Environ. Sci., Hokkaido	8 (1)	39~46	June 1985

A Study on Regional Allocation of Investments for Redistribution Policy of Population

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Abstract

This paper presents the regional allocation model of public investments for the redistribution policy of population and one detailed simulation concentrating on the controllability of the degree of local autonomy. In addition, we shall consider the simple transport policy model with the conditions that total cost (production cost+shipping cost) of redistribution of plant location is less than the total cost of one supply with the mass production.

Key Words: Regional allocation, Redistribution policy, Transport policy model, Plant location.

1. Introduction

From the experiences of maldistribution reported in the developing countries, the most commonly perceived problem relevant to population distribution is urban crowding. Rapid urbanization throughout the region has produced a heavy strain on urban services. Then, the regional allocation of investments is becoming one of the most important problems for the redistribution policy of population.

In the author's papers[15,19,22], we have shown the detailed simulations of the regional income disparities concentrating on the minimum proportion of public investment, the regional rates of saving and the local autonomy rate. In addition, we considered the justice policy in which each region should not bring about any wide disparity at the end of the planning period.

In this paper, we shall formulate a more generalized model of regional allocation of public investments for the redistribution policy of population, and consider the detailed simulations concentrating on the controllability of the degree of local autonomy. In addition, we shall consider the transport policy for the redistribution of plant location.

2. Regional Allocation Model of Public Investments for the Redistribution Policy of Population

This chapter presents the mathematical formulation of the regional allocation model of public investments for the redistribution policy of population arising from the author's paper and one detailed simulation concentrating on the controllability of the local autonomy rate.

First, we shall present the mathematical formulation of the regional allocation model which holds the following conditions.

(1) The allocation of regional investment is designed to maximize the total outputs under the condition that the outputs per regional residents of each region should not bring about any wide disparity at the end of the planning period.

(2) The supply of funds available for investment will be limited to the sum of all savings in each region.

(3) The productivity of investment, saving ratio and degree of local autonomy are set by central government.

(4) The investment for the removal of the maximum income disparity is allowed only with the mutual consents of all regions.

We define the notations as follows:

P_j^i: the productivity of investment of region j at time i.

S_j^i: the saving ratio of region j at time i.

U_j^i: the proportion of investment shared by region j at time i.

$$(U_j^i=1,\ i=1,\cdots,N) \tag{2.1}$$

r: the degree of local autonomy.

M: the number of regions.

N: the length of planning period.

X_j^i: the regional income of region j at time i.

$$(X_j^i-X_j^{i-1}\geq 0,\ i=1,\cdots,N,\ j=1,\cdots,M) \tag{2.2}$$

$C_j=X_j$: the regional income of region j at initial time.

L_j^i: the regional resident of region j at time i.

D_j^i: the minimum proportion of investment shared by region j at time i.

$$(0\leq D_j^i\leq 1/M) \tag{2.3}$$

Z^i: the national income at time i.

$$Z^i=\sum_j^M X_j^i, \qquad (i=1,\cdots,N) \tag{2.4}$$

The performance equations from condition are as follows.

$$\sum_{j=1}^M (X_j^i-X_j^{i-1})/P_j^i=\sum_{j=1}^M S_j^i\cdot X_j^{i-1} \qquad (i=1,\cdots,N) \tag{2.5}$$

where,

$$X_j^i-X_j^{i-1}=P_j^i\cdot U_j^i\cdot(1-r)\left(\sum_{j=1}^M S_j^{i-1}\cdot X_j^{i-1}\right)+P_j^i\cdot r\cdot S_j^{i-1}\cdot X_j^{i-1} \tag{2.6}$$

The left-hand side represents total investment and the right-hand side represents total saving in the whole country at time i.

The boundary conditions are as follows:

$$C_j=X_j^0 \tag{2.7}$$

$$D_j^i \leq U_j^i \leq 1 - \sum_{k \neq j} D_k \tag{2.8}$$

The performance equations from condition (1) are as follows:

$$\frac{X_1^N}{L_1^N} = \cdots\cdots = \frac{X_M^N}{L_M^N} \tag{2.9}$$

$$J = Z^N \longrightarrow \mathrm{Max}\left(Z^N = \sum_j^M X_j^N\right) \tag{2.10}$$

D_r: the extreme point of controllability.

$[0, Dr]$: the feasible region of controllability.

Next, assume that $S_j^i = S$, $D_j^i = D_j$ and $P_j^i = P_j$ $(i=1, \cdots, N,\ j=1, \cdots, M)$, we shall translate the model into the following equations.

