Environ. Sci., Hokkaido	7 (1)	1~13	June 1984

A Study on Model Reference Adaptive Control in Economic Development (II)

Etsuo Yamamura
Department of Regional Planning, Division of Environmental Planning, Graduate School of Environmental Science, Hokkaido University, Sapporo, 060

Abstract

We have investigated the adaptive process of the economic development model with the uncertainty concerning the values of the model's parameters.

Firstly, we have investigated the economic development model with the uncertainty concerning the values of the developing regional economic model's parameters.

Secondly, we have considered the parametric identification of the economic development model.

Thirdly, we have considered the design of adaptive state observer of the economic development model.

The model reference adaptive tecnique is conceived to be a useful method in spacifying the adaptive process analysis of the economic development model with the uncertainty concerning of the values of the model's parameters.

Key words: Model reference adaptive control, Economic development, Parametric identification, Adaptive state observation.

1. Introduction

In the previous paper: A Study on Model Reference Adaptive Control in Economic Development (I),[1] we considered one of the adaptive processes of the economic development policy model by using the discrete model reference adaptive technique with the certainty concerning the values of the adjustable model's parameters.

In this paper, we shall consider one of the adaptive processes of the economic development model by using model reference adaptive technique with uncertainty concerning the values of the adjustable model's parameters and the variation of these parameters, and also the parametric identification and adaptive state observation.

2. Economic Development Model with the Uncertainty Concerning the Values of Parameters

In this chapter, we shall consider the economic development model with the uncertainty concerning the values of parameters. In the real economic development.

we have the difficulties related to the uncertainty concerning the values of economic model's parameters, especially, of the developing economic model's ones.

Then, we shall consider the following simple economic development model with the uncertainty concerning the values of the developing regional economic model's parameters.

The developed regional economic model:

$$y_M(t)=a_1 y_M(t-1)+u_M(t-1) \tag{1}$$

$y_M(t)$: the regional output per regional resident of the developed region at the instant t.

$u_M(t)$: the regional invesment per regional resident of the developed region at the instant t.

a_1: the parameter of developed region.

The developing regional economic model.

$$y_s(t)=\hat{a}_1 y_s(t-1)+u_s(t-1) \tag{2}$$

$y_s(t)$: the regional output per regional resident of the developing region at the instant t.

$u_s(t)$: the regional investment per regional resident of the developed region at the instant t.

$\hat{a}_1$: the uncertain parameter of developing region.

The objective equation is as follows.

$$\lim_{t\to\infty}(1+d_1 z^{-1})\, e(t+1)=\lim_{t\to\infty} e^o(t+1)=0 \tag{3}$$

Where,

$$u_s(t)=(1+d_1 z^{-1})\, y_M(t+1)-\hat{P}(t)\, ys(t) \tag{4}$$

The estimated value of $\hat{P}(t)$ from the value of the certain parameter of developing region is as follows.

$$P=d_1+\hat{a}_1 \tag{5}$$

$e^o(t)$ can be computed using equations (2) and (4). One obtains.

$$e^o(t)=\left[P-\hat{P}(t-1)\right]y_s(t-1) \tag{6}$$

Applying the following aid error signal[2,3]:

$$e_a(t)=\left[\hat{P}(t-1)-\hat{P}(t)\right]y_s(t-1) \tag{7}$$

One obtains,

$$e^*(t)=e_a(t)+e^o(t)=\left[P-\hat{P}(t)\right]y_s(t-1) \tag{8}$$

Then,

$$\lim_{t\to\infty} e^*(t)=0 \tag{9}$$

The adaptive algorithm is as follows.

$$\hat{P}(t) = \hat{P}(t-1) + f_1 y_s(t-1)\, e^*(t) \qquad f_1 > 0 \tag{10}$$

$e^*(t)$ can be computed using equations (7), (8) and (10), one obtains.

$$e^*(t) = \frac{e^o(t)}{1 + f_1 y_s^2(t-1)} \tag{11}$$

The equation (11) corresponds to the equation (4. 84) in the previous paper: A Study on Model Reference Adaptive Control in Economic development (I)[1].

In general case, consider the description in terms of differential operators.

