PRODUCTION FUNCTION FOR A DYNAMIC THEORY OF THE FARM FIRM

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1. Introduction

The objective of this paper is to present a new view for farm production, and then, estimate a production function suitable for a dynamic theory of the farm firm.[1)] Theory of the firm, and subsequent production function have been the most fundamental part in Economics, and there may seem to be little things, unless it is something very sophisticated, to be added anew. In fact, however, we have to say that "the theory of the firm" was constructed in the framework of price and distribution theory, and I believe that a new theory of the firm should be provided so that it could predict, explain and instruct decisions and behaviors of a firm itself.[2)]

I have attempted to contribute toward it at the part of growth theory of a farm firm, whose threshold was opened by Penrose.[3)] I am confident that "production function" will be an extremely powerful apparatus for this attempt.

As for production function, however, Johansen points the problem correctly saying, "Econometric research on production function is growing ever more sophisticated as far as functional form and statistical methodology are concerned. Nevertheless the results of this research are not very comforting. The increasing degree of sophistication has, by producing many widely diverging results, served to reveal and expose our ignorance in this field than to produce firmly established knowledge. My feeling is that this may be due to the fact, in spite of the sophistication in other respects, the basic notion of production function has remained almost untouched".[4)]

I have been extremely conscious of the same thing and make an effort in the following chapters to establish a systematic concept for the dynamic aspect of production and to present a new production function with estimated result.

2. Problem in the concept of production function.

Conventional production function in the theory of the firm shows, for a given level of technological knowledge, the maximum output which can be obtained from a given set of inputs. It implys that what are technologically inefficient - in the sense that, to produce the given quantity of output, they use more at least one input and at least as much of the other inputs as some others - are omitted from the definition range of the function.[5)] Schumpeter pointed this thing, saying that the production function is "a planning function in words of blue-prints where every element that is technologically variable can be changed at will".[6] Those ideas are based on the concept of static equilibrium; complete information and perfect adjustment. Therefore we may say that the production function is a static equilibrium production function.

The crux of the difficulty lies in the inability of static equilibrium concepts to explain continuous processes through time. In discussing the production of a firm we take note of the price system, and the theory of the firm provides a beautiful apparatus for analysing the effect of changes in relative prices on production. However, consider the case where an entrepreneur wishes to employ more highly mechanized technique because of a change in relative prices. The fact that the switching is a slow process raises the difficulty. Any adjustment to changing conditions takes a long period of time to work themselves out. While static equilibrium analysis tells us the equilibrium which would be reached in the end if no further changes occurred or if instantaneous adjustment were possible, in fact change is continuous and adjustment is delayed.

Technique A, for the given technological knowledge and prices system in the industry, may be most appropriate and is adapted by an entrepreneur currently. But, before even a fraction of industry is equipped with technique A, either technical knowledge or factor prices change, and technique B becomes more "appropriate" in the sense of equilibrium. Some progress is made again, and then technique C appears. In such circumstances, old, new and newer techniques are in use because before the adjustment to one technique has worked itself out an even newer technique has appeared.

This consideration suggests that firms are on an assortment of varied techniques of production at any moment in time. Intrafirmly it means that a firm does not always go along the "best technique", i.e., sometimes stands apart from the production function. In other words, conventional concept of production function has blind parts; both relatively "inefficient" firms or ill-informed firms at a given point in time and moments when a firm stands aside the "best technique".

The reasons for delayed adjustment are rather well known. One of them is shown by the statement of

Hicks; "--- an entrepreneur by investing in fixed capital equipment gives hostages to fortune. So long as the plant is in existence, the possibility of economizing by changing the method or scale of production is small".[7)] Little advantage of full adjustment may be due to inevitable imperfectness in the fixed equipment market--transaction cost, low salvage value, etc. -- and the Penrose effect (14). Second, institutional or environmental condition not equal to all firms shall diversify the assortment of production technique of farms at a moment, or make instantaneous adjustment to the new "best technique" rather disadvantageous for a firm. Particularly for farms, environmental factors, such as climate, type of soil, topography, irrigation condition, etc., and institutional restriction like financial policy are critical elements in production. Third, less concrete but never less important factor, entrepreneurial factor retards concurrent adjustment and diversifies production techniques currently used. Personal character, financial difference, imperfect diffusion of information, etc., are well known examples for it.

The fact that new techniques of production appear each following the other in succession, and there exist a lot of different technique of production side by side lets a single production function not sufficient to describe the real world of production. Production relation at one technique of production is probably different from that at the other production technique since each technique is developed under different level of technological knowledge and different prices relation, and adopted by farms of different condition, reflecting contemporary status. This fact nullifies the concept of static equilibrium production function for the dynamic theory of the firm, since actual production at any point of time is compatible with the production relation at a specific technique of production, and a study of the dynamic aspect of production in a farm requires a study of how a technique of production is switched from one to another through time. Here I assume that change of production technique transforms the production function in a parametric way while the fundamental form of the production function be remained the same.[8)]

3. A new view for farm production and corresponding production function.

1) Concept of production technique and ex-post

production function.

It is fundamentally assumed in this paper that the farm manager's decision with respect to production may be discriminated into two stages. The first is to choose a specific production technique, and the later is concerned with exploitation of the choosed technique. If we may represent the specific production technique of a farm firm with a variable Z, corresponding "production function" is given;

$$Q = F_Z\ (K,L,A),\ (K,L,A) \in D_Z \quad \ldots. \quad (\ 3\text{-}1\ )$$

where Q output

K capital equipment and materials input

L labor service

A acreage cropped

Following Johansen, I will call the function ex-post production function. This function indicates the input-output relation under the restrictions imposed by the choosed production technique Z. One of the restrictions is that the input combination has to belong to a sub-space D_Z in the (K,L,A) space. In other words, once a particular technique is choosed, substitution possibility between inputs is restricted in a small range. There are three reasons, even if not exhaustive, why the variable Z is necessary in specification of production function.

