<table>
<thead>
<tr>
<th>Title</th>
<th>Very long period seismic signals observed before the caldera formation with the 2000 Miyake-jima volcanic activity, Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Kobayashi, Tomokazu; Ohminato, Takao; Ida, Yoshiaki; Fujita, Eisuke</td>
</tr>
<tr>
<td>Citation</td>
<td>Journal of Geophysical Research. B, Solid earth, 114(2), B02211</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2009</td>
</tr>
<tr>
<td>Doc URL</td>
<td><a href="http://hdl.handle.net/2115/36094">http://hdl.handle.net/2115/36094</a></td>
</tr>
<tr>
<td>Rights</td>
<td>An edited version of this paper was published by AGU. Copyright 2009 American Geophysical Union.</td>
</tr>
<tr>
<td>Type</td>
<td>article (author version)</td>
</tr>
<tr>
<td>File Information</td>
<td>Kobayashi_JGR2008.pdf</td>
</tr>
</tbody>
</table>

Hokkaido University Collection of Scholarly and Academic Papers : HUSCAP
Very Long Period Seismic Signals

Observed before the Caldera Formation

with the 2000 Miyake-jima Volcanic Activity, Japan

by

1 Tomokazu Kobayashi, 2 Takao Ohminato, 3 Yoshiaki Ida, and 4 Eisuke Fujita

1 Institute of Seismology and Volcanology, University of Hokkaido, Sapporo, Japan

2 Earthquake Research Institute, University of Tokyo, Tokyo, Japan

3 Advance Soft Corporation, Tokyo, Japan

4 National Research Institute for Earth Science and Disaster Prevention, Tsukuba, Japan

Submitted to Journal of Geophysical Research
Abstract

Very long period (VLP) seismic signals, whose waveform consists of an initial impulsive signal and a later oscillatory wave with 0.2 or 0.4 Hz in dominant frequency, were observed before the caldera formation in the 2000 activity of Miyake-jima volcano, Japan. The results of waveform inversion show that the initial and later parts can be explained by a northward single-force of $1.5 \times 10^8$ N working at a depth of 2 km beneath the summit and a moment tensor solution at a depth of 5 km below and 2 km southwest of the summit with $\sim 10^{12}$ Nm, respectively. A clear positive correlation of the amplitudes between the two sources strongly suggests that the shallow single-force triggers the deeper moment source in spite of the several km distance between the two sources. To analyze the source time functions of the moment tensor that do not always oscillate in phase, we introduce a new method of moment tensor diagonalization which is performed in the frequency domain. According to the analysis, the two principal components have similar amplitudes and are greater than the third principal component, suggesting an axially symmetric oscillation. One of the possible systems is a combination of two cracks intersecting perpendicularly. Our interpretation is that the single-force was generated when magma containing rock blocks suddenly began to move in a choked subsurface magma path, and the resultant pressure waves propagated and excited a resonance oscillation of the two cracks.
Miyake-jima island, located about 200 km south of Tokyo, is one of the most active basalt volcanoes in Japan (Figure 1). In most historical events, basaltic magma erupted mainly from the flank of the volcano over a more or less constant interval [Tsukui and Suzuki, 1998], while the 2000 eruption took place in a quite different way after 17 years quiescence. The 2000 activity is divided into four stages on the basis of surface phenomena; i.e., magma intrusion stage, summit subsidence stage, explosion stage, and degassing stage [Nakada et al., 2005].

The volcanic activity started at 18:30 on 26 June (Japan Standard Time; JST) with an earthquake swarm that began beneath the summit (Figure 1) [Sakai et al., 2001; Uhira et al., 2005], indicating the propagation of dikes due to magma intrusions [Irwan et al., 2003; Ueda et al., 2005]. This is the beginning of the magma intrusion stage. According to geodetic observations during the intrusion stage, a dike that extended to the southwest from the southern edge of the summit area was formed with its top depth at about 2 km. The first dike opening, however, ceased approximately two hours after the onset. Immediately after that, a new dike perpendicular to the first dike began to propagate northwestwardly from the southwest side of the island. This intrusion further extended offshore of the island by the next day culminating in a small submarine eruption [Kaneko et al., 2005], and continued to
migrate to the vicinity of Nii-jima and Kozu-shima islands. The earthquake swarm beneath the island died out by the next day and the activity in the island became relatively quiescent.

Small volcanic earthquakes began to be recorded beneath the summit at the beginning of July and the volcanic activity stepped into the next stage, that is, the summit subsidence stage. On 8 July, a summit eruption occurred accompanied by a subsidence of the summit area leading to a formation of a caldera. The caldera intermittently enlarged mainly from July to early August. The final size of the caldera was 1.6 km in diameter and 450 m in depth [Geshi et al., 2002]. As the subsidence activity of the summit area faded out, intermittent explosive eruptions began to occur (explosion stage) and eventually led to the largest explosion on 18 August. After the largest eruption, the volcanic activity was characterized by a strong volcanic gas emission (degassing stage). By the end of August and early September, the volcano began to emit substantial quantities of sulfur dioxide, at times exceeding 100,000 metric tons of SO₂ per day [Kazahaya et al., 2004].

During the 2000 volcanic activity, many types of volcanic earthquakes were observed, reflecting magmatic activity under the volcano such as magma intrusion [e.g., Sakai et al., 2001; Uhira et al., 2005], caldera development [e.g., Kikuchi et al., 2001; Kumagai et al., 2001; Kobayashi et al., 2003], and summit eruptions [e.g., Kobayashi et al., 2005]. In the magma intrusion stage, we detected very long period seismic signals containing
an initial impulsive wave followed by a monotonous oscillation with a period of a few seconds.

Figure 2 shows an unfiltered seismogram for half a day recorded by a broadband seismometer at station KAS on 30 June, 2000. In particular, seismic signals oscillating with a few seconds in period and lasting for approximately one minute can be identified in the figure. We call these events very long period (VLP) events in this study. The VLP events are clearly recognized for a few days after the earthquake swarm on 26 June, and the number of the events gradually increases by the time of the first summit eruption on 8 July. However, the events are hardly identified after the summit eruption. Several events are identified exceptionally on 9 July, but the waveform feature changes in that the initial impulsive waves are obscure and the later oscillatory part decays within a few tens of seconds.

Various types of VLP events have been observed in and around volcanoes worldwide. The VLP events observed at active volcanoes are generally considered to represent the behavior of volcanic fluids in magmatic and/or hydrothermal systems. Compared with short-period events, the VLP events have waveforms that are less contaminated by reflected and refracted waves associated with complex velocity and density structures of volcanoes due to their relatively long wavelengths, and are more easily used to extract source information. Owing to the merits of the VLP waveforms, the analysis of the
events observed at Miyake-jima provides useful information regarding the magma pluming system of Miyake-jima volcano and the magmatic processes occurring before the caldera formation at this volcano.

The method for inversion of VLP waveforms applied to obtain the source location and mechanism follows that of Ohminato et al. [1998] and Kumagai et al. [2003]. Most of the published studies dealing with the moment tensor inversion of VLP events have dealt with the case that an excitation of a VLP event can be explained by a relatively simple mechanism that consists of a single-force, a moment tensor, or a combination of the force and moment applied at a point source. However, the VLP events we consider cannot be explained satisfactorily by these simple mechanisms, and thus more sophisticated techniques, such as a method developed in this paper, are required for their analyses.

We determine the source locations and mechanisms of the VLP events observed before the first summit eruption on 8 July 2000 in Miyake-jima volcano. We first show some basic features of these VLP waveforms and the observation system in section 2. In section 3, we conduct waveform inversions to find the source location and mechanism of the VLP events. In this analysis we suggest that the initial impulsive pulse and the later oscillatory part of the waveforms are produced by two sources at different locations. In section 4, we investigate the relationship between the two sources and decompose the moment tensor
solution of the later oscillatory part into three dipoles that are not always oscillating in phase.

For this analysis, we develop a new technique. In section 5, a series of numerical tests for the
resolution of the inversion method are conducted. Finally, we discuss the physical meaning of
the VLP events in relation to the magma plumbing system and the volcanic processes in
Miyake-jima.

2. Data and Characteristics of the VLP Seismic Signals

For the waveform analyses in this study, we use broadband seismograms recorded
on a seismic network composed of seven stations. The network configuration is shown in
Figure 1. The solid circles in the figure indicate the locations of the broadband seismic
stations. The broadband seismometers at CND, KAS, and OFN were deployed by the
Earthquake Research Institute (ERI), University of Tokyo. The instrument at each station is a
Guralp CMG-3T, which has a flat response up to 100 s. The broadband seismometers of ERI
were operated from the afternoon on 29 June. The other four stations MKA, MKK, MKS, and
MKT were deployed by the National Research Institute for Earth Science and Disaster
Prevention (NIED), and the instrument at each NIED station is a Streckeisen STS-2, which
has a flat response up to 120 s. These four seismometers had already been in operation before
26 June. All seismometers are velocity sensors. Three component seismograms are available
for all stations, except at stations CND and OFN. The horizontal components at CND and OFN are not available throughout the whole period and until 2 July, respectively.

