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Introduction to the Regional Economic Growth Model

—Focus on the Controllability of the Regional Income Disparity—

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Abstract

The intension of this paper is to bring out the concept of several regional economic growth models which focus attention on the controllability of regional income disparity viz-the Neoclassical-type model, the Rahman-type model and the Model reference adaptive model. Among three models, the Model reference adaptive technique is conceived to be a useful method in specifying the quantitative analysis of the adaptive process of regional economic development.

Key Words: Regional economic growth model, Regional income disparity, Neoclassical-type model, Rahman-type model, Model reference adaptive model.

1. Introduction

The regional income disparity has been of great interest to many scholars and policy makers as well in the history of every stage of development. Being the major input in the process of regional development, deep analysis in the field stemmed not only from academic motivation but also from the need for sound policy formulation.

The intension of this paper is to bring out the concept of several regional economic growth models which focus attention on the controllability of regional income disparity viz-the Neoclassical-type model, the Rahman-type model and the Model reference adaptive model.

2. Neoclassical-Type Model

The Neoclassical-type model is based on the growth theory by Samuelson and Solow. This chapter reports two models such as Fukuchi model and Mera model.

2.1 *Fukuchi model*[1)]

This model represents a national economy which is divided into n regions with the following assumptions such as all regions have a Cobb-Douglas-type production function, the regional growth rates of the production factors diverge from the overall average in proportion to the differences in productivity of the respective factors in the respective regions, and the factors are fully employed.

The mathematical formation is as follows:

$$Y_{i,t}=\alpha_0 K_{i,t}^{\alpha_1} L_{i,t}^{\alpha_2} \qquad (\alpha_1+\alpha_2=1,\ \alpha_1,\ \alpha_2>0) \quad (i=1, 2, \cdots, n) \tag{2. 1}$$

$$\Delta K_{i,t}=K_{i,t}-K_{i,t-1}=(r+\eta\cdot\lambda_{i,t-1})\,K_{i,t-1} \quad (i=1, 2, \cdots, n) \tag{2. 2}$$

$$\Delta L_{i,t}=L_{i,t}-L_{i,t-1}=(\mu+\rho \quad {}_{i-1})\,L_{i,t-1} \quad (i=1, 2, \cdots, n) \tag{2. 3}$$

$$\lambda_{i,t}=\left\{\left(\frac{\partial Y}{\partial K}\right)_{i,t}-\left(\frac{\overline{\partial Y}}{\partial K}\right)_t\right\}\div\left(\frac{\overline{\partial Y}}{\partial K}\right)_t \quad (i=1, 2, \cdots, n) \tag{2. 4}$$

$$\quad_{i,t}=\left\{\left(\frac{\partial Y}{\partial L}\right)_{i,t}-\left(\frac{\overline{\partial Y}}{\partial L}\right)_t\right\}\div\left(\frac{\overline{\partial Y}}{\partial L}\right)_t \quad (i=1, 2, \cdots, n) \tag{2. 5}$$

$$\left(\frac{\overline{\partial Y}}{\partial K}\right)_t=\alpha_1\left(\frac{\bar{Y}_t}{\bar{K}_t}\right) \tag{2. 6}$$

$$\left(\frac{\overline{\partial Y}}{\partial L}\right)_t=\alpha_2\left(\frac{\bar{Y}_t}{\bar{L}_t}\right) \tag{2. 7}$$

$$\hat{Y}_t=\alpha_0 \bar{K}_t^{\alpha_1} \bar{L}_t^{\alpha_2} \tag{2. 8}$$

$$\bar{K}_t=\frac{1}{n}\sum_{i=1}^{n} K_{i,t} \tag{2. 9}$$

$$\bar{L}_t=\frac{1}{n}\sum_{i=1}^{n} L_{i,t} \tag{2. 10}$$

Notations

- $K_{i,t}$: the capital stock in the i-th region at time t.
- $L_{i,t}$: the labor force in the i-th region at time t.
- $Y_{i,t}$: the output in the i-th region at time t.
- η : the adjustment coefficients of the interregional movement of capital.
- ρ : the adjustment coefficients of the interregional movement of labor.
- $\bar{K}_t$: overall average inputs of capital at time t.
- $\bar{L}_t$: overall average inputs of labor at time t.
- $\hat{Y}_t$: macroscopic average output at time t.
- $\lambda_{i,t}$: the divergences of the marginal productivities of capital at time t.
- $\pi_{i,t}$: the divergences of the marginal productivities of labor at time t.
- γ : the average rates of growth of capital.
- μ : the average rates of growth of labor.