The first is due to the problem of measurement and aggregation. Measurement of labor and land is rather straightforward. Although each class of labor and land should be regarded as a separate factor of production, we may be allowed to measure them in man-hour and acre respectively and aggregate them with the units[9)], unless we are particularly interested in the differential quality of labor and land. The major problem is the measurement of capital and materials input. As usual, we may think of a farm firm as a price taker in the capital equipment and materials input market, and assume that the heterogeneous goods may be aggregated on the basis of the given market prices. This aggregation, however, lets the variable Z inevitable in a production function. The same amount of capital in our convenient unit can be shaped in different form, and can be composed of different combination of elements, in other words, the "same amount" of capital can give rise to various different output.

Second, the same capital equipment is not associated with a single mode of cultivation and not a single level of output is to correspond to each amount of capital. Date of seeding, planting and dressing is an important factor of cultivation mode, and have great influence on output.

Third is due to what is called "worker effect" (18). According to the skill of operation for each cultivation activity, various different level of output may be acquired from the same set of (K,L,A). For example, square and evenness of plowing and dressing per hour are not same for all operators.

Consequently, production technique is characterized with following three factors; I - set of capital equipment, M - Set of modes of cultivation, S - set of skills for each operation.[10)] Expressing it with a vector,

$$Z = \{I \; M \; S\} \qquad \ldots\ldots \qquad (3\text{-}2)$$

Substance of technique and its definition may be a controversial problem. But here it is taken into account only to give the function (3-1) concrete content and relate it to the dynamic aspect of production of a farm firm. In a mathematical sense, it is nothing but an auxiliary variable to give one-to-one correspondence between output and input set.

It is of importance for one not to confuse the further argument in this paper to distinguish between a technological change and a change in production technique.[11] A technique is a utilized method of production, and a change of technique is an alternation of the vector (3-2). It often occurs in a firm without any technological change. On the while, technological change is an advance in the pool of technical knowledge, may it be defined either in a firm or in an industry but usually in an industry, and expands the potentiality for changes in technique. It is not necessarily connected with a change in production technique in a firm.

2) Ex-ante production function and its concept.

We may define a set U, which includes all possible techniques of production that can be obtained if we take into account all shape of capital equipment and mode of cultivation to be possible with present knowledge.

The word of present knowledge makes a little ambiguity for the set U. We can discriminate four stages of present knowledge which differ in proximity to a firm.[12] At one level is the knowledge of pure scientists dealing with basic principles of pure science; genetics, biochemistry, physics etc., At second level is the knowledge of applied scientists concerned with application of those principles to production; agronomy, engineering etc.. At the third stage, there is level of knowledge of farm managers concerned with practical operation of production; plowing, seeding and harvesting etc.. Unless the transmission of knowledge is perfect to all farm managers, there will be the final stage of knowledge, that of a particular farm manager. Which level is relevant depends on the purpose of the study. The fourth is pertinent to the study aiming at explaining actual decision and production of a farm firm through time. For cross-sectional studies, the third level is appropriate. Technological progress in Economics means expansion of the knowledge at the second level and the level is of importance for macro phenomena.

If we assume that the ex-post production function (3-1) is transformed parametrically by the technique adopted, all functions included in the set U can be enveloped by,

$$Q = F(K,A,L/Z), \qquad Z \in U \qquad \ldots\ldots \quad (3\text{-}3)$$

This function is to summarize the relevant technological knowledge at a certain point of time. Following Johansen, it will be called ex-ante production function. This is relevant to the first stage of decision concerned with production, that is, choice of production technique.

3) A flow of new knowledge and its effect on the production function.

A flow of new technical knowledge will take three kinds of form. The first kind appears embodied in new capital equipments. The second group as new mode of cultivation. The last form is improvement of operation skill. Any of new knowledge can be realized by a suitable mixture of these types. This is true, if we see, e.g., that utilization of plowing machine shall be connected with deep plowing and heavy fertilization cultivation, intensive pest control with pesticide is realized with high speed spraying machine. It is not denied, however, that some of them may appear in separated form: Dressing method, i.e., date, frequency and mixture etc., is improved to be better for his soil condition, reflecting his experiences even in absence of new capital equipment. Operation skill is improved by learning by doing without any new mode of cultivation and any new equipment.

Considering the flow of new knowledge in terms of time unit, not as knowledge unit as before, is useful for understanding the dynamic aspect of production. A major complication for this, and indeed for all production theory, arises out of the difference between production process and production of commodity. Technique of production substancially refers to process, while to understand its economic implications we must think in terms of commodity to which prices, costs can be ascribed. The difficulty, related with it, is that the production process of a farm product is composed of many sub-processes, e.g., plowing, seeding, pest controlling and others, with semi independent capital equipment, mode and skill for each sub-process.

The whole production technique of a farm product, therefore, consists of n sub-techniques and,

$$Z = \left\{ Z_1,\ Z_2,\ Z_3, ---, Z_n \right\} \qquad \ldots.. \quad (3\text{-}4)$$

Each subscript means the number of sub-process and (3-4) denotes the whole production is composed of n sub-processes. Reflecting the idea of (3-2),

$$Z = \begin{bmatrix} I_1, & I_2, - - -, I_n \\ M_1, & M_2, - - -, M_n \\ S_1, & S_2, - - -, S_n \end{bmatrix} \qquad \ldots.. \quad (3\text{-}5)$$

The overall technique of production of a farm product changes through peacemeal changes in these component sub-processes, and the component technique for each sub-process through peacemeal change in its three elements. Through the above consideration, we can postulate that new knowledge flown into during a period almost always consist of the three kinds of form, since even if a certain separated form of new knowledge may be realized in one sub-process, other form of new knowledge is likely realized in other sub-processes.

The effect of new knowledge on the ex-ante function is straightforward. A flow of new technical knowledge provides a new technique of production, and the range of the set U is expanded. Such effect is all and exhaustive. Therefore, the specification (3-3) is immediately appropriate and almost sufficient as far as the parametric role of the Z is sustained in the new Z. Ex-ante production function at time t is obtained by dating the set U,

$$Q = F(K, A, L/Z), \qquad Z \in U_t \quad \ldots\ldots \quad (3\text{-}6)$$

4) Technique of production and equipment-labor ratio.