The observed VLP seismic signals consist of two parts. Each of the VLP seismic signals begins with an impulsive waveform and is followed by a monotonous oscillation. Additionally, the VLP events are classified into two types according to the dominant frequency of the oscillatory part; one with the dominant frequency of 0.2 Hz, and the other with 0.4 Hz. The events with a dominant frequency of 0.4 Hz are observed more frequently than the 0.2 Hz type, but clear difference in temporal pattern of occurrence cannot be recognized between the two types. In the following, we will describe two typical events for these types, and mention their basic features in detail.

We first show an example of the VLP events with a dominant frequency of 0.2 Hz in Figure 3a. The three component velocity waveforms observed at station KAS are displayed with their amplitude spectra. Seismograms in the top panels are obtained by applying a low-pass filter with a cutoff frequency of 0.8 Hz to a VLP event observed on 1 July. The origin time on the horizontal axis corresponds to 21:37:50 (JST). We call this VLP event “Event 1” hereafter. An impulsive waveform, whose pulse width is about 1 s, is clearly identified in the initial part of the waveform. The initial pulse is dominant in the horizontal components, particularly the NS component, rather than the vertical component. The
dominant energy in the horizontal components is common to the other stations. The later portion of the seismic signals consists of a monotonous oscillation with about a 5 s period.

The amplitude spectra of this VLP event are shown at the bottom of Figure 3a. A sharp peak can be recognized at 0.2 Hz in each component, corresponding to the visible monotonic later phase in the waveform. Another frequency peak is identified at 0.4 Hz and several other peaks can also be recognized at around 0.6 and 0.8 Hz, suggesting that they may be harmonic overtones corresponding to higher modes of resonance. The amplitude of the dominant frequency component is on the order of $10^{-4}$ to $10^{-3}$ m/s, which is significantly larger than that of the microseismic noise that is approximately on the order of $10^{-6}$ to $10^{-5}$ m/s during the analysis period.

Figure 3b shows a VLP event with a dominant frequency of 0.4 Hz, observed on 3 July. Seismograms in the top of Figure 3b are low-pass filtered with a 0.8 Hz cutoff. The origin time on the horizontal axis is 18:00:50. We call this VLP event “Event 2” hereafter. An impulsive initial part is clearly identified in the NS component as is observed in Event 1. As shown in the bottom panels, a spectral peak at 0.4 Hz corresponding to the later part of the waveform, is dominant rather than the peak at 0.2 Hz, and several other frequency peaks are recognized as well.

Hereafter, we expediently call the impulsive portion the “initial part”, and the
oscillatory signal the “later part”.

3. Source Location and Mechanism

3.1 Single Source or Composite Source?

To determine the location and the mechanism of the VLP source, we conduct waveform inversions over a domain centered on the volcano. In this analysis, synthetic waveforms for trial point sources placed at a grid point in three-dimensional space are computed, and are compared with the observed waveforms. The source point and source time functions that give the best waveform fit are regarded as the optimal estimate of the source for each seismic event. For this waveform inversion, we use the method developed by Ohminato et al. [1998].

We use a one-dimensional smoothed velocity structure model, obtained using a layered-velocity structure composed of four velocity layers in the deeper part [Sakai et al., 2001] and a slow-velocity layer of 1.5 km/s at the top [Kobayashi et al., 2003]. For the construction of the smoothed structure, we interpolate the arithmetic mean of the velocity values in the upper and lower sides of each discontinuity boundary. The $P$ wave velocities $V_p$ are 1.50, 1.85, 2.90, 4.80, and 6.35 km/s at 0.0, 0.1, 0.5, 2.5, and 12.5 km in depth, respectively. We assume that the shear wave velocity is equal to $V_p/\sqrt{3}$, and the density is
2700 kg/m$^3$.

The Green’s functions are calculated for the velocity structure with station elevation corrections. For the calculation, no topographic effect is taken into account except for the station elevation corrections. The reason why we ignore the effect of topography is as follows. There are two cases where a topographic effect is important. One is when a source is shallow and waveforms at the surface are significantly distorted by surface waves. The other case is when stations are close to strong topography such as the summit of volcano. In our case, the sources expected from the particle motion analysis are deep enough to ignore the surface wave excitation, and the stations are not close to the summit.

Gaussian pulses with a pulse width of 0.6 s convolved with the Green’s functions are used as elementary source time functions and the source time histories are represented by superimposing the elementary functions spaced at 0.4 s intervals. Unknown amplitudes of the elementary functions are determined so as to minimize the weighted sum of the squared differences between the observed and synthetic waveforms over the entire time window for all stations. The residual errors are influenced by weights given to seismic data, namely, different error definitions might produce serious difference on the inversion results. The weights should be given to seismic data according to a quality of noise, but it is difficult to know in advance how noise is included in seismic data. Ohminato et al. [1998] propose two
definitions of squared error for evaluation of the misfits between the observations and the synthetics; “definition A” and “definition B”, which emphasize different aspects of the noise included in the data. In this study, we employ the two errors for the waveform inversion and investigate whether or not the difference of error definitions give rise to serious influences on the inversion results for the data we consider. Resultantly, both the obtained source positions and the source time functions are essentially indistinguishable between the two error definitions. Thus, in this study, we will show the results obtained from the definition A only.

This procedure is repeated for all the grid points in the searched domain. The searched domain is from -5.0 to 4.0 km in the NS-direction, -4.0 to 5.0 km in the EW-direction, and 10.0 km in vertical extent, with the horizontal center at the summit of Miyake-jima (34.081°N, 139.530°E). The grid search is conducted first with coarse grids of 0.5 km, and then with finer meshes of 0.1 km in a domain of 1 km × 1 km × 1 km surrounding the source location determined in the previous step for the final estimate of the source location. Before conducting the waveform inversions, we apply a low-pass filter with a cutoff frequency of 0.8 Hz to the observed waveform and then resample them every 0.2 s.

We first apply the waveform inversion method to the entire waveform assuming a single point source. We conduct the above inversion procedure for three types of source mechanisms; a single-force with three components, a moment tensor with six components,
and a combination of a moment tensor and a single-force with nine total components. The
estimated residual errors between the observations and synthetics for each VLP event for each
source type are listed in Table 1. The solution derived from the combination of moment and
force gives the minimum residual errors for both VLP events, but the lowest residuals do not
always mean the optimal solution because of the different number of free parameters.

Akaike’s Information Criterion (AIC) [Akaike, 1980] is a useful tool to estimate the optimal
number of free parameters. In this study, we evaluate AIC values as one of the indicators for
the selection of optimal source mechanism although the conditions required for the use of
AIC may not necessarily be satisfied in certain cases [Sakamoto et al., 1986]. Consequently,
the source type with a combination of moment and force yields the minimum values of the
AIC for both the events (Table 1), that is, the inversion results show that both the moment and
force significantly contribute to produce the VLP events. However, investigating the
inversion results in detail, there are several unnatural features so that we cannot accept the
obtained solutions without question, as we enumerate below.

Figures 4a and 4b show the source time functions obtained from the waveform
inversion assuming the combined source mechanisms for Event 1 and Event 2, respectively.
The Cartesian coordinates are set with x axis positive eastward, y axis positive northward, and
z axis positive upward. The left six and right three diagrams give the source time functions of
the moment tensor and the single-force components, respectively. The results show that the
source mechanisms appear to be rather complicated. For the resulting moment tensor
solutions, the source time functions are dissimilar and there is a significant phase difference
among the components. The eigenvectors obtained by a sample-by-sample eigenvalue
analysis of the six moment tensor components show that the principal axes of oscillation
rotate with time and are not fixed spatially, and that the ratios among the three eigenvalues
are not constant but vary with time. The complex features do not indicate an oscillation of a
simple-shaped source such as a crack and a cylinder [Chouet; 1996]. The single-force
solutions are also complex for both the events. They have oscillatory time histories with
dominant period of a few seconds for all the components, and the time histories are dissimilar
among the three components, suggesting that the force works in different orientations over
time. These complexities of the source time functions prevent an easy interpretation of the
obtained solutions in terms of a realistic source process.

The locations having minimum residuals for the combined solutions involving both
single-force and moment tensor components are indicated by circles in Figure 5. The best
solutions are positioned at the depth of 2 km beneath the summit area for both the events. If
both the moment tensor and force components significantly work at the same position, the
source locations for both the moment only source type and the force only source type are also
expected to be determined at or near the same locations as the combination of moment and force source type. For the case of force only source, the optimal sources are located at a depth of about 2 km beneath the summit (diamonds). However, the best source locations for the moment only source type are not determined at the shallow beneath the summit area but a depth of approximately 5 km (triangles). Does this discrepancy in source locations have some suggestive meaning?