The reference model

$$A_M(z^{-1})\, y_M(t) = z^{-d} B_M(z^{-1})\, u_M(t) \tag{12}$$

Where,

$$A_M(z^{-1}) = 1 + a_{M_1} z^{-1} + \cdots\cdots + a_{M_n} z^{-N} \tag{13}$$

$$B_M(Z^{-1}) = b_{M_0} + b_{M_1} z + \cdots\cdots + b_{M_n} z^{-N} \tag{14}$$

The adjustable model

$$A_s(z^{-1})\, y_s(t) = z^{-d} B_s(z^{-1})\, u_s(t) \qquad d > 0,\ y_s(0) \neq 0 \tag{15}$$

Where,

$$A_s(z^{-1}) = 1 + a_1 z^{-1} + \cdots\cdots + a_n z^{-n} \tag{16}$$

$$B_s(z^{-1}) = b_0 + b_1 z^{-1} + \cdots\cdots + b_m z^{-m} \qquad b_0 \neq 0 \tag{17}$$

If the following equations we satisfied, the model reference adaptive control is also satisfied.

$$\begin{aligned} D(z^{-1})\left[y_s(t+d) - y_M(t+d)\right] &= D(z^{-1})\, e(t+d) \\ &= e^o(t+d) = 0 \end{aligned} \tag{18}$$

$$e(t) = y_s(t) - y_M(t) \tag{19}$$

(a) $d=1$

$D(z^{-1})\, e(t+1)$ obtains the following equation at $d=1$

$$\begin{aligned} D(z^{-1})\, e(t+1) = {} & B_s(z^{-1})\, u_s(t) + \left[D(z^{-1}) - A(z^{-1})\right] y_s(t+1) \\ & - D(z^{-1})\, y_M(t+1) \end{aligned} \tag{20}$$

If the equation (18) is satisfied, the equation (20) is zero.

$$\begin{aligned} & B_s(z^{-1})\, u_s(t) + \left[D(z^{-1}) - A_s(z^{-1})\right] y_s(t+1) - D(z^{-1})\, y_M(t+1) \\ & = b_0 u(t) + P_0^T \phi_0(t) - D(z^{-1})\, y_M(t+1) = 0 \end{aligned} \tag{21}$$

Where,

$$P_0^T = [b_1, \cdots, b_m,\ d_1 - a_1, \cdots, d_n - a_n] \tag{22}$$

$$\phi_0^T(t)=\left[u_s(t-1),\ \cdots,\ u_s(t-m),\ y_s(t),\ \cdots,\ y_s(t-n+1)\right] \tag{23}$$

$u_s(t)$ obtains from the equation (21)

$$\begin{aligned} u_s(t) &= \left\{D(z^{-1})\,y_M(t+1)-P_0^T\phi_0(t)\right\}\Big/b_0 \\ &= \left\{D(z^{-1})\,y_M(t+1)-\left[D(z^{-1})-As(z^{-1})\right]y_s(t+1)\right\}\Big/B_s(z^{-1}) \\ &= \left\{D(z^{-1})\,y_M(t+1)-\left[\sum_{i=1}^{n}(d_i-a_i)\,z^{-i}\right]y_s(t+1)\right\}\Big/B(z^{-1}) \end{aligned} \tag{24}$$

The equation (24) represents the following equation.

$$\begin{aligned} u_s(t) = f\Big[&u_s(t-1),\ \cdots,\ u_s(t-m),\ y_s(t),\ \cdots,\ y(t-n+1), \\ &y_n(t+1),\ \cdots,\ y_M(t-n+1)\Big] \end{aligned} \tag{25}$$

$u_s(t)$ is feasible equation, because $u_s(t)$ depends on the last input, output untill present time and y $(t+1)$ obtained by equation $A_M(z^{-1})\,y_M(t+1)=B_M(z^{-1})\,u_M(t)$.

(b) $d>1$,

$$D(z^{-1})=A_S(z^{-1})\,S(z^{-1})+z^{-d}R(z^{-1}) \tag{26}$$

Where,

$$S(z^{-1})=1+s_1z^{-1}+\cdots\cdots+s_{d-1}z^{-(d-1)} \tag{27}$$

$$R(z^{-1})=r_0+r_1z^{-1}+\cdots\cdots+r_{n-1}z^{-(n-1)} \tag{28}$$

The equation (18) obtains as follows using the equation (26) and $A_s(z^{-1})\,y_S(t+d)=B_s(z^{-1})\,u_s(t)$.

$$\begin{aligned} D(z^{-1})\,e(t+d) &= As(z^{-1})\,S(z^{-1})\,y_s(t+d)+z^{-1}R(z^{-1})\,y_s(t+d) \\ &\qquad -D(z^{-1})\,y_M(t+d) \\ &= B_s(z^{-1})\,S(z^{-1})\,u_s(t)+R(z^{-1})\,y_s(t) \\ &\qquad -D(z^{-1})\,y_M(t+d)=0 \end{aligned} \tag{29}$$