Capital equipment should be regarded as something more than the physical embodiment of free capital. Its physical form also implicates a certain amount of free capital and embodies a certain method of operation. For example, the free capital, measured by initial cost, of five hundred shovels and one cultivator may be identical, but plowing with five hundred shovels needs five hundred labors while one cultivator needs only one or a few operators. This characteristics of capital equipment implies that as new knowledge embodied in new capital equipment is flown into, many cannot be effectively used without a new capital-labor proportion.

Same conclusion for new capital equipment can be attained by noting that invention of new equipment and entrepreneurial choice for it are basically economic phenomena and economic decision. No equipment producer goes to the trouble and expense of developing and manufacturing equipments which do not meet the needs of their market, that is, equipments which do not realize less cost production technique under the given or expected factor prices system. Managerial choice between alternative equipments is also economic based decision in terms of cost and profit. Therefore, each equipment should reflect respective factor prices system at the time when it was produced and purchased, in other words, naturally embodies respective specific capital equipment-labor proportion.

Completely same reasoning is true for mode of operation. Even if the operation is done with same equipment, new mode of operation is developed and improved so as to save dearer factor. And the mode of operation is reflected partly on the capital-labor ratio.

The above discussions suggest us that a production technique Z may be partly represented by the capital-labor ratio. And the ex-ante production function (3-5) is rewritten,

$$Q = F(K, A, L/\frac{M}{L}) \quad ..... \quad (3\text{-}6)$$

where, M is capital equipment service per unit output and L is labor service per unit output.

5) Technique of production and time.

In the previous section it was asserted that technique of production can be represented, even if not exhaustive, by equipment-labor ratio. However, it is clear that there may be purely output augmentation change of technique which does not give any effect on the equipment-labor ratio.

Improvement of operation skill is likely almost pure output augmentation. As in the case where the speed of running an equipment increased, it may sometimes look to save a little more labor than equipment. But, it will probably save some maintenance and some extra loss in the end and labor saving effect is not so plausible. Some of new modes of cultivation give little effect on the equipment-labor ratio, and their major effect is output augmentation, e.g., some new materials input and change of date of some operations etc..

Skill of operation is improved by the process of learning by doing and its source is personal experiences of daily operation. As a result, change of production technique due to improvement of operation skill can be sufficiently represented by time. Pure output augmentating new modes of cultivation are oftenly developed through a kind of adjustment process, reflecting personal experiences, and thus, they are also possibly represented by time. Ex-ante production function of (3-6) is improved to,

$$Q = F(K,\ L,\ A/\tfrac{M}{L},\ t) \qquad \ldots\ldots \quad (3\text{-}7).$$

6) Specification of the production function.

My next task is to construct a concrete form of production function, incorporating the ideas discussed hitherto. I start from the ex-post function. I assume here that expostal production relation is described with a small segment of Cobb-Douglas type function.[13]

$$Q = WL^{\alpha} A^{\beta} K^{\gamma}\ ,\ \ (K,\ L,\ A) \in D_Z \qquad \ldots\ldots \quad (3\text{-}8)$$

Incorporating the ideas of section 4 of this chapter to proceed to the ex-ante function, we get,

$$\begin{aligned} \alpha &= \alpha(m) \\ \beta &= \beta(m) \qquad \ldots\ldots \quad (3\text{-}9) \\ \gamma &= \gamma(m) \end{aligned}$$

where $m = \frac{M}{L}$

The selection of functional type is a decision to be made in each case, since there is no strong theoretical preferrence for a particular type. Even if it will be a trial and error decision, there is an efficient guide line.

Former stage of mechanization is characterized by substitution of machinery power for human power, and advances which make this possible must have an inherent characteristic of labor-saving. And once mechanization is progressed to an extent, new machinery progress is made to improve the operation efficiency of the existing production method (15). We can thus postulate that production elasticity of (3-8) shall be diminishingly (de)increasing monotonic function, provided that the farm firms are under the former stage of mechanization.

Setting them as linear logarithmic functions,

$\alpha = a_o + a\text{In}m$

$\beta = b_o + a\text{In}m$ ..... (3-10)

$\gamma = c_o + a\text{In}m$

We can easily find that ex-postal return to scale ε,

$\mathcal{E} = e_o + e\text{In}m$

where $e_o = a_o + b_o + c_o$ and $e = a + b + c$

Straightforwardly, if e = o, the ex-ante function is homogeneous and return to scale is "constant" without regard to production technique adopted.

I mentioned in section 5 of this chapter the possibility of pure output augmentating change of production technique. And, I set the intercept parameter W as a function of time,

$W = W(t)$ ..... (3-11)

$W(t)$ is possibly either $W_o e^{\lambda t}$ or $W_o t^{\lambda}$.

Consequently, the final ex-ante function will be,[14)]

$Q = W(t) L^{\alpha(m)} A^{\beta(m)} K^{\gamma(m)}$ ..... (3-12)

4. Classical problems of statistical estimation.

I intend to estimate the ex-ante function developed in the previous chapter with the LS estimation method. Writing the ex-ante function (3-12) in a logarithmic form including a disturbance U,

$\text{In } Q_{it} = \text{In}W_{oi} + t + a_o\text{In}_{it} + a\text{In}m_{it}\text{In}L_{it} +$

$$b_o \text{In}A_{it} + b_o \text{In}m_{it}\text{In}A_{it} + c_o \text{In}K_{it} +$$
$$c\text{In}m_{it}\text{In}K_{it} + U_{it} \quad \ldots\ldots \quad (4\text{-}1)$$

In advance of further progress, some classical problems of estimating the parameters in the production function (4-1) are investigated.[15)] Without trying to make an exhaustive survey for them, I will examine them briefly to the extent that is deserved to be the preliminary step for my estimation.

1) Small variation of observations.