Figures 6a and 6b show the minimum residual errors as a function of depth for the three types of assumed source mechanisms for Event 1 and Event 2, respectively. If we focus on the curves corresponding to the combined source mechanism of moment and force (circles), it is noted that the residual error does not converge monotonically to a global minimum at a certain depth but has several local minima. The residual curves have a global minimum at a shallower depth of 2 km and have one small dent (arrows in Figure 6) at a depth of about 5 km. The depth of global minimum at the shallow seems to correspond to that of global minimum of the single-force only solution (diamonds), and the small dent at 5 km seems to correspond to the global minimum for the moment tensor only solution (triangles).

Investigating the spatial relationship of the positions having the global/local minima in more detail, we find that there are good spatial consistencies between the source locations (Figure 5). The locations corresponding to shallower minima (~2 km) for the combined mechanism
are quite close to the best source positions for the force only source. On the other hand, the
locations corresponding to the deeper local minima (~5 km), which are indicated by stars in
Figure 5, are determined near the source locations for the moment only source. This suggests
that the VLP event is produced by two separated sources at which the moment and the force
work individually.

In this context it is recalled that each waveform is composed of the two parts; the
initial impulse and the following oscillation. The particle motions in the case of the initial
impulsive parts point to a shallow depth of approximately 0 to 1 km, while those in the case
of the later oscillatory waves tend to point to the deeper part. These features of the particle
motions are suggestive of the existence of two spatially separated sources.

The best source location and mechanism are determined under the assumption of a
one-source model where both the single-force and the moment tensor components work at the
same point. However, there are several questionable points in the solutions of one-source
model as mentioned above which suggest that this assumption is not necessarily compatible
with the real feature of the seismic source of the VLP events. Particularly considering the
characteristics of the error distribution and the particle motions, a VLP event is possibly
excited by two separated sources; the actual source may consist of a single-force source at
about 2 km and a moment tensor source at about 5 km. The idea of two separated sources is
well worth consideration although the abovementioned features may not necessarily be
decisive reasons that completely reject the possibility of one-source model. In the following
section, a possibility of this two-source model will be examined in detail.

3.2 VLP Source Location and Mechanism of Two Sources

In this section, we explore the possibility of the two-source model that the moment
and the force work at different locations individually. It is, however, difficult to constrain the
source locations and the source time functions of the two sources simultaneously from the
observed waveforms of the VLP events alone because of the trade-off between the locations
and the mechanisms. Thus we make use of the waveform property of the VLP events as a
constraint on the source; we divide the waveform into these two parts, which are treated
individually to represent the single-force and the moment tensor.

For each of the initial and later parts of the VLP waveform, we conduct the same
waveform inversion analyses as described in the previous section. We separate the initial and
later parts of the waveforms by using a cosine taper with a length of 2.5 s. In Figure 3 the
taper window is shown by a gray vertical bar in which the taper amplitude diminishes left to
right from unity to zero for the initial part, and right to left for the later part.

The initial and later parts are individually fitted to a single-force solution or a
moment tensor solution. The estimated residual errors and AIC values for the initial and later
parts are listed in Table 2. For both the initial parts of Events 1 and 2, the AIC value of the single-force is smaller than that of the moment. The residual error is also lower for the single-force in spite of the lower number of free parameters. For the later part, the moment solution gives smaller AIC values for both events compared to the single-force. Therefore the AIC evaluation suggests that the force and moment solutions primarily excite the initial and later parts of the waveforms, respectively.

Figure 7 shows the obtained source locations of the single-force and moment solutions by stars and circles, respectively. Contours surrounding the stars and circles represent the range of the residual errors within 125% of the minimum error. The optimum source location of the single-force is at (34.082°N, 139.526°E, 1.8 km below sea level (b.s.l.)) for Event 1 and (34.083°N, 139.526°E, 1.9 km b.s.l.) for Event 2, while that of the moment is (34.059°N, 139.518°E, 5.5 km b.s.l.) for Event 1 and (34.083°N, 139.528°E, 5.1 km b.s.l.) for Event 2. It is noted that the source positions estimated by the two-source model agree well with those estimated by the one-source model for each of the assumed source mechanisms. The optimal source locations for the initial part are determined at a depth of ~2 km beneath the summit area. These positions coincide with those obtained when we inverted the entire waveform assuming the force only source. The best solutions for the later parts are located at the depth of ~5 km, which is good agreement with the results assuming the moment only
The locations of the force solutions are confined to within a small area at a depth of about 2 km beneath the summit, but the locations of the deeper moment solutions are substantially different between Event 1 and Event 2. The error contour of Event 2 is elongated to the south toward the source location of Event 1. This suggests that the calculated source location for Event 2 may not be reliable and that the true source may be located at the same point as the Event 1. To examine this idea, we apply the inversion analysis to the band-pass filtered waveform of Event 2. The 0.2 Hz component is extracted by applying a band-pass filter with a passing band of 0.1 to 0.3 Hz. We add to Figure 7 the band-passed solution of Event 1 to compare it to the result of Event 2 under the same condition. As expected, the band-passed solution for Event 2 is close to that of Event 1 as shown in Figure 7 (34.058°N, 139.518°E, 5.6 km b.s.l.). The source location estimated using all the frequency components lower than 0.8 Hz for Event 2 is different from the location of its 0.2 Hz component. It is likely that the higher frequency components of the seismic waves, including those at 0.4 Hz, are substantially affected by inhomogeneous velocity and/or density structures and distort the moments in our inversion analysis. The idea that the two events should have the same moment source is consistent with the observation that the spectral peaks of these two events in Figure 3 are quite similar and are interpreted to represent
harmonic overtones of the same resonator.

The source time histories of the single-force components obtained for the initial part show pulse-like signals which mainly work northwardly with a pulse width of 1.5 s. The moment solutions have oscillatory time histories and dominant diagonal components. Also for the solution using the separated waveforms, the source time functions are dissimilar to each other, and there are phase differences among the components with several tens of degrees. The scatter of principal axis orientations and the ratios of eigenvalues are still so large that the source mechanism cannot be explained by an oscillation of a simple-shaped and spatially-fixed source.

In the above analysis of the source locations and the source time functions, the time that separates the waveform into the initial and later parts is selected rather intuitively and the divided waveforms are distorted by the applied cosine taper for the separation. This problem can be avoided if the source time functions of both the shallower single-force source and the deeper moment can be constrained by the entire waveforms without any separation. Unfortunately, this simple method does not work well because of the trade-off between the two sources. In order to avoid the effects of the trade-off, we use a simple constraint that the single-force source works for the time before a threshold time $T_c$ at the shallower point and the moment tensor source works after $T_c$ at a deeper point. The possibilities that the moment
source is effective before $T_c$ and the single-force source is effective after $T_c$ are neglected in this calculation. To get a better estimate of $T_c$, we calculate the source time function and the residual error regarding its fitness to the observed waveforms for variable values of $T_c$. In this analysis, we use only Event 1 and assume the same locations for the shallow and deep sources as determined above. Figure 8 shows the residual errors as a function of $T_c$. The figure shows that the two-source model has a clear minimum, confirming that the single-force acts first at the shallow point and then a moment tensor source acts at the deeper point.

Figure 9a shows the waveform matches between observed waveforms (dotted line) and synthetic waveforms (solid line) for the optimal solution of Event 1. The synthetics fit the observations even at the joint between the initial and later parts, which confirms that the separation of the waveforms has not resulted in any serious effects on our analysis. The left three and right six diagrams in Figure 9b give the source time functions of the single-forces and the moment tensor components, respectively. The single-force solution that accounts for the initial part of the waveform has a dominant NS component compared to the EW and UD components. The force works to the north for about 1.5 s with magnitude of $1.5 \times 10^8$ N. For the moment solution, the diagonal components are dominant, indicating an existence of volumetric variation. Among the diagonal components, the contribution of horizontal components is larger than the vertical component. The amplitudes of the diagonal
components are on the order of $10^{12}$ Nm. Here it is noted that the source time functions do not always oscillate in phase from component to component, and the time series of the eigenvectors show a significant temporal variation in the orientation of the three axes. Since a possibility that the source time histories of the moment tensor components might be in phase and stable at different locations cannot be discarded, we investigate the source time functions calculated at grids adjacent to the optimal source location. At all the grids in proximity to the optimal location, however, the source time functions have significant phase differences of several tens of degrees, and the eigenvectors are not stable.

