is characterized by

$$\frac{\alpha}{\alpha\beta+1-\beta}p_j(\mu)A_jL_j^{\alpha-1} = e'(h_j), \tag{49}$$

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta}p_j(\mu)A_jL_j^{\alpha} = (1+r)\left[c_j+b+\frac{\beta}{1-\beta}\theta_j k_j\right] \tag{50}$$

$$\frac{(r+\lambda)k_j}{q(\theta_j)}+\beta\theta_j k_j = (1-\beta)\left[e'(h_j)h_j - e(h_j) - b\right], \tag{51}$$

$k_1\theta_1 = k_2\theta_2$, $m(U_1, V_1) = \lambda(1-U_1)\mu$, $m(U_2, V_2) = \lambda(1-U_2)(1-\mu)$, $\theta_1 = V_1/U_1$, $\theta_2 = V_2/U_2$. Thus, the equilibrium is the solution to the system of 11 equations in 11 unknowns, L_1, L_2, h_1, h_2, θ_1, θ_2, U_1, U_2, V_1, V_2, and μ.

Proposition 15 *Suppose that all industries share the same β, λ, and $e(h)$. Interindustry dispersion in working hours arises if and only if $k_1 \neq k_2$. In particular, $k_1 > k_2$ implies $h_1 > h_2$. Technology differentials cannot account for differentials in hours of work.*

Proof. Equation (51) implies a positive relationship between h_j and the left-hand side of (51). Given (48), the second term on the left-hand side of (51) is the same for industries 1 and 2. Therefore, we need only compare $(r+\lambda)k_1/q(\theta_1)$ and $(r+\lambda)k_2/q(\theta_2)$. Thus, we have

$$\frac{(r+\lambda)k_1}{q(\theta_1)} - \frac{(r+\lambda)k_2}{q(\theta_2)} = \frac{(r+\lambda)}{q(\theta_1)}\frac{k_2\theta_2}{\theta_1} - \frac{(r+\lambda)k_2}{q(\theta_2)} = k_2\theta_2\left[\frac{r+\lambda}{\theta_1 q(\theta_1)} - \frac{r+\lambda}{\theta_2 q(\theta_2)}\right] > 0.$$

This follows because $k_1 > k_2 \Leftrightarrow \theta_2 > \theta_1 \Leftrightarrow \theta_2 q(\theta_2) > \theta_1 q(\theta_1)$. This establishes that the left-hand side of (51) is larger for industry 1 than for industry 2. ■

If labor markets are segmented between industries, and workers choose between them, then firms face different degrees of labor market tightness. As Acemoglu (2001) points out, firms that incur higher recruitment costs (or equipment costs) pay higher wages. Thus, more workers apply to work in these high-paying industries, and labor markets are slacker in these industries. Proposition 15 establishes that "good jobs" require longer hours of work. Interestingly, differences in technology A_j cannot account for working hours differentials, although they do account for interindustry differences in total labor inputs L_j. Since employment in each firm satisfies $l_j = L_j/h_j$, both technology and recruitment costs account for differences in firm size across industries.

5 Conclusion

In this paper, we analyzed the choice of employment and hours of work in a dynamic model with search frictions and bargaining. We showed that the bargaining outcome defines the earnings profile faced by the firm when it chooses its composition of employment and hours of work. We found that differences in recruitment costs are responsible for dispersion in hours of work, both within and across industries. In particular, "good jobs" (Acemoglu, 2001) pay better and require longer hours of work. Interestingly, technology differentials cannot account for working hours differentials.

An advantage of our approach is that the earnings profile is derived as part of an equilibrium. Since the earnings profile reflects both the production technology and the worker's disutility from labor, hours of work may be determined either by the firm's value maximization or by a worker's labor–leisure choice. In addition, the derived earnings profile is consistent with the empirical labor supply curve. We found that, because of Smith's (1999) overemployment effect, hours of work are below their optimal level. Further, the Hosios condition is not sufficient for restoring efficiency. To restore efficiency under concave production technology, the firm must appropriate the whole bargaining surplus. This is a general feature of large firm models with search and bargaining. Investigation of the robustness of this inefficiency result in richer environments belongs to future search.