If we take the static equilibrium concept we can not expect enough independent variation in observations. Firms do not select input combination randomly but rather choose a production technique and subsequent input combination according to a set of decision rules. For example, provided all firms use the same decision rule of profit maximization, they tend to produce at a small area on the production function unless there are sufficient inter-firm variation in price system. This problem is of course well known, and it may be partly resolved by simultaneous equation estimation, where the production function parameters are estimated along-side parameters of induced demand equations for input factors. On the concepts which are adopted in this paper, such supplementary equations should be too complicated form to be used in practice. On the contrary, on the account of the concepts, i.e., diversification of production technique at a moment, we naturally deserve to hope a wide variation in observations.

2) Simultaneous equation bias.

Even if there will be sufficient spread in observations, here we encounter the simultaneous bias in applying single equation system method, and the estimates are both biased and inconsistent. Once a production technique is choosed, levels of inputs are to be dependently determined, related with output level, by a decision rule as mentioned in the previous section. On the while, output is a function of inputs and the independent variables,input factors, are to be functionally related to the disturbance term Uit in (4-1). One of the prerequisite conditions for the LS method to yield unbiased estimates is violated. This bias persists even for infinitely large samples, that is, the estimates are inconsistent.[16)] It may be also resolved by simultaneous equation system method. It was already mentioned in the previous section that such method is not appreciated in my case.

On the while, we can be confident that such bias will not be encountered in my case, noting that an entrepreneur's decision for input level is related to the anticipated output rather than the actual output (3, 11). This is the case if input levels are decided before the realized output is known, as in agriculture where the disturbances of production can not be forcasted in advance and input decisions are not affected by the disturbance in production.[17)]

3) Specification bias.

Three sources of bias related to my estimation, even if not exhaustive, will be under what is called specification bias; measurement error, aggregation error and exclusion of relevant variables. They will be investigated one by one in the following.

a) Measurement error bias: Intermediate goods such as manure, self-produced in the farm and then used for the final product, are very susceptible to measurement error, which causes both bias and inconsistency.[18)] Further investigation shows the bias is negative for the parameter of erroneously measured factor, and we can expect the parameter for material goods might be a little underestimated.

b) Aggregation error bias: A lot of input used for production are usually categolized into a few group and then aggregated group by group. For example, family labors and hired labors, skilled labors and unskilled labors, men labors and women labors, are aggregated to be the input of labor, mineral fertilizer and organic fertilizer, pesticide and fertilizer, are aggregated to be another input of material input.

Unless the Leontief's separability theorem, say if the marginal rate of substitution between two inputs is independent of other inputs, the two inputs may be combined into an intermediate good which is then combined with other inputs to form the final output, holds true, aggregation for sub-group of input may completely disturb my estimation whatever method of aggregation may be used.[19)] If the Leontief's theorem holds true, it is a very controversial matter what type of aggregation function should be used.[20)] Aggregation method used in this paper is as usual,

$L = \sum L_i$ for labor input

$K = \sum P_i K_i$ for materials and equipment input

where P_i is the price of the material.

This method holds efficient if and only if i) elasticity of substitution within group is either indefinite or zero, and ii) MRTS within group in physical unit is same with their price ratio for material inputs, while unity for labor.[21] Otherwise, L and K contain aggregation error, and the parameters for K and L tend to be estimated biased downward.

c) Exclusion of relevant variables: If we exclude a relevant variable which is correlated with other included variables, the estimates will be both biased and inconsistent. What is most likely excluded are the managerial and the environmental factors. One may suppose that the managerial and environmental factor can be specified as a separated input factor. If one add another assumption that they are constant in a farm for a period, their effect on production will be completely absorbed in the intercept term W_o in my estimation. The remained problem in statistical estimation will be to take into account that the term is different from one farm to another.[22] It should be noted, however, that a part of managerial role is taken into account by including the variable m and t in my specification. It is reflecting my understanding that managerial role in a farm is realized at actual production either through his decisions regarding to choice of production technique or through his skill of operation at his farm field, and that managerial factor should be incorporated into a production function in the form of parameter of technique.[8]

Letting the intercept term W_o as well as the parameter be different from farm to farm is to note the fact that the effect of managerial role and adhesive environmental condition is specific to each farm.

5. Empirical results.

1) Data and variables.

The argument presented in the preceding chapters is applied in an empirical work of 27 Japanese rice farms which were surveyed for more than four years. The data were collected for the period 1952-1965 from Farm Management Record compiled by Ministry of Agriculture.[24] It gave me 183 cases.

Variables included are;

Q; output in kg.

L; labor input in hours.

A; acreage cropped to rice in "tanbo".

K; capital equipment service and materials input like fertilizer and pesticide etc. in "yen".

Land is measured in tanbo planted to rice. There may be difference in their fertility. This problem is pertinently resolved by letting the intercept term be specific to each farm. Furthermore, a certain soil condition may retard adoption of some technique in a case. Such a thing does not make a serious trouble in my case since I included technique variable m and t in my specification.

Labor service is measured in hours spent for direct cultivation activities. Indirect labor used for producing intermediate goods like self-produced fertilizer is not accounted as labor service but included in estimating the value of the intermediate good, which then composes materials input term.

Capital equipment service is estimated with the depreciation value of its stock and the maintenance expenditure. Materials input consists of expenses for fertilizer, pesticide, fuel and electricity, and estimated value of self-supplied intermediate goods.[25)26)] The variable K is the arithmetic summation of the two.[27)]

Output is measured in physical unit, kg harvested. The effect of weather fluctuation was excluded by using weather index, which was estimated for each district.[28]

2) Preliminary investigation.

A preliminary investigation for my data and the period is made in advance of statistical estimation. The data period is very remarkable period. Under the rapid change of economic structure, production technique in a farm is supposed to be impelled to be changed as much, and the period should provide a good data to test my argument.

Each farm in my data is plotted on the diagram of labor and land productivity. It is Fig. 1. The left one is for 1952 and the right for 1965. One see that the dots did not distribute in a narrow space in either year. And also one see that the scattering was widened remarkably in 1965. This observation will be a suggestion for ever widening variation in production technique. Fig. 2 sets out the variation of two productivities of a sample farm over the period. It supports my anticipation for wide variation of production technique over time.