4. Source Properties

4.1 Amplitudes of the Initial and Later Parts of the VLP Waveforms

According to the results of the previous section, the initial and later parts of the VLP waveforms can be explained by two separate sources. In order to examine the relationship between these two sources, we compare the amplitudes of the initial and later parts of the waveforms as shown in Figure 10. In this figure, we plot the maximum amplitudes of 23 VLP events observed from 30 June to 5 July, 2000 at station KAS, the nearest station to the summit. The amplitudes of the later parts are compared with those of the initial parts for the two frequency components of 0.2 and 0.4 Hz. Band-pass filters with passing bands between
0.1 and 0.3 Hz, and 0.3 and 0.6 Hz, are used for 0.2 and 0.4 Hz components, respectively. For the initial impulsive signals, a low-pass filter with a 0.8 Hz cutoff is used. There is a clear positive correlation between the amplitudes of the initial and later parts, which suggests the presence of some physical cause-and-effect relationship between the two sources. Considering the observation that the force solutions producing the initial parts always precede the moment solutions by a few seconds, it is highly possible that the shallower source of the single-force triggers the following oscillatory deeper moment source.

4.2 Temporal Variation of Peak Frequency and $Q^{-1}$ Value

We next investigate the temporal variation of the waveform properties. Figures 11a and 11b show the temporal variation of peak frequency (solid circles) and $Q^{-1}$ value derived from the half width of the spectral peaks (open circles) for the 0.2 and 0.4 Hz components, respectively. The left and right vertical axes represent scales of peak frequency and $Q^{-1}$ value, respectively. We plot the data from the same 23 VLP events as analyzed in the previous section. The peak frequency of the 0.2 Hz component shows a change with time that is fitted to a regression line (solid line) of 12% decrease from 30 June to 5 July, 2000 (Figure 11a). A similar trend is also found in the data of the 0.4 Hz component with a regression line (solid line) of 30% decrease for the same period. Dotted lines in Figures 11a and 11b represent the
decrease rates determined from the other components, i.e., 0.4 and 0.2 Hz components, respectively. The characteristic frequencies decrease between 12 and 30% over a period of six days. On the other hand, we cannot obtain any clear trend from the data of the $Q^I$ values by applying a linear fitting. This may be because they have much greater scatter.

4.3. Moment Tensor Diagonalization

In general, a moment tensor can be decomposed into three eigenvalues and eigenvectors by a suitable rotation of the coordinate axes. The ratios among three eigenvalues represent the geometry of the source while the eigenvectors specify the orientation of the source [Chouet, 1996]. This kind of eigenvalue decomposition assumes that the time histories of the six moment tensor components are similar to each other and the amplitude ratios among the components are constant irrespective of time. When this assumption holds, the three eigenvalues and the corresponding eigenvectors can be extracted solely from the amplitudes of the six moment tensor components. In early studies, since the time histories of six moment tensor components are approximately similar to each other, the maximum peak/trough amplitudes for the components were used to evaluate the eigenvalues and the corresponding eigenvectors [e.g., Ohminato et al., 1998; Kumagai et al., 2001, 2003; Chouet et al., 2003, 2005]. However, our case is not so simple as the previous studies. The best
source time histories obtained in our analysis do not show synchronous vibrations but contain some phase shift, i.e., different time lapses of vibrations among the six components, so that the conventional treatment cannot be applied to the dissimilar waveforms. To better describe the source properties, we have to take into consideration the differences of not only the amplitudes but also the phases in the time histories. For this purpose, analyses in the frequency domain are more convenient than those in the time domain because we can extract more directly and easily the source information regarding both the amplitudes and the phases. In the rest of this section, we employ a new approach in which complex spectra of source time functions are analyzed in the frequency domain.

Six panels in Figure 12a show the normalized amplitude spectra of the source time functions for the six moment tensor components for Event 1 depicted in Figure 9b. The frequency peaks at 0.2 and 0.4 Hz are easily recognized. Amplitudes of the diagonal components, $M_{xx}$, $M_{yy}$ and $M_{zz}$, are significantly greater than those of the non-diagonal components, $M_{xy}$, $M_{yz}$ and $M_{zx}$, for the 0.2 Hz peaks, while the amplitudes of all the six components are comparable to one another for 0.4 Hz peaks.

In Figure 12b, we plot the complex spectral values of the moment tensor components of the 0.2 and 0.4 Hz peaks, respectively. The horizontal and vertical axes represent the real and imaginary parts of the spectral values, respectively. The complex peak
values of each moment component give both the amplitude and phase of the oscillation. In particular, the ratio of the imaginary part to the real one defines the phase of the oscillation. If the peak values of all moment components are distributed along a single line on the complex plane, all the moment components should oscillate synchronously without any phase shift. Figure 12b shows, however, that the peak values of actual moment components are not well aligned, indicating that there are phase differences among oscillations.

To represent the observed amplitudes and phases of oscillations, we provide a new method of moment tensor diagonalization which is performed in the frequency domain. In this method, a seismic source is assumed to be represented by a superposition of three independently oscillating dipoles which are spatially fixed at the same point and are perpendicular to each other. For the complex moment tensor components at each frequency, the complex eigenvalues, which represent the amplitudes and phases of the three dipole oscillations, can be obtained by a geometrical rotation of the coordinate axes in a least squares approach (See Appendix A). One of candidates for a seismic source that oscillates with phase differences among the principal components is a composite model; two or more seismic sources are combined and oscillate independently. For simplicity, we assume that the three principle axes of such sources are common in the proposed model. One of the simplest source models of such a type is a combination of two cracks that cross each other at right.
angles and vibrate with different phases.

Figures 13a and 13b show the evaluated three eigenvalues on the complex plain (top) and the corresponding real eigenvectors (bottom) for the 0.2 and 0.4 Hz components, respectively. The eigenvalues are referred to as $M1$, $M2$, and $M3$ and are plotted by black colored circles with solid error bars. We find in Figure 13 that the magnitude of $M3$ tends to be significantly smaller than that of $M1$ and $M2$, which are more or less similar. This indicates that most of the oscillation occurs perpendicular to the $M3$ axis and almost symmetrically around the axis. For the 0.2 Hz components, the $M3$ axis is inclined to the direction of N25°E by 29° from the vertical axis. The phase difference between the three components ranges to 50°. For the 0.4 Hz components, the $M3$ axis is more inclined from the vertical and the phase difference is almost 130° although the results may be less reliable.

5. Resolution of the Waveform Inversion Method

The source locations for the later parts of the VLP events are determined at a deep southern region of the island, whose horizontal location is slightly outside the observation network deployed in Miyake-jima and its depth is approximately equivalent to the extent of the network. A question may arise how well the estimated source locations and mechanisms can be estimated from the data and network. To evaluate the resolution of our waveform
inversion, we conduct numerical tests in which we assess how precise the source time
functions and source location can be reconstructed through our waveform inversion for a
known source locating at the same position as the estimated moment source.

5.1 Procedure of Numerical Tests

We first construct theoretical seismic waveforms radiated from a given source,
which are calculated for the seven seismic stations so as to mimic the actual seismic
observations in Miyake-jima (Figure 1). We do not use the waveforms of horizontal
components at stations OFN and CND so as to be in the same calculation condition as the
analysis of Event 1. In this test, we use an oscillation of a crack source, locating at the deep
moment source of Event 1 (Figure 7). Six components of a moment tensor \( M(t) \) can be
expressed as a function constituted of a time history of crack oscillation \( \dot{M}_o(t) \), an orientation
of crack \( \theta \) and \( \delta \) (Figure 14a), and the Lamé coefficients \( \lambda \) and \( \mu \) of the country rock [Chouet,
1996]. To represent the elementary source time function \( \dot{M}_o(t) \), we use an oscillatory wave
with the length of 50 s that consists of a cosine function oscillating with a frequency of 0.2
Hz, and pad zeros to the cosine function from 0 to 10 s and apply a hanning window, whose
function is represented by \( 0.5 - 0.5 \cos(2\pi/T_i) \) with \( T_i = 40 \) s, to the cosine function from 10 to
50 s. The resulting \( \dot{M}_o(t) \) is shown in Figure 15. In this test, we assume a condition of Lamé
coefficients $\lambda = 2\mu$. To imitate actual seismic signals, we add white noises to the theoretical seismic waveforms. We here use normal random number $N(0, \sigma^2)$ as the white noise, where $5\%$ of the maximum amplitude among the theoretical waves is given to the variance $\sigma^2$. Then we apply the waveform inversion to the theoretical waveforms including the noise in the same manner as in chapter 3, and finally compare the inferred source locations and source time functions to the original ones.

5.2 Reconstruction of Source Time Functions for a single crack

In this test, we assume five types of cracks with different orientations; crack1) a vertical dike striking EW ($\theta = 0^\circ$ and $\delta = 90^\circ$), crack2) a vertical dike striking NW-SE ($\theta = 45^\circ$ and $\delta = 90^\circ$), crack3) a vertical dike striking NS ($\theta = 90^\circ$ and $\delta = 90^\circ$), crack4) a vertical dike striking NE-SW ($\theta = 135^\circ$ and $\delta = 90^\circ$), and crack5) a horizontal sill ($\delta = 0^\circ$). The angles of $\theta$ and $\delta$ are measured from north clockwise and from the vertical, respectively. Figure 16 shows the calculated source time functions derived from the inversion assuming the moment tensor mechanism only (solid lines) and the original source time functions (dotted lines). The sum of squared residual (SSR) between the inferred and original source time functions is shown at the right top corner on each figure. The source time functions are reconstructed well except at both the sides of each trace probably because of the relatively
low signal-to-noise ratio. The results show no phase differences among the six components for all the cases, indicating that the phase differences of moment solutions are not produced by insufficient inversion resolution due to the network configuration nor an artificial effect of a low-pass filter applied to the seismic waveforms.