The equalities (2.6) and inequalities (2.8) can be replaced in the terms of inequalities of X_j^i variables instead of U_j^i variables.

$$X_j^i - (1+P_j \cdot r \cdot S_j^{i-1}) \cdot X_j^{i-1} - D_j \cdot P_j \cdot (1-r) \sum_{j=1}^M S_j^{i-1} \cdot X_j^{i-1} \geq 0 \tag{2.11}$$

$$X_j^i - (1+P_j \cdot r \cdot S_j^{i-1}) \cdot X_j^{i-1} - (1 - \sum_{k \neq j} D_k)\, P_j(1-r)\left(\sum_{j=1}^M S_j^{i-1} \cdot X_j^{i-1}\right) \leq 0$$
$$(i=1, \cdots, N) \tag{2.12}$$

The equation (1) can be replaced in the following equations.

$$\sum_{j=1}^M \left\{X_j^i - \left(1+P_j \cdot r \cdot S_j^{i-1}\right) \cdot X_j^{i-1}\right\} / \sum_{j=1}^M P_j \cdot (1-r) \cdot \left(\sum_{j=1}^M S_j^{i-1} X_j^{i-1}\right) = 1$$
$$(i=1, \cdots, N) \tag{2.13}$$

The boundary conditions are as follows:

$$C_j = X_j^0 \tag{2.14}$$

$$0 \leq D_j \leq 1/M \tag{2.15}$$

$$\frac{X_1^N}{L_1^N} = \cdots\cdots = \frac{X_M^N}{L_M^N} \tag{2.16}$$

Next, we have considered the following main results from the simulation analysis based on the model mentioned above.

(1) The extreme point of controllability shows a rapidly decreasing rate by the increases of disparities of the productivities of investments and the redistribution policy of population.

(2) The extreme point of controllability shows a rapidly decreasing rate by the increases of differences of the regional incomes at initial time and the redistribution policy of population.

(3) When the planning period time decreases, the extreme point of controllability shows a rapidly decreasing rate by the increases of differences of the regional incomes at initial time and the redistribution policy of population.

(4) It is imposible to redistribute of population as the degree of local autonomy only increases, but it is possible to redistribute of population as the saving ratio of

low productivity region increases.

(5) The national income based on the redistribution policy of population shows a rapidly decreasing rate as the minimum proportion of investment increases, but the national income shows a small decreasing rate as degree of the local autonomy and the saving ratio of low productivity region increases.

Next, we shall consider one typical simulations of the model of two-region case to clear the meanings of the results mentioned above. In the model, the productivity of investments and saving ratio are assumed to be a constant over time.

The data used in the computation is shown as follows:

$$P_1 = 1.400 \qquad P_2 = 1.200$$
$$X_1^0 = X_2^0 = 10 \qquad L_1^0 = L_2^0 = 10$$
$$S = 0.200 \qquad N = 8$$

Where, the minimum proportion of investment, the degree of local autonomy rate and the saving ratio of region 2 are variables.

In the Figure 1, the real lines represent the national income at N=8 with the degree of local autonomy $r=0.8$ and the minimum proportion of investment $D_j=0$, and the dotted lines represent the national income at $N=8$ with the degree of local autonomy $r=0$ and the minimum proportion of investment $D_j=0.4$. Two simulations have the same controllability in the feasible space.

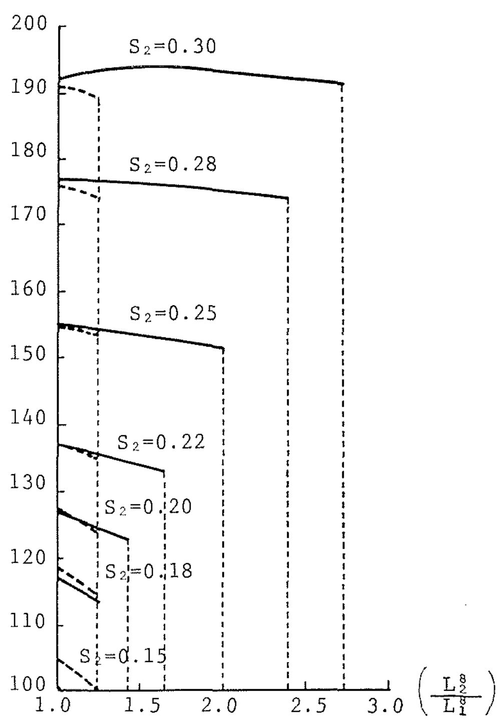

Figure 1. Simulation of Redistribution Policy of Population.

From the facts presented in the graphs, it is imposible to redistribute the population as the saving ratio of region 2 decreases, but it is possible to redistribute the population with the local autonomy policy as the saving ratio of region 2 increases.

It is clear from the facts presented above that the local autonomy policy and the saving ratio of developing region play an important role in the redistribution population.

3. Transport Policy Model for Redistribution of Plant Location

In this chapter, we shall consider the simple transport policy model with the conditions that the total cost (production cost+shipping cost) of N supplies and N demands is less than the one of one supply and N demands.

We define the notations as follows:

x_{ij}: transport volume from region i to region j

t_{ij}: the cost of shipping one unit from region i to region j

t_{ii}: the cost of shipping one unit in region i

$$(t_{i,i} < t_{i,j} \qquad i,\ j=1, \cdots, N) \tag{3.1}$$

N: the number of regions

S_i: the supply of region i

D_i: the demand of region i

C_i: the production cost per one unit locating at region i

$$C_i = -a \cdot S_i + b \qquad a,\ b: \text{constants} \tag{3.2}$$

The production cost function represents a linear function for the agglomerative profit of mass production.