$u_s(t)$ obtains from the equation (29).

$$u_s(t)=\left\{D(z^{-1})\,y_M(t+d)-R(z^{-1})\,y_s(t)\right\}\Big/B_s(z^{-1})\,S(z^{-1}) \tag{30}$$

or,

$$\begin{aligned} u_s(t) &= \left[D(z^{-1})\,y_M(t+d)-R(z^{-1})\,y_s(t)-B_T(z^{-1})\,u_s(t)\right]\Big/b_0 \\ &= \left[D(z^{-1})\,y_M(t+d)-P_0^T\phi_0(t)\right]\Big/b_0 \end{aligned} \tag{31}$$

Where,

$$B_T(z^{-1})=B_s(z^{-1})\,S(z^{-1})-b_0 \tag{32}$$

$$P_0^T=[b_0s_1+b_1,\ b_0s_2+b_1s_1+b_2,\ \cdots b_ms_{d-1},\ r_0,\ \cdots r_{n-1}] \tag{33}$$

$$\phi_0^T=\left[u_s(t-1),\ \cdots,\ u_s(t-m-d+1),\ y_s(t),\ \cdots,\ y_s(t-n+1)\right] \tag{34}$$

$u_s(t)$ is feasible equation because one is not depended on the uncertain value. Consider the following control law substitute to the equation (31).

$$u_s(t)=\left[D(z^{-1})\,y_M(t+d)-\hat{P}_0(t)\,\phi_0(t)\right]\Big/\hat{b}_0(t) \tag{35}$$

$$\lim_{t\to\infty} D(z^{-1})\,e(t+d)=\lim_{t\to\infty} e^0(t+d)=0 \tag{36}$$

The following equation obtains from the equation (35).

$$D(z^{-1})\,y_M(t+d)=\hat{b}_0(t)\,u_s(t)+\hat{P}_0(t)\,\phi_0(t) \tag{37}$$

The following equation obtains from the equations (12), (26), (33) and (34).

$$D(z^{-1})\,y_s(t+d)=b_0 u_s(t)+P_0{}^T\phi_0(t) \tag{38}$$

Substracting equation (37) from (38), we obtain

$$D(z^{-1})\left[y_s(t+d)-y_M(t+d)\right]=e^0(t+d)=\left[P-\hat{P}(t)\right]\phi(t) \tag{39}$$

or,

$$e^o(t)=\left[P-\hat{P}(t-d)\right]\phi(t-d) \tag{40}$$

Where,

$$\phi^T(t)=\left[u_s(t)\ ;\ \phi_0{}^T(t)\right] \tag{41}$$

$$\hat{P}^T(t)=\left[\hat{b}_0(t),\ \hat{P}_0(t)\right] \tag{42}$$

$e^*(t)$ obtains from the aid error signal such as

$$e_a(t)=\left[\hat{P}(t-d)-\hat{P}(t)\right]^T\phi(t-d) \tag{43}$$

$$e^*(t)=e_a(t)+e^0(t)=\left[P-\hat{P}(t)\right]^T\phi(t-d) \tag{44}$$

The adaptive algorithm satisfied, $\lim_{t\to\infty} e^*(t)=0$ is as follows.

$$\hat{P}(t)=\hat{P}(t-1)+F(t-1)\,\phi(t-d)\,e^*(t) \tag{45}$$

Where,

$$\left.\begin{aligned}&F^{-1}(t)=\lambda_1(t)\,F^{-1}(t-1)+\lambda_2(t)\,\phi(t-d)\,\phi^T(t-d)\\&F(0)>0\,,\qquad 0<\lambda_1(t)\leq 1\,,\qquad 0\leq\lambda_2(t)<2\end{aligned}\right\} \tag{46}$$

$e^*(t)$ obtains from the equations (40), (43), (44) and (45).

$$\begin{aligned}e^*(t)&=e^o(t)+\Big[\hat{P}(t-d)-\hat{P}(t-1)\,\phi(t-d)\\&\qquad-\phi^T(t-d)\,F(t-1)\,\phi(t-d)\,e^*(t)\\&=\Big[P^T\phi(t-d)-\hat{P}^T(t-1)\,\phi(t-d)\\&\qquad-\phi^T(t-d)\,F(t-1)\,\phi(t-d)\,e^*(t)\Big]\end{aligned} \tag{47}$$

From the equations (47) and (38),

$$e^*(t) = \left[D(z^{-1})\, y_s(t) - \hat{P}^T(t-1)\, \phi(t-d)\right] \Big/ \left[1+\phi^T(t-d)\, F(t-1)\, \phi(t-d)\right]$$
$$= \hat{e}(t) \Big/ \left[1+\phi^T(t-d)\, F(t-1)\, \phi(t-d)\right] \tag{48}$$

Then, $e^*(t)$ can be calculated without using $\hat{P}^T(t)$.