A limitation of our approach is that the basic model with free entry cannot support an equilibrium with heterogeneous firms. We modified the basic model by replacing the free entry assumption with endogenous entry. Generally, one needs a richer model of multiple industries to account for the heterogeneity of job characteristics in equilibrium, and much of this line of research is left for future research. A possible direction is suggested by Cahuc et al. (2008).

In this paper, we did not consider the heterogeneity of workers. Kuhn and Lozano (2005) documented that the observed increase in hours of work in the US was concentrated among skilled men. A deeper understanding of dispersion in hours of work requires a model that incorporates heterogeneous skills. It would be interesting to incorporate skilled and unskilled workers into the model of this paper to investigate whether it can explain Kuhn and Lozano's (2005) findings.

In this paper, we focused on the steady state. However, the choice of employment and hours of work is particularly important for understanding business cycles. It is well documented that during a recession, firms cut employment rather than reduce hours of work sufficiently to maintain the level of employment. Since employment adjustment is costly, it seems rational to hoard labor during a recession if favorable business conditions return shortly (Burnside et al., 1993; Bertola and Caballero, 1994). This suggests that firms that fire their employees face large and persistent negative shocks during a recession. A useful task for future work is to extend the model of this paper to explain the substitution between employment and hours of work over the business cycle.

Appendix

A Proof of Proposition 1

Rewrite (13) as

$$W_l(h,l) + \frac{1}{\beta l} W(h,l) = \frac{f'(hl)\,h}{l} + \frac{1-\beta}{\beta l}\left[e(h) + \frac{r}{1+r} J^U\right].$$

Let $y(l) \equiv W(h,l)$, $a(l) \equiv B/l \equiv \beta^{-1}/l$, and

$$g(l) \equiv \frac{f'(hl)\,h}{l} + \frac{1-\beta}{\beta l}\left[e(h) + \frac{r}{1+r} J^U\right] \equiv \pi(l) + \frac{\Pi}{l}.$$

Then the equation can be rewritten more compactly as $y' + a(l)y = g(l)$, which is a linear ordinary differential equation with variable coefficients, and its solution is known to be easily derived. A useful reference is Bellman and Cooke (1995, pp. 40–41.), from which the general solution to this differential equation is given by

$$y(l) = \exp\left\{-\int_0^l a(s)ds\right\}\left[\int_0^l \exp\left\{\int_0^i a(s)ds\right\} g(i)di + C\right],$$

where C is a constant. Since $y(l)$ is the earnings function, it is (economically) reasonable to assume $y(0) = 0$, which leads to $C = 0$. Thus,

$$y(l) = \exp\left\{-\int_0^l a(s)ds\right\}\int_0^l \exp\left\{\int_0^i a(s)ds\right\} g(i)di = \int_0^l \exp\left\{-\int_i^l a(s)ds\right\} g(i)di.$$

Since

$$\exp\left\{-\int_i^l a(s)ds\right\} = \exp\left\{-B\int_i^l \frac{1}{s}ds\right\} = \exp\left\{-B\left[\ln s\right]_i^l\right\} = \exp\left\{-B\left[\ln l - \ln i\right]\right\} = l^{-B} i^B,$$

we obtain

$$y(l) = \int_0^l l^{-B} i^B \left[\pi(i) + \frac{\Pi}{i}\right] di = l^{-B}\int_0^l i^B \left[\pi(i)\right] di + \frac{\Pi}{B}.$$

Thus, the earnings function we are looking for is

$$W(h,l) = l^{-1/\beta}\int_0^l i^{\frac{1}{\beta}-1}\left[f'(hi)\,h\right] di + (1-\beta)\left[e(h) + \frac{r}{1+r} J^U\right],$$

which is a generalization of the wage functions derived by Bertola and Caballero (1994), Bertola and Garibaldi (2001) and Cahuc and Wasmer (2001).