To investigate how well our inversion method can distinguish the types of source mechanisms, we evaluate the AIC values for the results obtained assuming three single-force components only and six moment tensor components only. The AIC values for the force only case are evaluated to be -4702, -2480, -429, -2811, and -11751 for the crack1, crack2, crack3, crack4, and crack5, respectively, while for the moment only case the values are -18421, -19935, -18670, -19497, and -21489, respectively. Additionally, we conduct numerical tests in the same manner for three cases giving source time functions assuming force only source; 1) a force working in EW direction, 2) a force working in NS direction, 3) a force working in UD direction. Also in this test, the source is put at the deep moment source of Event 1 (Figure 7) and the same elementary source time function $M_s(t)$ are used (Figure 15). The inversion results show that the AIC values for the force only case are evaluated to be -17312, -18113, and -20579 for the three types, respectively, while for the moment only case the values are -3572, -8376, and -15313, respectively. These results show that the inversion is able to choose the correct mechanism type with significant difference in AIC values for all the cases.
Thus our inversion has sufficient resolution to distinguish the force and the moment that contribute to the excitation of seismic signals even when the source is positioned near the edge of the observation network.

We carry out grid searches in the same manner as in chapter 3 to confirm how well our inversion can determine the source location. The differences from the true source position $\Delta x, \Delta y, \Delta z$ are shown at the bottom of each figure. For all the cases, the source locations are determined within the discrepancy of 200 m at maximum.

5.3 Reconstruction of Source Time Functions with Phase Differences

We next investigate how precisely the waveform inversion and the proposed moment tensor diagonalization methods can extract the given source properties from incoherent source time functions. To represent the source time histories having phase differences among the components, we assume two cracks that cross each other at right angles and vibrate at the same frequency and amplitude but with different phases (Figure 14b). Angles of $\eta$ and $\zeta$, which are measured from north clockwise and from the vertical, respectively, represent the orientation of symmetric axis, and $\omega$ is the rotation angle around the symmetric axis counterclockwise. Here $\zeta = 0^\circ$ and $\omega = 0^\circ$ describe a pair of vertical cracks striking in NS and EW directions. We use a phase difference of $3\pi/4$ between the oscillations of the two cracks.
We assume three types of configurations of intersecting two cracks; type1) NS and EW striking vertical dikes ($\zeta = 0^\circ$ and $\omega = 0^\circ$), type2) NE-SW and NW-SE striking vertical dikes ($\zeta = 0^\circ$ and $\omega = 45^\circ$), and type3) type1 rotated by $\zeta = 29^\circ$ and $\eta = 25^\circ$ so as to be analogous to the result obtained from the real data by applying the proposed moment diagonalization approach (Figure 13a). Figures 17a, 17b, and 17c show the source time functions for the type1, type2, and type3, respectively, derived from the inversion assuming the moment tensor mechanism only. The black colored solid and dotted lines represent the calculated and the given source time functions, respectively. The results show that our inversion can reproduce the given source time histories despite the existence of phase difference. The AIC values for the moment tensor only source type are evaluated to be -21130, -22071, and -20323 for the type1, type2, and type3, respectively, while for the single-force only source the values are -3167, -2435, and -930, respectively. The inversion has sufficient resolution to distinguish between the two mechanism types for the source location even when the source time histories are different from component to component. The gray colored lines show the source time histories inverted using a Green’s function representing an oscillation of a cylindrical-shaped source that is equivalent to the intersecting two cracks oscillating in phase. We can find that for all the cases the cylinder model cannot explain both the phase and amplitude appropriately for all the cases, even thought its geometrical feature of axial symmetry is
A grid search is carried out to investigate the spatial resolution for the determination of source locations. The differences from the true source position $\Delta x$, $\Delta y$, $\Delta z$ are shown at the bottom of each figure. In spite of the more complex source time histories, the waveform inversion can determine the source positions with the errors of 100 m (one grid) at maximum for each direction for all the cases.

The bottom panels of Figure 17 show the three eigenvalues on the complex plain evaluated by using the moment tensor diagonalization method proposed in section 4.3. Circles and stars represent the inferred and the corresponding theoretical values, respectively. The phase differences among the three principal components are reproduced well with the error of 7° at maximum. The estimated orientations of symmetric axis ($M3$) $\zeta'$ and $\eta'$ are shown at the left bottom corner of each panel. For all the three cases the inclination from the vertical axis $\zeta$ can be replicated with the error of 2-3°.

6. Discussion

The features of the VLP events studied in this paper are summarized as follows.

1. The VLP events are constituted of two parts; the initial part – an impulsive pulse which is dominant in the horizontal components, and the later part – a very long period
oscillation with 0.2 or 0.4 Hz in dominant frequency.

2. The initial part can be excited by a single-force solution working northwardly at a depth of 2 km beneath the summit, while the later part is produced by a moment tensor solution at 5 km depth and 2 km southwest of the summit.

3. The source of the moment tensor solution can be described as the oscillations that take place with almost the same amplitudes but somewhat different phases around a nearly vertical axis.

4. The shallow impulsive source is thought to trigger the deep oscillatory source.

5. Characteristic peak frequencies of oscillatory waveforms tend to decrease by approximately 10 to 30% over a period of six days.

6. **Physical Interpretation of the Force Solution**

   A single-force generally arises from an exchange of momentum between the source region and the Earth [Takei and Kumazawa, 1994]. According to our inversion analysis, the single-force acted northwardly on the earth. A northward force can be a reaction force of either a southward acceleration or a northward deceleration of motion. It is thus suggested that the single-force solution is associated with accelerating or decelerating mass transport in the interior of the volcano. For example, the observed single-force of $1.5 \times 10^8$ N can arise if
a spherical-shaped rock mass with the density of 2700 kg/m³ and the radius of 10 m is accelerated to 20 m/s to the south over a period of 1.5 s. The counter-force corresponding to the termination of the southward movement of mass is expected to be generated at a shallow depth because the force is almost horizontal. However, the source time function of the single-force does not involve a clear signal corresponding to the southward motion that counterbalances the southward acceleration. This may be because the southward movement of the mass decelerated too slowly in the viscous magma to release seismic energy efficiently.

6.2 Physical Model of the Moment Solution

As shown in Figure 12a, the moment tensor solution has multiple spectral peaks with harmonic overtones. This strongly suggests that the source is resonating. Since the oscillation is symmetric with respect to the $M3$ axis, the resonator has an axially symmetrical nature. The simplest geometry of such a resonator is a cylinder, but its size must be as large as 10 km so as to produce low-frequency oscillation of 0.2 Hz [Biot, 1952; Fujita and Ida, 2003]. On the other hand, a low-frequency resonance can be easily obtained from slow waves trapped in a fluid-filled crack [Chouet, 1986; Ferrazzini and Aki, 1987]. For example, when the crack stiffness $C$ that controls the physical condition of resonance is 100 [Figure 17 in Chouet, 1986], a 1.3 km long square crack with an aperture of 6.5 m can generate an
oscillation of 0.2 Hz for a fundamental mode, assuming 2.6 km/s and 6.0 km/s for the sound speeds of the magma and the country rock, respectively, with the common density of 2700 kg/m$^3$ [Morrissey and Chouet, 2001].

Geodetic analyses, in which the crustal deformation associated with the earthquake swarm on 26 June has been investigated using GPS and tilt data, revealed two dikes striking NE-SW and NW-SE, whose locations agree well with the hypocentral traces [Irwan et al., 2003; Ueda et al., 2005]. Additionally, the geodetic analyses also suggest a deep pressure source near the intersection of the two cracks (Figure 1), and the location is in good agreement with the hypocenter of the moment tensor source. From these observations, a combination of two intersecting cracks vibrating independently may meet the symmetric nature of the seismic source with phase differences. We thus examine this intersecting two-crack model in more detail. The gray colored squares with error bars of dotted lines in Figure 13 show the eigenvalues $M_1$, $M_2$, and $M_3$ that are calculated from the two-crack model by a least squares approach (Appendix B). For the calculation, we assume a condition of Lamé coefficients $\lambda = 2\mu$ because this condition would be more appropriate for volcanic rock near magma temperature [Murase and McBirney, 1973]. As shown in the figure, the orthogonal crack model is able to reproduce the observed complex eigenvalues of the moment tensor. The ratios of oscillation amplitudes between the two cracks $a/b$ are evaluated
as 1.18 and 1.09 for the 0.2 and 0.4 Hz components, respectively. These ratios are so close to 1 that the oscillations may effectively be regarded as axially symmetric. In this calculation, the amplitude of the crack oscillation in the maximum principal component is estimated to be $9.4 \times 10^{11}$ Nm in the time domain, which gives the fluid pressure change of $4.9 \times 10^4$ Pa in the cracks.