3. Parametric Identification of Economic Development Model

In this chapter, we shall consider the parametric identification of the developing economic model.

The parametric identification has two aspects of parametric identification such as the recursive parametric identification and the parameter tracking.

The recursive parameter identification likes to obtain an estimation of the parameters, as the process develops, without being obliged to use all the past input/output data at each step, and the parameter tracking likes to track the values of the parameters of a process which can vary during operation. In order to obtain good results in the recusive parametric identification and the parameter tracking, a certain degree of structural identification must be a prior achieved.

This can be done either from the economic analysis of the developed region to be identified or from an off-line identification based on a previous set of input/output data.

The recursive identification of the dynamic parameters as well as the tracking of the developed region when they are time-varying can both be formulated as a model reference adaptive problem.[4,5] The developed regional economic model to be identified represents the reference model. The developing regional economic model is constituted by an adjustable model having the structure of the economic model whose parameters are driven by an adaptation mechanism which implements an identification algorithm.

Depending on the configuration of the adjustable model, the signals which are fed in, and how the error between the outputs of the reference model and the adjustable model is obtained, one can distinguish three basic configurations such as the output error method, the equation error method, and the input error method which are illustrated in Figure 1.

These configurations corresponds to the three basic structures of model reference adaptive systems such as the parallel, series parallel and series type, and the correspondence is indicated in Table 1.

The equation error method is the most used configuration for recursive identification using algorithms derived from statistical considerations. From statistical considerations, the recursive identification algorithms for equation error configurations assuring in most of the eases unbiased parameter estimates have been derived. However, the convergence of the identification with regard to the initial parameter error is often difficult to prove.[6]

On the other hand, the model reference adaptive system which are designed from stability considerations, when used for identification, allow us to specify the conditions of the convergence with respect to initial parameter error, but the performance in the presence of measurement noise needs to be examined separately. Note also that the methodology based on the use of hyperstability and positivity