The mechanism of the equations mentioned above is interpreted as follows:

(1) We abtain $\left(\frac{\Delta K_{i,t}}{K_{i,t-1}}, \frac{\Delta L_{i,t}}{L_{i,t-1}}\right)$ according to the divergences of the marginal productivities of capital and labor at time $t-1(\lambda_{i,t-1}, \pi_{i,t-1})$

(2) We obtain $\left\{\left(\frac{\partial Y}{\partial K}\right)_{i,t}, \left(\frac{\partial Y}{\partial L}\right)_{i,t}\right\}$ according to $(\Delta K_{i,t}, \Delta L_{i,t})$ and $(K_{i,t}, L_{i,t})$

(3) We obtain $(\lambda_{i,t}, \pi_{i,t})$ according to the overall average input of capital and labor at time $t(\bar{K}_t, \bar{L}_t)$, macroscopic average output at time $t(\hat{Y}_t)$ and $\left\{\left(\frac{\bar{Y}}{K}\right)_t, \left(\frac{\bar{Y}}{L}\right)_t\right\}$

In the model, the marginal rates of growth of capital and labor in the developed region must decrease according to moving the capital and labor from the developing region to developed region for decreasing the regional income disparities such as

$$\left\{\frac{1}{N}\Sigma\left[\left(\frac{Y}{L}\right)_{i,t}-\left(\frac{\bar{Y}}{L}\right)_t\right]^2\right\}^{\frac{1}{2}}$$

Then, the globally stable condition is if and only if,

$$\alpha_1(\gamma-\mu)<\alpha_1\rho+\alpha_2\eta<2+\alpha_1(\gamma-\mu) \tag{2.11}$$

The application of the analysis to Japanese economy in the 1950's and early 1960's revealed that in spite of an increase in and which is favorable for the reduction of income disparities, the stability condition was not satisfied because of too high a rate of growth.

Fukuchi attempted to extend this model to a corresponding weak and strong stability conditions of per-capita income in a multi-regional economy.[2]

2.2 *Mera model*[3]

This model is assumed to be perfectly mobile interregionally, i. e., the physical location of every unit of labor and capital can be changed instantaneously without cost. In such a system physical distribution of productive factors is determined for any instant of time by the profit maximizing motive.

The growth path that maximizes the aggregate efficiency of the system, z^1. The growth is determined by soluing the following problem: To maximize

$$z^1=\lim_{T\to\infty}\int_0^T \frac{(1-s)\,[A_1F_1(L_1, K_1)+A_2F_2(L_2, K_2)]}{L_1+L_2}\, e^{-\rho t}dt \tag{2.12}$$

subject to

$$L=L_1+L_2=(L_{10}+L_{20})\,e^{\lambda t} \tag{2.13}$$

$$K=K_1+K_2 \tag{2.14}$$

$$\dot{K}=s\left[A_1F_1(L_1, K_1)+A_2F_2(L_2, K_2)\right]-\delta(K_1+K_2),\ K_1, K_2\geq 0 \tag{2.15}$$

$$0\leq s\leq 1 \tag{2.16}$$

$$K_1(0)+\mathrm{K}_2(0)=K_0 \tag{2.17}$$

Notations

K_i: the capital located all in region i. $(i=1, 2)$

L_i : the labor located all in region i. $(i=1, 2)$
F_i : the production function of i region. $(i=1, 2)$
A_i : the technology efficiency of i region $(i=1, 2)$
δ : the depreciation expense rate.
s : the saving rate.

Compare the above problem with one of a system of two isolated regions without subsidy programs. The aggregate efficiency of the system of isolated regions is defined by :

$$z^2=\int_0^\infty \frac{(1-s_1)\, A_1F_1(L_1, K_1)+(1-s_2)\, A_2F_2(L_2, K_2)}{L_1+L_2}\, e^{-\rho t}dt \tag{2. 18}$$

Let

$$s=s_1\, q+s_2(1-q) \tag{2. 19}$$

$$q=\frac{A_1F_1}{A_1F_1+A_2F_2} \tag{2. 20}$$

Next, three additioned constraints is as follows :

$$L_1=L_{10}e^{\lambda t} \tag{2. 21}$$

$$-\delta K_1<\dot{K}_1<A_1F_1-\delta K_1 \tag{2. 22}$$

$$K_1(0)=K_{10} \tag{2. 23}$$

Therefore, the aggregate efficiency is greater with mobile factors than with immobile factors

$$z^1 \geq z^2$$

Furthermore, Mera tried to analyze the disparities of regional income arising from the regional production functions. The case of Japan can be concluded that when per worker incomes are equalized among prefectures or major sectors and regions through the redistribution of social capital, the national income would be less by about 30 percent in the short run and by about 12 percent in the long run than would be achieved through the traditional distribution of social capital, and that the national income cannot be increased much by redistributing social capital from the observed patern. Therefore, although the interregional income equalization is a difficult task and costly in terms of foregone productions, the improvements in interregional income differential can be achieved without sacrificing much in the aggregate production by a proper mix of available policy measures.