We have discussed the source properties and the physical meaning of the deep moment source obtained under the assumption of two-source model that a single-force source and a moment tensor source individually work at different locations. The two-source model can explain the problem of why the solutions obtained from the one-source model have complicated error distributions and of why the characteristics of particle motions are different between the initial and the later parts. However, the complexities of the source time functions are not resolved even under the assumption of two-source model without further constraints. In this sense, it may be difficult to completely reject the possibility of one-source model and justify the two-source model solely from the results of the waveform inversion analysis in chapter 3. However, we suppose that the two-source model is more appropriate to the physical interpretation in the magma plumbing system of Miyake-jima. As discussed above, the locations of the deep moment source derived from the two-source model agree well with the locations of dikes and pressure sources inferred from geodetic observations. The
agreement of the locations enables us to interpret the physical meaning of the moment tensor source. On the other hand, no dike or pressure source that is capable of exciting resonant oscillations with periods as long as 5 s has been detected by geodetic observations at the best source locations inferred from the one-source model. The absence of the volumetric source at shallow depths prevents a reasonable interpretation of a physical mechanism for the moment solution. Thus, the two-source model is preferable to the one-source model not only from the complex features of inversion solutions but also from the physical and volcanological viewpoints for Miyake-jima.

6.3 Sources of VLP Events

From the positive correlation of amplitudes between the initial and later parts of VLP waveforms (Figure 10), it has been concluded that shallower source with a single-force triggered the deeper moment source. Assuming that the triggering signal travels 4 km from the shallower source to the deeper one with the sound speed of 2.6 km/s in basaltic magma, it takes about 1.5 s to reach the deeper source. This time delay is in harmony with the observations that the later oscillations started within a few seconds of the occurrence of the initial pulses. This suggests that the triggering signal is carried by an elastic wave in a conduit filled with magma. Following the estimates in the section 6.1, the pressure change associated
with the initial pulse is $5 \times 10^5$ Pa, if a single-force of $1.5 \times 10^8$ N worked on the cross-sectional area of a spherical mass, $3 \times 10^2$ m$^2$. This pressure value is an order of magnitude larger than the vibrational pressure change of $4.9 \times 10^4$ Pa in the cracks. The pressure difference may be interpreted to represent decay in pressure during transmission of the pulse signal in the magma.

It has been shown in Figure 11 that the characteristic frequency of the VLP waveforms decreased by approximately 10 to 30 % over a period of six days. The $Q^{-1}$ data is too scattered to completely justify the idea that the $Q^{-1}$ also decreases with time. However, considering that the decrease lines of 12 % and 30 % (thin solid lines) shown in Figures 11a and 11b do not significantly deviate from the $Q^{-1}$ distribution, we may guess that both the characteristic frequencies and attenuation factors of the source have decreased at the same rate. If this is true, the decreases may be interpreted as follows. The eigen frequencies of a resonance system increase with the sound speed of the resonator. The attenuation factors also change approximately in proportion to the shift of the sound speed because the attenuation is determined by the acoustic impedance between the resonator and the surrounding elastic media [Aki et al., 1977; Aki and Richards, 1980]. The observed changes of frequency and attenuation are thus explained by decreased sound speed of magma in the resonator, which is likely attributed to an increase of bubbles in magma by 0.2-0.8 % in volume fraction for pure
basaltic magma. This trend may reflect that the magmatic activity was gradually enhanced before the start of the summit eruption on 8 July with the magma becoming more bubbly.

### 6.4 Relation to the Magma Plumbing System

We now interpret the mechanisms of the VLP events in relation to the magma plumbing system and the volcanic process of Miyake-jima (Figure 18a) during the magma intrusion stage of the 2000 eruption. According to gravity and magnetic observations, a void space had formed beneath the summit a few days prior to the first summit eruption on 8 July [Sasai et al., 2002; Furuya et al., 2003]. The location of the void is indicated by a solid star in Figure 1 and its depth was estimated to be 1.7 km from the gravity analysis. Taking into consideration that the earthquake swarm on 26 June started at a depth of a few km beneath the summit [Sakai et al., 2001; Uhira et al., 2005], the vacant space is considered to have developed with the voluminous draining of magma from a shallow source beneath the summit. A notable point is that the shallow source of VLP events is quite close to the position of the void space, suggesting that the single-force is likely to have arisen during the magma outflow process from the summit to the south.

Our possible scenario regarding the magma draining stage leading to the caldera formation is as follows (Figure 18b). The magma flowed through two cracks that crossed
almost perpendicular to each other about 2-3 km southwest from the summit. A voluminous
outflow of magma from shallow depths beneath the summit area resulted in a gravitational
instability beneath the summit, leading to collapses of crustal rocks and resultantly forming a
void space. The top portion of the volcanic conduit beneath the newly formed void was filled
with remained and/or ascending magma and blocks of collapsed rocks, and the mixture of
magma and rocks drained southward. Small pieces of rocks were transported without
interruption, but some of large size blocks were caught at a choked part in the magma path,
temporarily slowing down the flow of magma. The block was then suddenly released due to a
pressure increase behind the constriction, and moved southwardly. Accelerated motion of the
block and magma produced a northward horizontal single-force, and the resultant pressure
wave propagated to the deep source through the magma, triggering a resonant oscillation of
the intersecting two cracks.

The developing void spaces due to the draining of magma facilitated collapses in the
volcanic conduit. When the subsurface rock of the summit area was not able to sustain its
own weight any longer, it collapsed down en masse associated with the subsidence of summit.
The surface phenomenon of this collapse was observed at the first summit eruption on 8 July.
After the first eruption on 8 July, the VLP events are hardly identified. This is probably
because the collapse of the void associated with the large summit subsidence substantially
changed the magma plumbing system, and the triggering system that had excited the resonant oscillation was broken.
Appendix A: Calculation of the Amplitudes and Phases of a Complex Moment Tensor

A complex moment tensor matrix \( M(t) \) can be transformed into a complex diagonal matrix \( \Lambda(t) \) by a real orthogonal matrix \( V \), as

\[
M_s(t) = V \Lambda(t) V^T, \quad (A1)
\]

where the complex diagonal elements of \( \Lambda(t) \) represent the amplitudes and phases of three independent oscillations and \( V \) represents a rotation of coordinate axes in a real space for the diagonalization. Here the superscript \( T \) denotes a transpose matrix. By applying a Fourier Transform to the source time histories \( M_s(t) \), we can obtain the moment tensor in the frequency domain \( \tilde{M}_s(\omega) \) as follows.

\[
\tilde{M}_s(\omega) = V \tilde{\Lambda}(\omega) V^T. \quad (A2)
\]

Here \( \tilde{\Lambda}(\omega) \) represents a complex diagonal matrix in the frequency domain, and \( V \) is a real orthogonal matrix that is independent of the frequency.

When we assume that an observed complex moment tensor \( \tilde{M}_o \) can be modeled by \( \tilde{M}_s \), we obtain the following observation equation.
\[ \tilde{M}_o = \tilde{M}_s + \Delta \tilde{M}, \quad (A3) \]

where \( \Delta \tilde{M} \) is a complex matrix representing an observation error. The variable \( \omega \) is omitted hereafter for simplicity. We can extract \( \tilde{M}_s \) from \( \tilde{M}_o \) by minimizing the Euclidean norm of the observation error \( |\Delta \tilde{M}|^2 \) that is defined as a summation of products of a moment tensor element and the corresponding complex conjugate transpose. Using a condition necessary to minimize \( |\Delta \tilde{M}|^2 \), i.e., \( \partial |\Delta \tilde{M}|^2 / \partial \gamma_k = 0 \) \((k = 1, 2, 3)\), we can derive the following relationship. Here \( \gamma_k \) are complex eigenvalues of \( \tilde{\Lambda} \).

\[ \gamma_k = (V^T \tilde{M}_o V)_{kk} \quad (k = 1, 2, 3). \quad (A4) \]

Introducing a squared residual error as \( e^2 \equiv |\Delta \tilde{M}|^2 / |\tilde{M}_o|^2 \), the error \( e^2 \) can be expressed using equation (A4) as follows,

\[ e^2 = 1 - \sum_{kk} \left| (V^T \tilde{M}_o V)_{kk} \right|^2 \bigg/ \text{tr}[\tilde{M}_o^* \tilde{M}_o], \quad (A5) \]

where the superscript * denotes a complex conjugate transpose matrix. For the
determination of \( \tilde{\Lambda} \) and \( V \), the rotation angles of \( V \) are first determined by a numerical search so as to minimize the residual error \( e^2 \), and then the eigenvalues \( \gamma_k \) are determined from the equation (A4). To quantify the uncertainty of the solutions, we regard 

\[ \pm e \cdot \sqrt{\sum_k |\gamma_k|^2} \]

as the error estimate of the predicted principal value in this study.
Appendix B: Intersecting Two-Crack Model

We consider a moment tensor source associated with two cracks that cross each other at right angles and vibrate at the same frequency but with different phases. The net moment tensor can be written in the frequency domain by a linear combination of two moment tensors for the two cracks [Chouet, 1996] as

\[
\begin{align*}
\tilde{\Lambda} &= a \begin{pmatrix} \lambda + 2\mu & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \exp(i\phi_1) + b \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda + 2\mu & 0 \\ 0 & 0 & \lambda \end{pmatrix} \exp(i\phi_2) , \\
&= \Lambda_a + \Lambda_b , \quad (B1)
\end{align*}
\]

where \(a\) and \(b\), and \(\phi_1\) and \(\phi_2\) indicate the amplitudes and phases of oscillations of the two cracks, respectively. Here \(\lambda\) and \(\mu\) are the Lamé coefficients of the country rock. In (B1) the planes of the two cracks are assumed to be parallel to the axis of the third principal component.