Fig. 3 presents the trend of labor input per unit land and the portion of machinery power for the period 1952-1974. We can read in this diagram that the period is divided into two phases.[28] The former

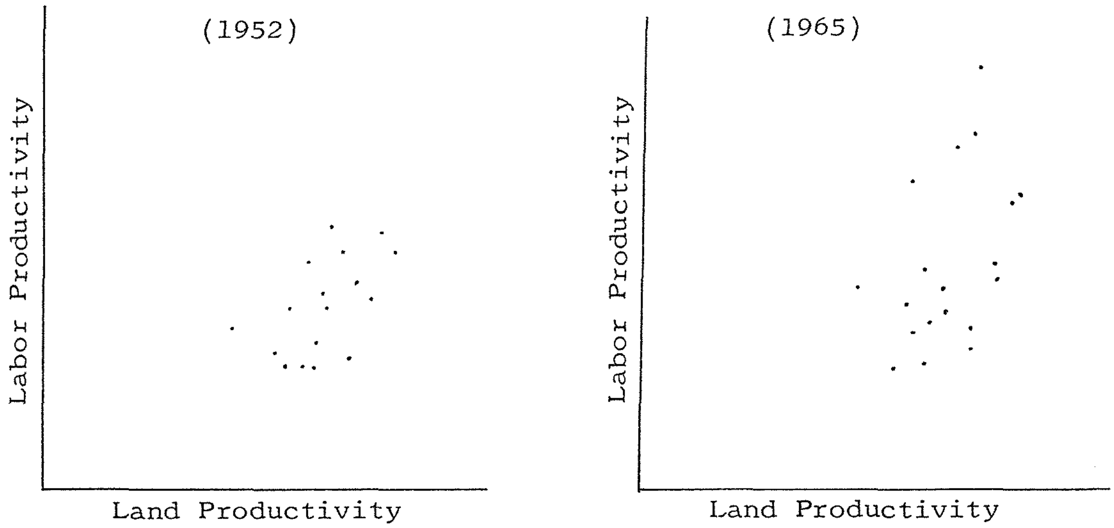

Fig. 1. Distribution of labor and land productivity at a moment in time.

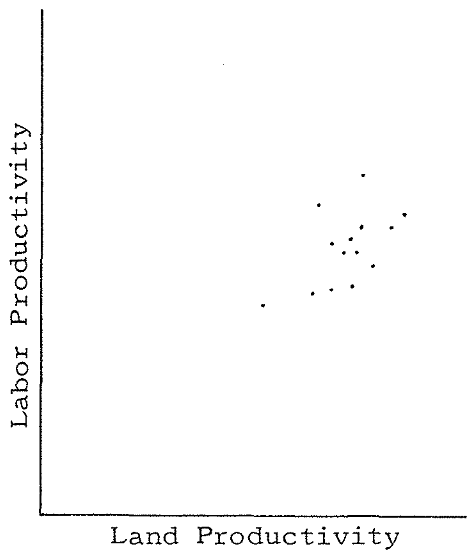

Fig. 2. Transformation of labor and land productivity of a farm over time.

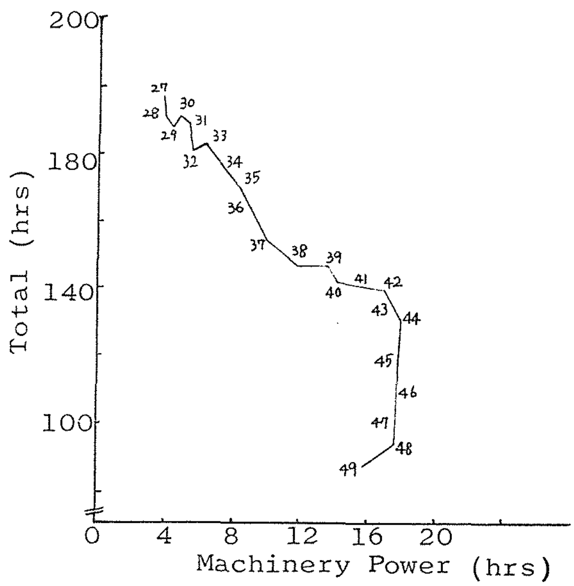

Fig. 3. Trend of labor input per unit land and the portion of machinery power.

phase is the period (1952-55) - (1962-64): During the period cultivator and spraying machine were diffused very extensively and a lot of human power were saved. We can observe that labor input per unit land decreased accompanying a very high speed expansion of machinery power utilization. Whereas during the second period (1962-64) - (1974) high efficiency machinery input such as tracter and combine substituted for old and inefficient machine. As a result, save of labor did not accompany increase of machinery power utilization hour. We can postulate that my data period is up to the typical former stage of mechanization which is characterized by substitution of machinery power for human power as discussed already in chapter 3. The second period may be a little advanced stage of mechanization. This fact reinforces our specification of (3-9) as diminishingly (de)increasing monotonic functions, linear logarithmic one. And we can firmly expect that very labor saving effect will be estimated during the period.