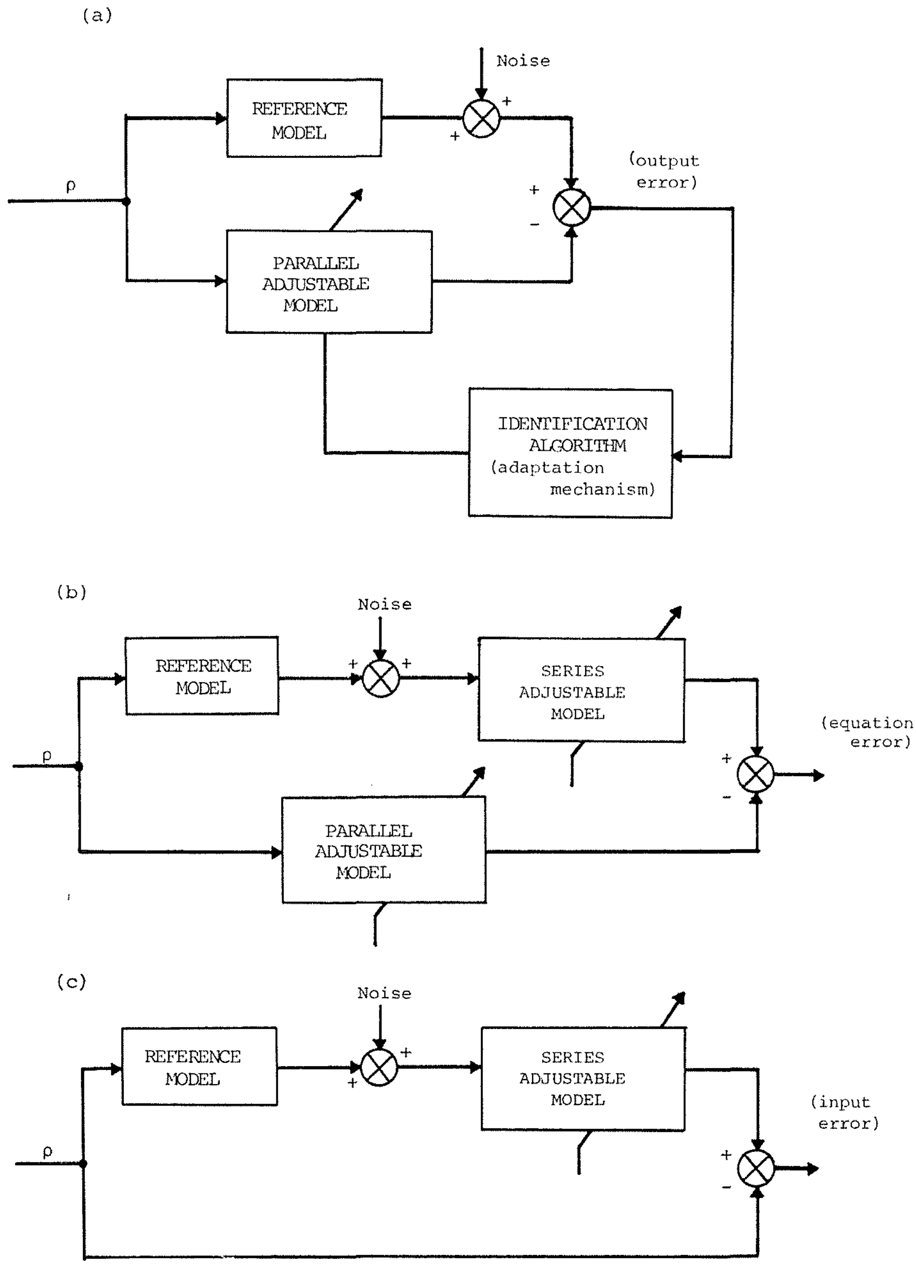

Figure 1. Three basic identification structures and their correspondence with MRAS's. (a) Output error method (parallel MRAS's). (b) Equation error method (seriesparallel MRAS's). (c) Input error method (series MRAS's).

Table 1. Correspondence Between Identification Structures and Model Reference Adaptive System Structures

Identification Structure	Model Reference Adaptive System	Terminology used in Chapter 3
Output error method	Parallel	Parallel identifier
Equation error method	Series-Parallel	Series-Parallel identifier
Input error method	Series	Series identifier

concepts for designing the model reference adaptive system can also be applied to the analysis of the stability properties of the identifiers derived from statistical considerations.[7,8]

In using recursive identifiers for discrete time processes, one can distinguish two basic situations.

Case A : Frequent changes in the values of the parameters of the reference model to be identified occur. The identifiers are supposed to be able to track the values of these parameters when such changes occur.

Case B : Recursive identification of linear time-invariant discrete-time processes is desired. In this case, the unknown parameters of the adjustable model are supposed to be constant over a long time period.

Case A identification can be solved satisfactorily in most of the cases using discrete-time adaptation algorithms of Theorem 1 in the paper : A Study on Model Reference Adaptive Control in Economic Development (I),[1] which have constant adaptation gains.

Case B identification can be solved using either Theorem 1 adaptation algorithms or Theorem 2 adaptations algorithms, which have decreasing adaptation gains since both have a recursive character.

However, if a parallel configuration is used, the algorithms of Theorem 2 can lead to an unbiased parameter estimation in the presence of measurement noise. This is not the case for the algorithms of Theorem 1, where in general a compromise between precision and speed of convergence must be made. Therefore for the Case B identification, algorithms of Theorem 2 are more suitable. Note also that the algorithms of Theorem 2 can eventually be used for Case A identification of the parameter changes are not too frequent. In this case, one can detect, by examining the generalized error, a parameter change, and we reinitialize the adaptation gains at a high value.

4. Design of Adaptive State Observer of Economic Development Model

As mentioned in chapter 3 of the paper : A Study on Model Reference Adaptive Control in Economic Development (I),[1] when the model to be controlled with inaccessible states has unknown parameters or they vary during operation, adaptive state observers must be built.[9,10,11] In this case from the linear asymptotic observer one can derive a model reference adaptive system structure which permits observation of the states of the model.[12]

In this chapter, we shall consider the Parallel Adaptive State Observer. We assure that the system whose states we wish to observe is in the observable canonical form described as follows.