The amplitudes and phases of oscillations can be fitted to an observed complex moment tensor by a least squares approach. We here minimize the following sum of squared residual \(E^2\) between the diagonal components of the observation \((M_1, M_2, M_3)\) and those of the model \((M_{c1}, M_{c2}, M_{c3})\).

\[
E^2 = |M_1 - M_{c1}|^2 + |M_2 - M_{c2}|^2 + |M_3 - M_{c3}|^2 , \quad (B2)
\]
where $M3$ component is the smallest value among the three components. Introducing complex coefficients $\alpha = a \cdot \exp(i\phi_1)$ and $\beta = b \cdot \exp(i\phi_2)$, the conditions necessary to minimize the residual $E^2$ are written as follows,

$$\frac{\partial E^2}{\partial \alpha} = 0, \quad \frac{\partial E^2}{\partial \overline{\alpha}} = 0, \quad \frac{\partial E^2}{\partial \beta} = 0, \quad \frac{\partial E^2}{\partial \overline{\beta}} = 0,$$

(B3)

where $\overline{\alpha}$ and $\overline{\beta}$ are the complex conjugates of $\alpha$ and $\beta$, respectively. From these conditions, we obtain the following relations.

$$\begin{align*}
(3\lambda^2 + 4\lambda \mu + 4\mu^2)\alpha + (3\lambda^2 + 4\lambda \mu \beta &= (\lambda + 2\mu)M_1 + \lambda M_2 + \lambda M_3 \\
(3\lambda^2 + 4\lambda \mu + 4\mu^2)\overline{\alpha} + (3\lambda^2 + 4\lambda \mu \overline{\beta} &= (\lambda + 2\mu)\overline{M}_1 + \lambda \overline{M}_2 + \lambda \overline{M}_3 \\
(3\lambda^2 + 4\lambda \mu)\overline{\alpha} + (3\lambda^2 + 4\lambda \mu + 4\mu^2)\beta &= \lambda M_1 + (\lambda + 2\mu)M_2 + \lambda M_3 \\
(3\lambda^2 + 4\lambda \mu)\overline{\beta} + (3\lambda^2 + 4\lambda \mu + 4\mu^2)\overline{\beta} &= \lambda \overline{M} + (\lambda + 2\mu)\overline{M}_2 + \lambda \overline{M}_3
\end{align*}$$

(B4)

Note that only two of the four relations are independent. Consequently, we can derive the following solutions.

$$\begin{align*}
\alpha &= \frac{2\mu(4\mu^2 + 6\lambda \mu + 3\lambda^2)M_1 - 2\lambda \mu(2\mu + 3\lambda)M_2 + 4\lambda \mu^2 M_3}{8\mu^2 (2\mu^2 + 4\lambda \mu + 3\lambda^2)} \\
\beta &= \frac{-2\lambda \mu(2\mu + 3\lambda)M_1 + 2\mu(4\mu^2 + 6\lambda \mu + 3\lambda^2)M_2 + 4\lambda \mu^2 M_3}{8\mu^2 (2\mu^2 + 4\lambda \mu + 3\lambda^2)}.
\end{align*}$$

(B5)
Acknowledgments. We are grateful to Earthquake Observation Center, the Earthquake Research Institute, University of Tokyo for providing hypocenters data of the earthquake swarm. We used GMT (Generic Mapping Tools) provided by Wessel and Smith [1991] for figure construction. We thank two reviewers (Dr. P. Dawson and an anonymous reviewer), the associate editor (Dr. Rapolla) and the editor (Dr. Taylor) for their helpful comments to improve our manuscript.
References


by J. M. Bernardo, M. H. DeGroot, D. V. Lindley, and A. F. M. Smith, pp. 143-166,

University Press, Valencia, Spain.

Aki K., M. Fehler, and S. Das (1977), Source mechanism of volcanic tremor: fluid-driven


Res.*, 2, 259-287.


Freeman, New York.

Biot M. A. (1952), Propagation of elastic waves in a cylindrical bore containing a fluid, *J.


Chouet B. (1986), Dynamics of a fluid-driven crack in three dimensions by the finite


Chouet B. (1996), New methods and future trends in seismological volcano monitoring, in


Chouet B., P. Dawson, T. Ohminato, M. Martini, G. Saccorotti, F. Giudicepietro, G. D. Luca,

G. Milana, and R. Scarpa (2003), Source mechanisms of explosions at Stromboli Volcano,


Figure Captions

Figure 1. Broadband seismic stations (solid circles) used for the present study. The location of Miyake-jima is shown at the top left with the tectonic setting. Gray dots represent the hypocenters of the earthquake swarm occurring between 18:00 on 26 to 12:00 on 27 June, 2000 (JST). The outline of the caldera developed during the 2000 eruptions is shown on the summit area by gray curves. A solid star indicates the location of void space inferred to be formed prior to the first summit eruption on 8 July [Furuya et al., 2003]. Open and solid squares indicate the central locations of deep pressure sources determined by Irwan et al. [2003] and Ueda et al. [2005], respectively, on the basis of the ground deformation associated with the June 26 earthquake swarm.

Figure 2. UD-component of velocity seismogram at station KAS from 12:00 to 24:00 on 30 June, 2000 (JST). Ellipses indicate very long period (VLP) events.

Figure 3. Three component velocity waveforms observed on 1 July (a) and 3 July, 2000 (b) at station KAS. The origin time in the horizontal axis is 21:37:50 for (a) and 18:00:50 for (b) (JST). Vertical gray colored bars represent the time window in which a cosine taper is applied
to divide the waveforms into two parts. The corresponding amplitude spectra are shown below the waveforms. Two spectral peaks around 0.2 and 0.4 Hz are clearly identified.

**Figure 4.** Source time functions obtained for Event 1 (a) and Event 2 (b) by assuming the combination mechanism that both the force and moment work at the same point. The source time functions of six moment tensor \((M_{xx}, M_{yy}, M_{zz}, M_{xy}, M_{yz}, M_{zx})\) and three single-force \((F_x, F_y, F_z)\) components are shown.

**Figure 5.** Source locations of VLP events obtained from three source types; three single-force components only (diamonds), six moment tensor components only (triangles), and the combination mechanism of the force and moment (circles) for Event 1 (solid symbols) and Event 2 (open symbols). Stars indicate the locations corresponding to the deeper local minima for the combination mechanism which are indicated by arrows in Figure 6.

**Figure 6.** Residual errors of waveform inversion as a function of depth for Event 1 (a) and Event 2 (b). The residual errors are shown for the sources of three single-force components (diamonds), six moment tensor components (triangles) and a combination of moment tensor...
and single-forces (circles). The residual errors for the combination mechanism have local
minima at the depth of ~5 km, indicated by arrows.

Figure 7. Source locations of VLP events obtained from the two-point waveform inversion
analyses. The stars and circles represent the hypocenters determined from the initial and later
parts of VLP waveforms, respectively. Contours surrounding the stars and circles represent
the range of the residual errors within 125 % of the minimum error. The later parts are
analyzed for the two cases; one using only 0.2 Hz component of the waveform (the gray
circle and dotted contour for the source location) and the other using all frequency range less
than 0.8 Hz (the black circle and solid contour).

Figure 8. Residual error as a function of the threshold time \( T_c \) before which only a
single-force solution works and after which only a moment tensor works. The shallow and
deep source locations of Event 1 (see Figure 5) that have been determined for the initial and
later parts, respectively, are assumed for the calculation. The single-force source and the
moment tensor source are placed at the shallower and deeper source locations, respectively.

Figure 9. Waveform matches and source time functions for the best fit solution of Event 1
derived from the two-source model. (a) Dotted and solid lines represent the observed and synthetic seismograms, respectively. (b) The source time functions are shown for the single-force \((F_x, F_y, F_z)\) and the moment tensor \((M_{xx}, M_{yy}, M_{zz}, M_{xy}, M_{yz}, M_{zx})\) located at different points (see Figure 7).