3) Statistical Models

My basic model is,

$$InQ_{it} = W_o + InL_{it} + InA_{it} + InK_{it} + u_{it} \quad \cdots (5\text{-}1)$$

where, subscript i denotes i th farm, and t denotes t th year's production of i th farm. Letting the intercept term W_o be different from farm to farm,

$$InQ_{it} = W_{oi} + InL_{it} + InA_{it} + InK_{it} + u_{it} \quad \cdots \quad (5\text{-}2)$$

Introducing the technique variable m,

$$InQ_{it} = W_{oi} + a_o InL_{it} + aInm_{it} InL_{it} + b_o InA_{it} + bInm_{it} InA_{it} + c_o InK_{it} + cInm_{it} InK_{it} + u_{it} \quad \cdots\cdots\cdot \quad (5\text{-}3)$$

Adding t as a secondary technique variable, and letting its parameter be specific each farm,

$$InQ_{it} = W_{oi} + \lambda_i t + a_o InL_{it} + aInm_{it} InL_{it} + b_o InA_{it} + bInm_{it} InA_{it} + c_o InK_{it} + cInm_{it} InK_{it} + u_{it} \ldots (5\text{-}4)$$

It is convenient for further development to write the models compactly in matrix notation. Model (5-1) will be written

$$Y = X\beta + U \quad \cdots\cdots \quad (5\text{-}5)$$

Here Y is an (nxl) column vector made up f sub vectors, X is a (nxk) matrix of independent variables and β a (kxl) parameter vector. U is (nxl) vector disturbance. OLS estimates for β is given by,

$$\beta = (X'X)^{-1} X'Y \quad \cdots\cdots \quad (5\text{-}6)$$

For model (5-2 and (5-3)

$$\begin{aligned} Y &= D_1 \delta + X\beta + U \\ &= \begin{bmatrix} D_1 & X \end{bmatrix} \begin{bmatrix} \delta \\ \beta \end{bmatrix} + U \quad \cdots\cdots \quad (5\text{-}7) \end{aligned}$$

where D is a $(nx(f-1))$ matrix of dummy variables.

$$D = \begin{pmatrix} \left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} P_1 \text{ elements} & \left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} & \cdots\cdots & \left\{\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right. \\ \left.\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}\right\} P_2 \text{ elements} & \left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} & \cdots\cdots & \left\{\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right. \\ \left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} P_3 \text{ elements} & \left.\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}\right\} & \ddots & \left\{\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right. \\ \vdots & & & \vdots \\ \left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} P_f \text{ elements} & \left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} & \cdots\cdots & \left\{\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}\right. \end{pmatrix} \quad \cdots\cdots \quad (5\text{-}8)$$

Each column of D contains 27 sub-vectors, each of P_i elements. Applying LS method, estimates is given,

$$\begin{pmatrix} \delta \\ \beta \end{pmatrix} = \begin{pmatrix} D'D & D'X \\ X'D & X'X \end{pmatrix}^{-1} \begin{pmatrix} D'Y \\ X'Y \end{pmatrix} \quad \cdots\cdots \quad (5\text{-}9)$$

Model (5-4) is rewritten,

$$\begin{aligned} Y &= D_1\delta + D_2\lambda + X\beta + U \\ &= \begin{bmatrix} D_1 & D_2 & X \end{bmatrix} \begin{pmatrix} \delta \\ \lambda \\ \beta \end{pmatrix} + U \quad \cdots\cdots \quad (5\text{-}10) \end{aligned}$$

where D_2 is an (nxi) matrix,

$$D_2 = \begin{pmatrix} \left.\begin{matrix} 1 \\ 2 \\ \vdots \\ P_1 \end{matrix}\right\} & \left.\begin{matrix} 0 \\ \vdots \\ \vdots \\ 0 \end{matrix}\right\} & \left\{\begin{matrix} 0 \\ \vdots \\ \vdots \\ 0 \end{matrix}\right. \\ \left.\begin{matrix} 0 \\ \vdots \\ \vdots \\ 0 \end{matrix}\right\} & \left.\begin{matrix} 1 \\ 2 \\ \vdots \\ P_2 \end{matrix}\right\} & \left\{\begin{matrix} 0 \\ \vdots \\ \vdots \\ 0 \end{matrix}\right. \end{pmatrix} \quad \cdots\cdots \quad (5\text{-}11)$$

$$
\begin{pmatrix}
\left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} & \left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} & \left\{\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right. \\
\left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} & \left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} & \left\{\begin{matrix} 1 \\ 2 \\ \vdots \\ P_f \end{matrix}\right.
\end{pmatrix}
$$

LS estimater is obtained straightly,

$$
\begin{pmatrix} \delta \\ \lambda \\ \beta \end{pmatrix} = \begin{pmatrix} D_1'D_1 & D_1'D_2 & D_1'X \\ D_2'D_1 & D_2'D_2 & D_2'X \\ X'D & X'D & X'X \end{pmatrix}^{-1} \begin{pmatrix} D_1'Y \\ D_2'Y \\ X'Y \end{pmatrix} \cdots\cdots \quad (5\text{-}12)
$$

4) Estimated results.

The estimated results appear in Table 1. At first, I assumed all three production elasticities are affected by change of production technique in model (5-3)B. But the estimate of c had no statistical sense and the model (5-3) A was tried setting c zero. As for model (5-4), I assumed at first that pure output augmentating effect of technical change existed in all 27 farms to make the matrix D_2 contain 27 sub-vectors. But the estimates for 14 farms were not different from zero at 10% level, and the matrix D_2 shrinked to have only 13 sub-vectors. The result is presented under (5-4)A. Alternatively, setting s which were not significantly non-zero at 5% level, I had D_2 made up 7 sub-vectors to obtain the result

Table 1. Estimates of the Parameters of the Ex-ante Function.

Parameters	Model(5-1)	Model(5-2)	Model(5-3)		Model(5-4)	
			A	B	A	B
α	-0.10312 (0.04574)	0.13164 (0.07187)				
β	0.84963 (0.05650)	0.57985 (0.17740)				
γ	0.24273 (0.04189)	0.25076 (0.04594)				
$\varepsilon=\alpha+\beta+\gamma$	0.98923 (0.02720)	0.96225 (0.16211)				
a_o			0.39815 (0.16292	0.27719 (0.09839)	0.18910 (0.09679)	0.25816 (0.09000)
b_o			0.28771 (0.23334)	0.30637 (0.23238)	0.57912 (0.24126)	0.27068 (0.20614)
c_o			0.17481 (0.08281)	0.23874 (0.04634)	0 20931 (0.04507)	0.18833 (0.04138)
a			-0.08663 (0.05894)	-0.03699 (0.02520)	-0.03884 (0.02360)	-0 05925 (0.02307)
b			0.11617 (0.06673)	0.10823 (0.06616)	0.10231 (0.06061)	0.14825 (0.05980)
c			0.03184 (0.03418)			
e=a+b+c			0.06138 (0.03898)	0.07124 (0.04123)	0.06347 (0.03728)	0.08900 (0.05235)
Adjusted R^2	0.887	0.924	0.925	0.925	0.945	0.942

Notes: a. Standard errors are in parentheses.
b. D_1 of (5-4)A has 13 sub-vectors and D_2 of (5-4)B has 7 sub-vectors.

under (5-4)B.