3. Rahman-Type Model

Rahman-type model is treated by using some optimal techniques arising from Rahman model. This chapter reports four models such as Rahman model, Sakashita model, Yamamura model and Fujita model.

3.1 *Rahman model*[4,5]

This first oriented study of theoretical research on the regional income disparities is treated by a centralized economic model.

The aim of this model is to analyze, in aggregative terms, the logic of regional allocation of investment in a two-region economy where the following conditions hold :

(1) the central control and planning of investment are aimed at maximizing the national income at the end of the planning period.
(2) the regional sentiments demand that the process of economic growth should not bring about any wide disparity in the regional living standards.
(3) the productivity of investment and the rate of saving both differ in the two regions.
(4) the planned saving equals planned investment through central direction.

From the condition (4), we obtain the following equation.

$$\frac{(x_{t+1}-x_t)}{k_1}+\frac{(y_{t+1}-y_t)}{k_2}=s_1x_t+s_2y_t \qquad (t=1, 2, \cdots, T) \tag{3.1}$$

Notations

x_t : the regional income of region 1 at time t.
y_t : the regional income of region 2 at time t.
k_1 : the productivity of investment of region 1.
k_2 : the productivity of investment of region 2.
s_1 : the saving ratio of region 1.
s_2 : the saving ratio of region 2.
z_t : the national income at time t.

In the equation (3.1), the left-hand side represents the total investment and the right-hand side represents the total saving in the whole country at time t. And all the coefficients s_1, s_2, k_1 and k_2 are positive. Further, without loss of generality we assume region 1 to be the more productive of the two, so that to get a given increase of income less investment is required in region 1 than in region 2.

In other words, $k_1>k_2$. (3.2)

The non-disinvestment constraints is expressed as :

$$x_{t+1}\geq x_t \quad \text{and} \quad y_{t+1}\geq y_t \tag{3.3}$$

From the condition (2), the political constraints are expressed in the following form :

$$(y_{t+1}/x_{t+1})\geq\gamma_1 \quad \text{and} \quad (x_{t+1}/y_{t+1})\geq\gamma_2\ ;\ 0<\gamma_1, \gamma_2<1 \tag{3.4}$$

The problem is the maximize $z_T=x_T+y_T$, subject to the condition (3.2), (3.3) and (3.4).

The following proposition are analyzed by the application of Bellman's Principle of Optimality.

(1) In general, the optimum income combination lies at an extreme every time.

(2) If $s_1k_1 > s_2k_2$, then the optimum program favors the more productive region 1, throughout the entire plan- period.

(3) In order that the optimum program may favor the less productive region 2, in any time at all, s_2k_2 must exceed s_1k_1.

(4) If in any particular time the optimum position favors the less productive region, then in all earlier times, if any, the same region must be favored.

(5) Given $s_2k_2 > s_1k_1$, and a plan-period sufficiently large, the optimum program must favors the less productive region in a number of initial times.

3.2 *Sakashita model*[6]

Rahman model was modified into a mixed economy which consists of private and public sectors in the case of two-region by Sakashita model.

This model is considered a two-region economy is which the central government can impose current income taxes within a certain limit anytime it wishes.

The equations of Sakashita model are as follows;

$$\frac{dx}{dt} = \sigma_1\lambda(1-r)(s_1x+s_2y)+\delta_1ur(s_1x+s_2y) \tag{3.5}$$

$$\frac{dy}{dt} = \sigma_2(1-\lambda)(1-r)(s_1x+s_2y)+\delta_2(1-u)r(s_1x+s_2y) \tag{3.6}$$

Notations

x: the regional income of the first region.

y: the regional income of the second region.

c_i: the average propensities of consumption in region $i\,(i=1, 2)$

s_i: the saving ratio in region i, $(i=1, 2)$.

σ_i: the incremental output-capital ratio of the private investment in region $i(i=1, 2)$.

δ_i: the incremental output-capital ratio of the public investment in region $i(i=1, 2)$.

λ: the proportion of private investment shared to the first region.

u: the proportion of public investment shared to the first region.

and

$$\begin{cases} 0 \leq r \leq \theta \\ 0 \leq \lambda \leq 1 \\ 0 \leq u \leq 1 \end{cases} \tag{3.7}$$

σ_i, δ_i, s_i and λ are constant coefficients and u and r are instrumental variables for the central government.