$$\hat{x}(t+1)=\begin{bmatrix} \hat{a}_1(t+1)-l_1 & \vdots & I \\ \vdots & \vdots & \\ \cdots\cdots & \vdots & \cdots\cdots \\ \hat{a}_n(t+1)-l_n & \vdots & 0\cdots\cdots 0 \end{bmatrix}\hat{x}(t)+\begin{bmatrix} l_1 \\ \\ l_n \end{bmatrix}y(t)$$
$$+\begin{bmatrix} \hat{b}_1(t+1) \\ \vdots \\ \hat{b}_n(t+1) \end{bmatrix}u(t)+\hat{u}_a(t+1)+\hat{u}_b(t+1) \tag{49}$$

$$y(t)=[1,0\cdots 0]\,\hat{x}(t)=\hat{x}_1(t) \tag{50}$$

Where, $\hat{x}(t)$ is the state of the adjustable observation model, $\hat{y}_s(t)$ is the output of the observation model, $\hat{a}_i(t+1)$, $\hat{b}_i(t+1)$ are the adjustable parameters, $l_1, \cdots, l_n$ is a set of gains which will influence the characteristics of the observer, and $\hat{u}_a(t+1)$ and $\hat{u}_b(t+1)$ are additional transient adaptation signals having the property as follows.

$$\hat{u}_a(t)/_{\varepsilon(t)=0}=0 \qquad \hat{u}_b(t)/_{\varepsilon(t)=0}=0 \tag{51}$$
$$\left(\varepsilon(t)=y_s(t)-\hat{y}_s(t)\right)$$

These transient adaptation signal vectors have the following form.

$$\hat{u}_a(t+1)=\begin{bmatrix} u_a^1(t+1) \\ \vdots \\ u_a^{n-1}(t+1) \\ 0 \end{bmatrix}, \quad \hat{u}_b(t+1)=\begin{bmatrix} u_b^1(t+1) \\ \vdots \\ u_b^{n-1}(t+1) \\ 0 \end{bmatrix} \tag{52}$$

Our objective is to design an adaptation mechanism such that the following conditions are both satisfied.

$$\lim_{t\to\infty}\hat{e}(t)=\lim_{t\to\infty}\left[x(t)-\hat{x}(t)\right]=0 \tag{53}$$

$$\lim_{t\to\infty}\hat{a}_i(t)=a_i \qquad \lim_{t\to\infty}\hat{b}_i(t)=b_i \tag{54}$$
$$(i=1,\cdots,n)$$

Using the equations (49) and (50), one obtains

$$\hat{e}(t+1)=\begin{bmatrix} a_1-l_1 & \vdots & I \\ \vdots & \vdots & \\ \cdots\cdots & \cdots & \cdots\cdots \\ a_n-l_n & \vdots & 0\cdots 0 \end{bmatrix}\hat{e}(t)-\begin{bmatrix} \Delta a_1(t+1) \\ \vdots \\ \Delta a_n(t+1) \end{bmatrix}\hat{y}(t)$$
$$-\begin{bmatrix} \Delta b_1(t+1) \\ \vdots \\ \Delta b_n(t+1) \end{bmatrix}u(t)-\hat{u}_a(t+1)-\hat{u}_b(t+1) \tag{55}$$

$$\varepsilon(t)=[1,0\cdots 0]\,\hat{e}(t)=e_1(t) \tag{56}$$

Where,

$$\Delta a_i(t+1) = \hat{a}_i(t+1) - a_i \qquad (i=1, \cdots, n) \tag{57}$$

$$\Delta b_i(t+1) = \hat{b}_i(t+1) - b_i \qquad (i=1, \cdots, n) \tag{58}$$

The equations (55) and (56) lead to the following difference equation characterizing $\varepsilon(t)$.

$$\left[1 - \sum_{i=1}^{n} (a_i - l_i)\, z^{-1}\right] e(t) = -\left[\sum_{i=1}^{n} \Delta a_i(t-i+1)_i^{*}\hat{y}(t-i) + \sum_{i=1}^{n} \Delta b_i(t-i+1)\, u(t-i) + \sum_{i=1}^{n-2} u_a{}^{i+1}(t-i) + \sum_{i=0}^{n-1} u_b{}^{i+1}(t-i)\right] \tag{59}$$

The equation (59) is rewrited the following equation by a convenient choice of $\hat{u}_a(t+1)$, $\hat{u}_a(t+1)$, $w_i(t)$ and $r_i(t)$.

$$\left[1 - \sum_{i=1}^{n} (a_i - l_i)\, z^{-i}\right] e(t) = -\left(1 + \sum_{i=1}^{n-1} c_i z^{-i}\right) \left[\sum_{i=1}^{n} \Delta a_i(t)\, w_i(t) + \sum_{i=1}^{n} \Delta b_i(t)\, r_i(t)\right] \tag{60}$$

The equation (60) is rewrited the following equation.