From these results following observations emerge: First, all estimates are statistically significant with high probability. Specially, the coefficient of lnm lnL is significantly negative while that of lnm lnA is significantly positive. These results support my assertion that production function is transformed parametrically by change of production technique represented with equipment-labor ratio. Saying in terms of Economics, technical change to increase equipment-labor ratio was labor-saving and land-using. One may add another observation that technical change as above was capital-using compared with labor but capital-saving compared with land.

Second, to see statistical significance of ($e = a + b + c$), in other words, to test whether the production technique of high equipment-labor ratio augmented the return to scale or not, the standard error of e was calcurated with the dispersion matrix of (a, b, c). One see that e is positive at statistical significance level of 5%. I infer from it that return to scale increase as the production technique is changed to increase the equipment-labor ratio during the period.

Third, the form of production function are diversified very much among farm at a moment in time, and at the same time, the function is transformed, sometimes significantly, over time in a farm. One can observe this fact in Table 2. Production elasticity for labor becomes too small to be understood as plausible. It may reflect that my specification of the elasticity function as a monotonic one was not enough for very high equipment-labor ratio technique.

As a next step, I have tested the validity of alternative specifications over traditional one with F-test shown Table 3. One can observe that my specification has been improved significantly from model (5-1) to model (5-4) with high statistical fitness. Following two facts are observed in this test. First, from the fact that model (5-4)s which have t as a variable of technical change, are significantly improved than models(5-3) which do not have it, one can infer that the variable t is a good secondary variable to represent technique change.

Second, it is noted, by comparing mode (5-2) with (5-1) and (5-2) with (5-3), that the initial level of technical efficiency of farm manager as well as adhesive environmental factors are diversified widely.

Table 2. *Diversification of production function at a moment in time (A), and Transformation of production function of a farm over time (B).*

Parameters	1952	1954	1959	1962	1965
Labor (α)	0.18291	0.15727	0.14692	0.14244	0.08129
Land (β)	0.45896	0.52238	0.54902	0.56024	0.71324
Capital (γ)	0.18833	0.18833	0.18833	0.18833	0.18833
Return to Scale (ε)	0.83020	0.86828	0.88427	0.89101	0 98286

(A)

Parameters	Farm a	Farm b	Farm c	Farm d	Farm e
Labor (α)	0.10439	0.07453	0.11346	0.16238	0.10224
Land (β)	0.65543	0.73013	0.63275	0.51034	0.66080
Capital (γ)	0.18833	0.18833	0.18833	0.18833	0.18833
Return to Scale (ε)	0.94815	0.99299	0.93454	0.86105	0.95137

(B)

Table 3. F-ratios for test of significance of the alternative specification of production function.

F-ratios	Model No.	Model(5-1)	Model(5-2)	Model(5-3)B	Model(5-3)A	Model(5-4)B	Model(5-4)A
	Residual	3.898	2.249	2.177	2.164	1.616	1.470
Model No.	df	179	153	151	150	144	138
Model (5-1)			4.315**	4.263**	4.145**	5.810**	5.560**
(5-2)				2.250**	1.964*	6.267**	4.875**
(5-3)B					0.901	7.141**	5.106**
(5-3)A						8.139**	5.430**
(5-4)B							2.284**

Note: F-ratios marked with ** denotes less than 1% significance level and those marked with * denote less than 5% significance level.

References

1. Christensen, L. R., D W. Jorgenson and L. J. Lau: "Transendental Logarithmic Production Function", The Review of Economics and Statistics, Vol. 50, p.p. 28-45, 1973.

2. Griliches, Zvi: "Specification Bias in Estimation of Production Functions", J. Farm Economics, Vol. 39, pp. 8-20, 1957.

3. Hoch, Irving: "Simultaneous Equation Bias in the Context of the Cobb-Douglas Production Function", Econometrica, Vol. 26, pp. 566-577, 1958.

4. ____________: "Estimation of Production Function Parameters Combining Time Series and Cross Section Data", Econometrica, Vol. 30, pp. 34-52, 1962.

5. Johansen, L.: Production Function, North-Holland, 1972.

6. Johnston, J.: Econometric Methods, 2nd ed., MacGraw-Hill, 1972.

7. Lee, J. H.: "An Essay on the Theory of the Firm", Nogyo Keiei Kenkyu, Vol. 2, pp. 133-152, Hokkaido Univ., Sapporo, 1975.

8. ___________: "A Model of the Growth of the Managerial Farm Firm", The Review of Agricultural Economics, Vol. 32, Hokkaido Univ., Sapporo, 1976 (forthcoming).

9. Le T. Nghiep, S. Sakiura: "Aggregation of Self-Supplied and Commercial Fertilizer Inputs; Some Implications on Japanese Agricultural Stagnation of the 1930's", J. Faculty of Agriculture, Vol. 58, pp. 14-26, Hokkaido Univ., 1975.

10. Mansfield, E.: The Economics of Technological Change, W. W. Norton, 1968.

11. Massel, B. F.: "Elimination of Management Bias from Production Functions Fitted to Cross Section Data: A Model and an Application to African

Agriculture", Econometrica, Vol. 35, pp. 495-508, 1967.

12. Matsuura, A.: "The Trend of Mechanization and Work Simplication, and Productivity in Rice Farming", (author's translation), Agriculture and Horticulture, Vol. 50, pp. 1326-1330, Yokendo, Tokyo, 1975.