**Figure 10.** Relation between the maximum amplitudes of the initial part and the later part for the NS component of the 0.2 Hz (a) and 0.4 Hz (b) signals. Band-pass filters with passing bands between 0.1 and 0.3 Hz (a) and between 0.3 and 0.6 Hz (b) are applied to the original waveforms to obtain the amplitudes of the later parts.

**Figure 11.** Temporal variations of the peak frequencies (solid circles) and the \(Q^{-1}\) values (open circles) for the 0.2 Hz (a) and 0.4 Hz (b) components. The thick solid lines are regression lines determined by fitting to the peak frequency data for each frequency component, while dotted lines which are the decrease rates determined from the data of the other frequency component are shown for comparison. The decrease rates obtained from the line fittings for each peak frequency are plotted for the \(Q^{-1}\) values (thin lines).

**Figure 12.** Spectra of the six moment tensor components for Event 1 (see Figure 9). (a) Total
amplitude spectra of the six moment tensor components. (b) Real and imaginary spectral
amplitudes at the peak frequencies of 0.2 and 0.4 Hz for the six moment tensor components.

Figure 13. Eigenvalues (top) and eigenvectors (bottom) corresponding to the spectral peaks
of 0.2 Hz (a) and 0.4 Hz (b) (see Figure 12b). The eigenvalues are labeled as M1, M2 and M3
in order of the magnitude and indicated by black colored circles with solid error bars. The
eigenvalues and eigenvectors, which specify the physical nature and geometrical orientation
of the moment source of Event 1, are interpreted as oscillations of two fluid-filled cracks
intersecting each other perpendicularly. Gray colored squares with errors plotted by dotted
bars show the eigenvalues predicted by the two-crack model.

Figure 14. Definitions of source coordinates used for numerical tests. (a) Single crack. $\theta$ and
$\delta$ represent the orientation of the crack. (b) Two cracks that cross each other at right angles. $\zeta$
and $\eta$ represent the orientation of symmetric axis, and $\omega$ is the rotation angle around the
symmetric axis.

Figure 15. Elementary source time function $M_{\delta}(t)$ used for the construction of six moment
tensor components that represent a crack oscillation.
Figure 16. Given (dotted lines) and calculated (solid lines) source time functions for (a) crack1; a vertical dike striking EW ($\theta = 0^\circ$ and $\delta = 90^\circ$), (b) crack2; a vertical dike striking N45°W ($\theta = 45^\circ$ and $\delta = 90^\circ$), (c) crack3; a vertical dike striking NS ($\theta = 90^\circ$ and $\delta = 90^\circ$), (d) crack4; a vertical dike striking N45°E ($\theta = 135^\circ$ and $\delta = 90^\circ$), and (e) crack5; a horizontal sill ($\delta = 0^\circ$), respectively. The sum of squared residual (SSR) between the inferred and given source time functions are shown at the right top corner of each panel. The differences between the true source positions and the inferred positions obtained through a grid search ($\Delta x, \Delta y, \Delta z$) are shown at the bottom.

Figure 17. Source time functions (top six) and three eigenvalues on the complex plain (bottom) for three types of intersecting two cracks: (a) type1; NS and EW striking vertical dikes ($\zeta = 0^\circ$ and $\omega = 0^\circ$), (b) type2; N45°E and N45°W striking vertical dikes ($\zeta = 0^\circ$ and $\omega = 45^\circ$), and (c) type3; type1 rotated by $\zeta = 29^\circ$ and $\eta = 25^\circ$, respectively. Solid and dotted lines represent calculated and given source time functions, respectively. The sum of squared residual (SSR) between the inferred and given source time functions are shown at the right top corner. The differences between the true source positions and the inferred positions through a grid search ($\Delta x, \Delta y, \Delta z$) are shown at the bottom. Circles and stars on the bottom
panels represent the three eigenvalues on the complex plain inferred by the moment tensor diagonalization method and the corresponding theoretical values, respectively. These values are normalized. The evaluated orientations of symmetric axis (M3) $\zeta'$ and $\eta'$ are shown at the left bottom.

Figure 18. Interpretation of the VLP seismicity in relation to the 2000 activity of Miyake-jima volcano. (a) Chronology of the volcanic activity. A submarine eruption and summit eruptions occurred at the times shown by gray and black colored reversed triangles, respectively. Four stages of the volcanic activity are after Nakada et al. [2005]. (b) A proposed model for the occurrences of the VLP events in the volcanic circumstances inferred from various geophysical observations. The model is viewed from the southeast.
Table 1. Residual errors and AIC values derived from the waveform inversions using the entire VLP waveform including both the initial and later parts. The residual and AIC values are calculated for three source types; a single-force with three components, a moment tensor with six components, and a combination of a moment tensor and a single-force with nine components.

<table>
<thead>
<tr>
<th>Source type</th>
<th>Residual Error</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Event 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force only</td>
<td>0.163</td>
<td>-6949</td>
</tr>
<tr>
<td>Moment only</td>
<td>0.128</td>
<td>-7243</td>
</tr>
<tr>
<td>Moment and Force</td>
<td>0.037</td>
<td>-11785</td>
</tr>
<tr>
<td><strong>Event 2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force only</td>
<td>0.239</td>
<td>-4847</td>
</tr>
<tr>
<td>Moment only</td>
<td>0.218</td>
<td>-4594</td>
</tr>
<tr>
<td>Moment and Force</td>
<td>0.102</td>
<td>-6860</td>
</tr>
</tbody>
</table>
Table 2. Residual errors and AIC values derived from the waveform inversions. The residual and AIC values are calculated for two source types; a single-force with three components and a moment tensor with six components, using only the initial impulsive part or only the later oscillatory part of the VLP waveforms.

<table>
<thead>
<tr>
<th>Source Type</th>
<th>Residual Error</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial Part</td>
<td>Later Part</td>
</tr>
<tr>
<td>Event 1 Force only</td>
<td>0.202</td>
<td>0.120</td>
</tr>
<tr>
<td>Event 1 Moment only</td>
<td>0.217</td>
<td>0.084</td>
</tr>
<tr>
<td>Event 2 Force only</td>
<td>0.150</td>
<td>0.277</td>
</tr>
<tr>
<td>Event 2 Moment only</td>
<td>0.216</td>
<td>0.215</td>
</tr>
</tbody>
</table>
Figure 2
Figure 4

(a) <Event 1>

Mxx

Fx

Myy

Fy

Mzz

Fz

Mxy

Mxy

Myz

Myz

Mzx

Mzx

2 \times 10^{11} \text{(Nm)}

2 \times 10^8 \text{(N)}

10 \text{s}

(b) <Event 2>

Mxx

Fx

Myy

Fy

Mzz

Fz

Mxy

Mxy

Myz

Myz

Mzx

Mzx

5 \times 10^{10} \text{(Nm)}

5 \times 10^7 \text{(N)}

10 \text{s}
Figure 5

- Force & Moment
- Moment
- Force
- Force & Moment; Local Minimum [~5 km]
Figure 6

(a) Event 1

(b) Event 2

Residual Error

Depth (km)
Figure 7

[Diagram showing earthquake locations and depth solutions]
Figure 8

![Graph showing Residual Error vs. Tc (s)](image)
Figure 10

(a) 0.2 Hz component

(b) 0.4 Hz component
Figure 11

(a) 0.2 Hz component

(b) 0.4 Hz component
Figure 12

(a) Amplitude Spectrum

(b) Imaginary vs. Real components

- Mxx
- Mxy
- Myy
- Myz
- Mzz
- Mzx
Figure 13

(a) 0.2 Hz component

(b) 0.4 Hz component
Figure 14

(a) Single Crack

(b) Intersecting Two Cracks
Figure 15

![Graph of $M_0(t)$ over time (s). The graph shows oscillations labeled with a scale of 1 Nm.]
Given Calculation (Cylinder model)

**Figure 17**

(a) [type1]  SSR = 5.96

(b) [type2]  SSR = 11.45

(c) [type3]  SSR = 4.73

Given Calculation

<Err A>  (Δx, Δy, Δz) = (0, +100, +100) m
<Err B>  (Δx, Δy, Δz) = (0, +100, 0) m

<Err A>  (Δx, Δy, Δz) = (-100, +100, +100) m
<Err B>  (Δx, Δy, Δz) = (-100, 0, +100) m

ζ' = 2°  η' = 86°
ζ' = 2°  η' = 59°
ζ' = 26°  η' = 5°
Figure 18

(a) VLP

Earthquake swarm

Tectonic earthquake beneath the summit

Outside of Miyake-jima

Void formation

Caldera Formation

Stage

Magma Intrusion  Summit Subsidence  Explosion  Degassing

(b)

N

Sea Level

~ 2 km

~ 5 km

To off Northwest

Dike

[Resonance oscillation]