B Proof of Proposition 5

Equations (26) and (29) imply

$$\frac{L}{L^*} = \left[\frac{\alpha\beta + 1 - \beta}{1-\beta}\right]^{\frac{1}{\alpha}} \geq 1,$$

which replicates Smith's (1999) overemployment result. Similarly, (25) and (28) imply

$$\begin{aligned}
\frac{e'(h^*)}{e'(h)} &= (\alpha\beta + 1 - \beta)\left[\frac{L}{L^*}\right]^{1-\alpha} = (\alpha\beta + 1 - \beta)\left[\frac{\alpha\beta + 1 - \beta}{1-\beta}\right]^{\frac{1-\alpha}{\alpha}} \\
&= (1-\beta)\left[1 + \frac{\alpha\beta}{1-\beta}\right]^{\frac{1}{\alpha}} \equiv g(\beta),
\end{aligned}$$

where $0 < \alpha < 1$ and $0 \leq \beta \leq 1$. It is easy to show that

$$g'(\beta) = \frac{(1-\alpha)\beta}{1-\beta}\left[1 + \frac{\alpha\beta}{1-\beta}\right]^{\frac{1}{\alpha}-1} \geq 0$$

for all $\beta \in [0,1]$. Thus, $g(\beta)$ is monotonically increasing. Since $g(0) = 1$, by continuity, we establish that $e'(h^*)/e'(h) \geq 1$, or equivalently $h^* \geq h$. From $L/L^* = hl/h^*l^* \geq 1$, we have $l/l^* \geq h^*/h \geq 1$.

C Firm Heterogeneity under Free Entry

This section briefly discusses why the free entry assumption must be replaced with endogenous entry in the models of Section 4. Under free entry ($J(0) = 0$), the equilibrium conditions are:

$$\frac{\alpha}{\alpha\beta + 1 - \beta} A_j L_j^{\alpha-1} = e'(h_j), \tag{52}$$

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta + 1 - \beta} A_j L_j^{\alpha} = (1+r)c_j, \tag{53}$$

$$\frac{(r+\lambda)k_j}{q(\theta)} + \beta\theta k_j = (1-\beta)\left[e'(h_j)h_j - e(h_j) - b\right]. \tag{54}$$

Note that ϕ does not appear in (53), while (46) contains ϕ. It is easy to verify that (53) determines L_j, and given this, (52) determines h_j for $j = 1, 2$. However, (54) contradicts the fact that all firms face the same tightness θ.

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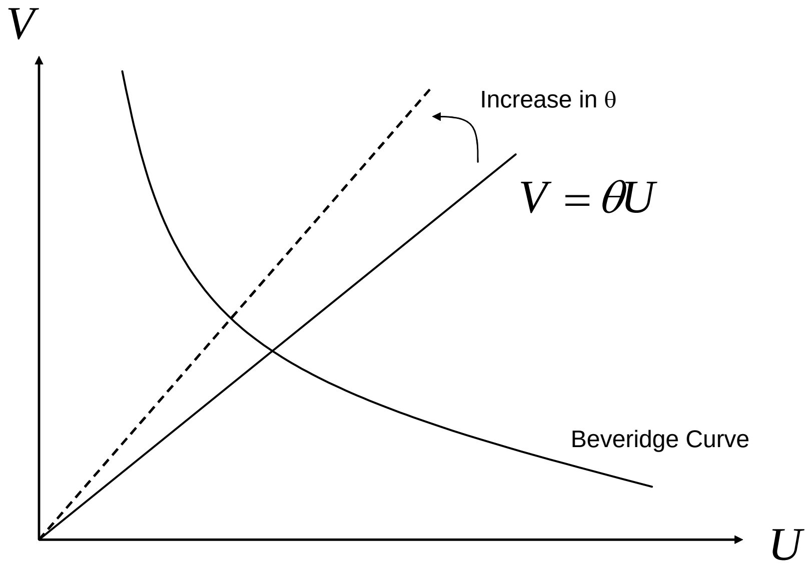