$$\varepsilon(t) = h(z^{-1}) \left[P - \hat{P}(t)\right]^T \phi(t-1) \tag{61}$$

Where,

$$h(z^{-1}) = \left[1 + \sum_{i=1}^{n} c_i z^{-i}\right] \Big/ \left[1 - \sum_{i=1}^{n} (a_i - l_i)\, z^{-i}\right] \tag{62}$$

$$P^T = [a_1,\ a_2,\ \cdots,\ a_n,\ b_1,\ \cdots,\ b_n] \tag{63}$$

$$\hat{P}^T(t) = \left[\hat{a}_1(t),\ \cdots,\ \hat{a}_n(t),\ \hat{b}_1(t),\ \cdots,\ \hat{b}_n(t)\right] \tag{64}$$

$$\phi^T(t-1) = \left[w_1(t),\ \cdots,\ w_n(t),\ r_1(t),\ \cdots,\ r_n(t)\right] \tag{65}$$

We assume that the equation (60) corresponds to an asymptotically hyperstable feedback system, and therefore we shall have

$$\lim_{t\to\infty} \varepsilon(t) = 0 \tag{66}$$

Now since the system of equations (55) and (56) is completely observable, the equations (66) will also imply that the equation (53) is satisfied if asymptotic identification of the parameters is achieved.

$$\left.\begin{aligned} &\lim_{t\to\infty} \Delta a_i(t+1) = \lim_{t\to\infty}\left[a_i(t+1) - a_i\right] = 0 \qquad (i=1, \cdots, n) \\ &\lim_{t\to\infty} \Delta b_i(t+1) = \lim_{t\to\infty}\left[b_i(t+1) - b_i\right] = 0 \qquad (i=1, \cdots, n) \end{aligned}\right\} \tag{67}$$

Because $\lim_{t\to\infty} \hat{u}_a(t+1) = 0$ and $\lim_{t\to\infty} \hat{u}_b(t+1) = 0$ if the equation (66) holds.

One conclude that if the input sequence is a sum of at least $n+1$ periodic signals of distinct frequencies, then the equation (66) will imply the verifications of the equation (67). Therefore, under the above condition equation (66) will also imply that $\lim_{t\to\infty} \hat{e}(t)=0$.

Under the hypothesis that equations (59) and (60) are zero-state equivalent, the design of an asymptotically adaptive state observer is transformed into the design of an asymptotically hyperstable feedback system, which leads to the equation (60).

The remaining design contains the determination of the adaptation signals $\hat{u}_a(t+1)$ and $\hat{u}_b(t+1)$ and of the auxiliary variables $w_i(t)$ and $r_i(t)$, and the determination of the adaptation algorithm for $\hat{a}_i(t)$ and $\hat{b}_i(t)$.

If the equations (59) and (60) is 2th, the two equations is rewrited the following equations.

$$\left[1-\sum_{i=1}^{2}(a_i-l_i)\,z^{-i}\right]\varepsilon(t) = -\left[\sum_{i=1}^{2}\Delta a_i(t-i+1)\,\hat{y}(t-i) + \sum_{i=1}^{2}\Delta b_i(t-i+1)\,u(t-i)+u_a{}^1(t)+u_b{}^1(t)\right] \tag{68}$$

$$\left[1-\sum_{i=1}^{2}(a_i-l_i)\,z^{-1}\right]\varepsilon(t) = -(1+c_1z^{-1})\left[\sum_{i=1}^{2}\Delta a_i(t)\,w_i(t) + \sum_{i=1}^{2}\Delta b_i(t)\,r_i(t)\right] \tag{69}$$

We can obtain the following the conditions by (68)=(69).

$$u_a{}^1(t) = -\left[\Delta a_1(t)-\Delta a_1(t-1)\right]c_1\,w_1(t-1) + \left[\Delta a_2(t)-\Delta a_2(t-1)\right]w_2(t) \tag{70}$$

$$u_b{}^1(t) = -\left[\Delta b_1(t)-\Delta b_1(t-1)\right]c_1\,r_1(t-1) + \left[\Delta b_2(t)-\Delta b_2(t-1)\right]r_2(t) \tag{71}$$

$$w_2(t) = w_1(t-1) \qquad r_2(t) = r_1(t-1) \tag{72}$$

If the adaptive matrix is the diagonal matrix, the following equations are obtained.

$$\Delta a_i(t) = \Delta a_i(t-1)+f_{ai}\,\varepsilon(t)\,w_i(t)\,, \qquad \underset{i=1,2}{f_{ai}}>0 \tag{73}$$

$$\Delta b_i(t) = \Delta b_i(t-1)+f_{bi}\,\varepsilon(t)\,r_i(t)\,, \qquad \underset{i=1,2}{f_{bi}}>0 \tag{74}$$