13. Mundrak, Yair: "Empirical Production Function Free of Management Bias", J. Farm Economics, Vol. 43, pp. 44-57, 1961.

14. Penrose, Edith: The Theory of the Growth of the Firm, Wiely, 1959.

15. Salter, W. E. G.: Productivity and Technical Change, 2nd ed., Cambridge Univ., 1969.

16. Ulveling, E. F., L. B. Fletcher: "A Cobb-Douglas Production Function with Variable Return to Scale", Ame. J. Agricultural Economics, Vol. 52, pp. 322-326, 1970.

17. Walters, A. A.: "Production and Cost Functions: An Economic Survey", Econometrica, Vol. 31, pp. 1-66, 1963.

18. Welch, F.: "Education in Production", J. Political Economy, Vol. 78, pp. 35-59, 1970.

19. Ministry of Agriculture and Forestry: Nogyo Keiei Chosa Hokokusho (Farm Management Record), 1952-1965.

20. ____________________: Noson Bukka Chingin Tokei (Statistics of Rural Wages and Prices), 1952-1965.

21. ____________________: Suito Sakkyo Hyohonhitsu Seisanryoku Chitaibetsu Ruinen Chosa Seiseki (A Survey for Yield in a Standard Paddy Land in Each District of Same Productivity), 1966.

(Notes)

1) Closely related ideas were proposed already by Johansen (5) and Salter (14), and their ideas gave me a lot of important suggestions.

2) Related argument was presented in my review for the theory of the firm (7).

3) See Penrose (14), and refer to my frontal attempt (8).

4) Johansen (5), pp. 1.

5) See Mansfield (10), pp. 12-15, and Walters (15).

6) Schumpeter, History of Economic Analysis, p. 1031.

7) Hicks, The Theory of Wages, p. 183.

8) As for estimation only, one alternative may be possible: Stratifying the sample by different technique and estimating separate function for each stratum. But, it is very difficult to find the pertinent criteria of stratification, and therefore the meaning of the production function estimated for each stratum is very dubious. Even if such criteria can be possibly found, we then face the problem that degree of freedom in estimation is severely lost. Aside from such respects of estimation, many interest inferences which could be otherwise obtained are lost by

abandoning the assumption of parametric relation and taking the alternative approach.

9) The problem of measurement also extended to output. One of the important features of technique is quality of output. It will be put, however, outside of my attention in this paper.

10) I think almost all feasures of a technique will be characterized by the set of capital equipment as the production process is mechanized (5, 15). For example, planting method is a factor belonging to M when the operation is done by bare hand, but it will be partly belong to I at the stage of machinery operation.

11) See Mansfield (10), pp. 10-12.

12) See Mansfield (10), p. 10, and Salter (15), pp. 13-16.

13) Cobb-Douglas function, even if the relevant range is confined to a small range in (3-1), implys high substitutability between factors within the confined range. As an alternative specification we can adopt a segment of CES function whose substitution elasticity is much smaller than unity. But, for its statistical estimation, one has to assume the equilibrium condition, which

conflicts with my critical assertion. I think logarithmic Tayor expansion and non-linear regression estimation method is possible.

14) Expanding the function logarithmically, one find it to be very similar to a restrictive transcendental logarithmic production function (1). Trans-log production function is given by,

$InQ = W + \Sigma w_i InX_i + 1/2 \Sigma\Sigma w_{ij} InX_i InX_j$.

Very similar function to mine is obtained from it if following restrictions are imposed,

a) group-wise additivity holds true, but commodity wise additivity within group not. In addition, b)

$w_M=0$, $w_{KA}=0$, $w_{KK}=0$, $w_{AA}=0$, $w_{MM}=0$, $w_{KL}= -w_{KM}$, $w_{LL}= -2w_{LM}$, $w_{LA}= -w_{AM}$.

15) For an extensive review for this matter, see Walter (17).

16) For details, see Johnston (6), pp. 341-351, and Hoch (3).

17) This justification hinges on what components are included in the disturbance term of the production function. Reflecting that I have insisted that there is wide variation in production technique as well as in environmental

condition both between farms and years, we may concede that the disturbance term includes some farm-specific factors. Input level is naturally expected to be related with those farm specific factor and we are again put into the problem of simultaneous equation bias in spite of introduction of the anticipated output concept. But it is originated from excluding a relevant variable of management and environment, it may be better to investigate in the framework of specification bias. Hoch (3) investigated it in the framework of simultaneous bias whereas Mundrak (13) and Griliches (2) in specification bias. Unless farm specific factors are independent of other input factors, exclusion of them causes both simultaneous and specification bias. Both biases depend on the correlation between the farm specific factors and the included factor inputs. It is very interesting to note that exclusion of managerial factor would not produce either bias in our estimation if the farm specific factors have no correlation with physical inputs.

18) See Johnston (6), pp. 281-291.

19) Griliches (2) investigated in details for a

particular case.

20) For example, Griliches (2) suggested to use geometric sum in estimating Cobb Douglas function. Welch (18) used CES function in aggregating skilled and unskilled labors. Le (9) applied the same method in aggregating self-supplied fertilizer and commercial fertilizer.

21) See Welch (18) and Le (9).

22) Hoch (4), Mundrak (13) and Massel (11) etc. used co-variance analysis methods for this problem.

23) See Lee (13).

24) Gathering and decoding records are accomplished by Statistics and Survey Div., Ministry of Agriculture and Forestry.

25) Estimation method for the value of self-supplied goods is referred to Farm Management Records (19).

26) Each expense is deflated to the price level of 1960, and then their arithmetic sum is used. Deflator is obtained from Statistics of Rural Wages and Prices (20).

27) This may be subject to criticism. But it was the best I could have. Separating them into two inputs made the coefficient of equipment input negative or made the both very unstable. My

procedure will be appreciated if the two inputs were highly substitutable during the period.

28) Yield per acre in standard paddy land in each district of same productivity is regressed on time. And theoretical output per acre will be calculated from the regression. The ratio between the actual output and the theoretical output is taken as weather indexes for the farm located in the district. I used the experimental station's data at first, but it was not successful.