Figure 1

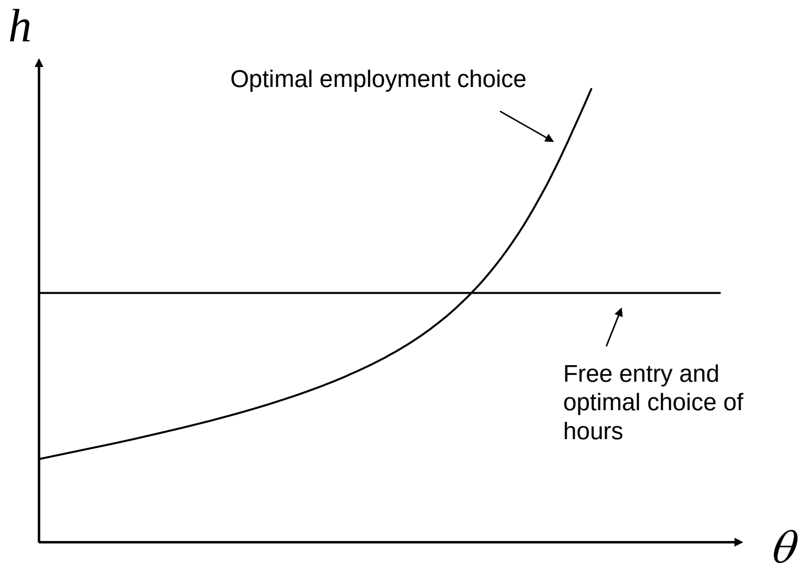

Figure 2

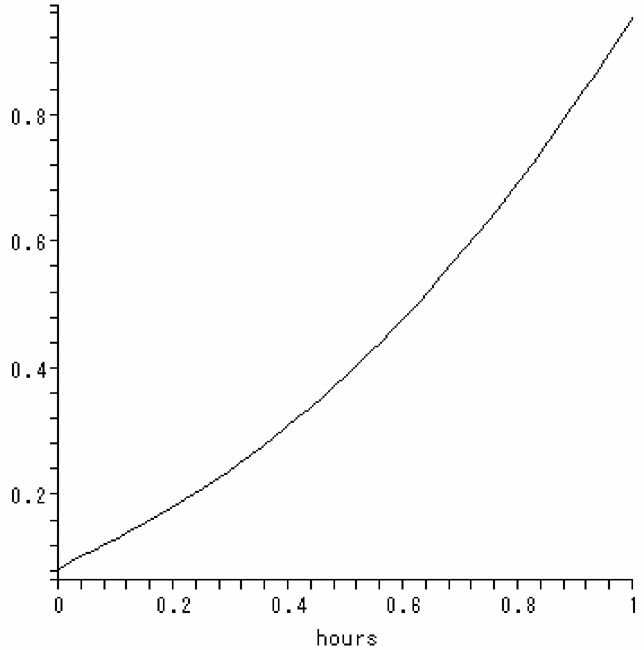

Figure 3

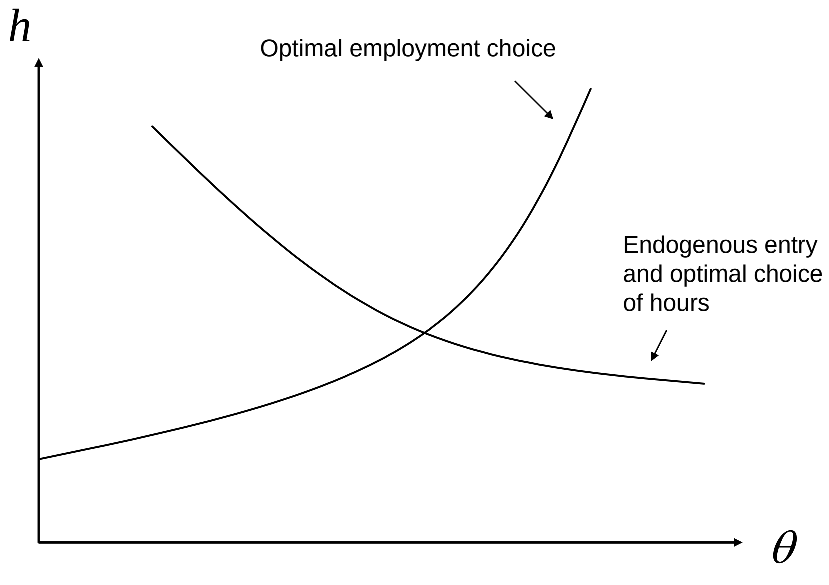

Figure 4