Assume the following condition:

$$\sum_{i=1}^{N} S_i = \sum_{i=1}^{N} D_i$$

The total cost (production cost+shipping cost) per one unit from region i to region j is as follows:

$$C_{ij} = t_{ij} + C_i = t_{ij} - a \cdot S_i + b \tag{3.4}$$

Next, the total cost of N supplies and N demands is compared with the one of one supply and N demands.

The total cost of one supply in the central place i_0 and N demands represents as follows:

$$C_1 = \sum_{j=1}^{N} C_{i_0 j} \cdot D_j = \sum_{j=1}^{N} t_{i_0 j} D_j - a\left(\sum_{i=1}^{N} S_i\right)\left(\sum_{j=1}^{N} D_j\right) + b\left(\sum_{j=1}^{N} D_j\right) \tag{3.5}$$

The total cost of N supplies and N demands represents as follows:

$$C_2 = \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} \cdot x_{ij} = \sum_{i=1}^{N} \sum_{j=1}^{N} t_{ij} x_{ij} - a \sum_{i=1}^{N} \sum_{j=1}^{N} S_i \cdot x_{ij} + b \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij} \tag{3.6}$$

where,

$$\begin{cases} S_i = \sum_{j=1}^{N} x_{ij} \qquad D_j = \sum_{j=1}^{N} x_{ij} \\ \sum_{i=1}^{M} S_i = \sum_{j=1}^{N} D_j = \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij} \end{cases} \tag{3.7}$$

Compare the two costs of C_1 and C_2

$$\boldsymbol{C}_2 - \boldsymbol{C}_1 = \left(\sum_{i=1}^{N} \sum_{j=1}^{N} t_{ij} \cdot x_{ij} - \sum_{j=1}^{N} t_{i_0 j} \cdot D_j \right) + a \left\{ \left(\sum_{i=1}^{N} S_i \right) \left(\sum_{j=1}^{N} D_j \right) - \sum_{i=1}^{N} S_i^2 \right\} \tag{3.8}$$

If $\boldsymbol{C}_2 - \boldsymbol{C}_1 < 0$, the total cost of N supplies and N demands is less than the one of one supply and N demands. Namely, the total cost of redistribution of plant location is less than the total cost of one supply with the mass production.

This condition is transformed into the following inequality.

$$a < \frac{\sum_{j=1}^{N} t_{i_0 j} \cdot D_j - \sum_{i=1}^{N} \sum_{j=1}^{N} t_{ij} \cdot x_{ij}}{\left(\sum_{i=1}^{N} S_i \right)^2 - \sum_{i=1}^{N} S_i^2} \tag{3.9}$$

The coefficient a represents the cost reductive rate with the mass production. If the right hand side of the inequality (3. 9) is larger than the coefficient a, it is possible to redistribute the plant location. The denominator of the inequality (3. 9) is positive and constant. Then, the numerator can become the larger value by the two methods that t_{ioj} increases and t_{ij} decreases. The method with the increase of t_{ioj} is very difficult. Then, we can adopt the method with the decrease of t_{ij}.

This method represents the transport policies such as the decrease of transport cost of intraregion and interregion by the improvement of transport facilities.

In our country, the large scale transportation network such as the highway and new railway is under construction from Tokyo to other region for the redistribution of plant location. But, this improvement method of transport facilities represents the decrease of $t_{i_0 j}$. Then, this improvement method is very difficult to redistribute the plant location.

The most important improvement method of transport facilities for the redistribution of plant location is the construction of the transport network of intraregion and interregion between the local regions by the highway and new railway.

4. Conclution

We considered the regional allocation model of public investments for the redistribution policy of population, and the transport policy model for the redistribution of plant location.

The main results are as follows.

(1) The extreme point of controllability shows a rapidly decreasing rate by the increases of disparities of productivities of investments and the redistribution policy of population.

(2) The extreme point of controllability shows a rapidly decreasing rate by the

increases of differences of the regional incomes at initial time and the redistribution policy of population.
(3) When the planning period time decreases, the extreme point of controllability shows a rapidly decreasing rate by the increases of differences of the regional incomes at initial time and the redistribution policy of population.
(4) It is impossible to redistribute of population as the degree of local autonomy only increases, but it is possible to redistribute of population as the saving ratio of low productivity region increases.
(5) The national income based on the redistribution policy of population shows a rapidly decreasing rate as the minimum proportion of investment increases, but the national income shows a small decreasing rate as the degree of local autonomy and the saving ratio of low productivity region increases.
(6) The most important improvement method of transport facilities for the redistribution of plant location is the construction of the transport network of intraregion and interregion between the local regions by the highway and new railway.

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(*Received 29 January 1985*)