The equations (70) and (73), (71) and (74) lead to the following equations.

$$u_a{}^1(t) = -f_{a_1}w_1(t)\,c_1\,w_1(t-1)\,\varepsilon(t)+f_{a_2}w_2{}^2(t)\,\varepsilon(t) \tag{75}$$

$$u_b{}^1(t) = -f_{b_1}r_1(t)\,c_1\,r_1(t-1)\,\varepsilon(t)+f_{b_2}r_2{}^2(t)\,\varepsilon(t) \tag{76}$$

The prior output of the adaptive observer is as follows.

$$\hat{y}^o(t) = \hat{x}_1{}^o(t) = \left(\hat{a}_i(t-1)-l_1\right)\hat{x}_1(t-1)+\hat{x}_2(t-1) + \hat{b}_1(t-1)\,u(t-1)+l_1y(t-1) \tag{77}$$

Where,

$$\hat{y}(t)-\hat{y}^o(t)=\left(y(t)-\hat{y}^o(t)\right)-\left(y(t)-\hat{y}(t)\right)$$
$$=\varepsilon^o(t)-\varepsilon(t) \tag{78}$$

Then,

$$\varepsilon^o(t)=\varepsilon(t)+\sum\left[\hat{a}_i(t)-\hat{a}_i(t-1)\right]w_i(t)$$
$$+\sum_{i=1}^{2}\left[\hat{b}_1(t)-\hat{b}_i(t-1)\right]r_i(t) \tag{79}$$

The equations (79), (73) and (74) lead to the following equations.

$$\varepsilon(t)=\varepsilon^o(t)\Big/\left[1+\sum_{i=1}^{2}f_{a_i}w_i^2(t)+\sum_{i=1}^{2}f_{b_i}r_i^2(t)\right] \tag{80}$$

The parallel adaptive observer is obtained by the equations such as (49), (50), (70), (71), (75), (76) and (80).

5. Conclusion

We have investigated the adaptive process of the economic development policy model by using the model reference adaptive technique with the uncertainty concerning the values of the adjustable model's parameters, and also the parametric identification and adaptive state observation.

The main results are as follows.

(1) The model reference adaptive technique is conceived to be a useful method in specifying the adaptive process of the economic development model with the uncertainty concerning the values of parameters.

(2) The recursive identification of the dynamic parameters as well as the tracking of the developed region when they are time-varying can both be formulated as a model reference adaptive problem.

(3) The model reference adaptive state observer is conceived to be a useful method in controlling with inaccessible states has unknown parameters or they vary during operation in Economic Development Model.

References

1) Yamamura, E. (1984): A Study on Model Reference Adaptive Control in Economic Development (I): Environmental Science, Hokkaido, Vol. 6, No. 2.
2) Ionescu, T. and R. Monopoli (1977): Discrete Model Reference Adaptive Control with an Augmented Error Signal: Automatica, Vol. 13, No. 5.
3) Lozano, L. and I. D. Landau, (1981): Redesign of Explicit and Implicit Discrete Time Model Reference Adaptive Control Schemes: International Journal of Control, Vol. 33, No. 2.
4) Åström, K. J. and P. Eykoff (1971): System Identification, A Survey: Automatica, 7.
5) Landau, I. D. (1974): A Survey of Model Reference Adaptive Techniques—Theory and Applications: Automatica, 10.
6) Mendel, J. M. (1973): Discrete Techniques of Parameter Estimation, The Equation Error Formulation: Marcel Dekker.

7) Landau, I. D. (1975): Algorithme d'identification récursive utilistant le concept de positivité: Rev. Fr. Autom. Intorm. Rech. Opérat, Ser. Jaune, J-2.
8) Bethoux, G. (1976): Approche unitaire des méthodes d' identification et de commande adaptive des procedes dynamiques; Thése 3 me Cycle, Institut National Polytechnique de Grenoble, Grenoble, July.
9) Caroll, R. L. and D. P. Lindorff (1973): An Adaptive Observer for Single-Output Linear Systems: IEEE Trans. on Automatic Control, Vol. AC-18, No. 5.
10) Lüders, G. and K. S. Narendra (1973): An Adaptive Observer and Identifier for a Linear System: IEEE Trans. on Automatic Control, Vol. AC-18, No. 5.
11) Kudva, P. and K. S. Narendra (1973): Synthesis of an Adaptive Observer Using Lyapunov's Direct Method: International Journal of Control, Vol. 18.
12) Y. D. Landau (1979): Adaptive Control, The Model Reference Approach: Marcel Dekker.

(*Received 27 February 1984*)