By the adoption of new variables, $\theta_1 = ur$ and $\theta_2 = (1-u)r$, the system of equations (3.5) and (3.6) is rewritten as

$$\frac{dx}{dt}=\left\{\sigma_1\lambda(1-\theta_1-\theta_2)+\delta_1\theta_1\right\}(s_1x+s_2y) \tag{3.8}$$

$$\frac{dy}{dt}=\left\{\sigma_2(1-\lambda)(1-\theta_1-\theta_2)+\delta_2\theta_2\right\}(s_1x+s_2y) \tag{3.9}$$

where

$$0\leq\theta_1, \qquad 0\leq\theta_2, \qquad \theta_1+\theta_2\leq\theta \tag{3.10}$$

The target is to maximize the national income at the final time period, $t=T$, starting with a set of initial conditions, x_0 and y_0 at $t=0$ under the constraints of (3.8) and (3.9).

$$\text{Maximize } \left\{x(T)+y(T)\right\}=\int_0^T\left(\frac{dx}{dt}+\frac{dy}{dt}\right)dt+(x_0+y_0) \tag{3.11}$$

This problem should be solved by the use of the Pontrijagin maximum principle. The Hamiltonian is defined as follows;

$$H=-\psi_0\left(\frac{dx}{dt}+\frac{dy}{dt}\right)+\psi_1\frac{dx}{dt}+\psi_2\frac{dy}{dt} \tag{3.12}$$

where

ψ_0, ψ_1 and ψ_2 are auxiliary variables whose behaviors are given by

$$\begin{cases}\dfrac{d\psi_0}{dt}=0\\[2mm] \dfrac{d\psi_1}{dt}=-\dfrac{\partial H}{\partial x}\\[2mm] \dfrac{d\psi_2}{dt}=-\dfrac{\partial H}{\partial y}\end{cases} \tag{3.13}$$

At $t=T$, we see that $\psi_0(T)=-1$ and $\psi_1(T)=\psi_2(T)=0$ by the transversality conditions at the terminal point. (3.12) and (3.13) are rewrited by the introduction of new auxiliary variables, $\phi_1=1+\psi_1$ and $\phi_2=1+\psi_2$.

$$\begin{aligned}H=\Big\{&[\sigma_1\lambda\phi_1+\sigma_2(1-\lambda)\phi_2]+[(\delta_1-\sigma_1\lambda)\phi_1-\sigma_2(1-\lambda)\phi_2]\,\theta_1\\ &+[-\sigma_1\lambda\phi_1+(\delta_2-\sigma_2(1-\lambda))\,\phi_2]\,\theta_2\Big\}(s_1x+s_2y)\end{aligned} \tag{3.14}$$

$$\begin{cases}\dfrac{d\phi_1}{dt}=-s_1\Big\{[\sigma_1\lambda(1-\theta_1-\theta_2)+\delta_1\theta_1]\,\phi_1+[\sigma_2(1-\lambda)(1-\theta_1-\theta_2)+\delta_2\theta_2]\,\phi_2\Big\}\\[2mm] \dfrac{d\phi_2}{dt}=-s_2\Big\{[\sigma_1\lambda(1-\theta_1-\theta_2)+\delta_1\theta_1]\,\phi_1+[\sigma_2(1-\lambda)(1-\theta_1-\theta_2)+\delta_2\theta_2]\,\phi_2\Big\}\end{cases} \tag{3.15}$$

The two equations in (3.15) are mutually dependent, so that ϕ_1 and ϕ_2 are linearly dependent on each other.

If the following relative sizes of parameters are assumed, the following properties of the optimum solution to the regional allocation problem are observed.

$$\begin{cases} \delta_1 < \delta_2 < \sigma_2 < \sigma_1 \\ s_1\delta_1 < s_1\sigma_1 < s_2\delta_2 < s_2\sigma_2 \\ s_1\sigma_1\lambda + s_2\sigma_2(1-\lambda) < s_2\delta_2 \end{cases} \tag{3.16}$$

(1) As indicated by assumptions (3. 16), the public investment in one of the regions will be justified during some phase of the planning period, even if it is less productive than the private investment in the same region. The reason is that the central government can directly control the interregional flow of public funds outside the market mechanism and can concentrate its public investment on the region with greater growth potential.

(2) If T is very large, the public investment should be almost persistently concentrated to the second region. as far as there is no change in the value of relevant parameters.

(3) The resultant optimum solution is very sensitive to the relative sizes of such parameters as s_i, σ_i, $\delta_i\,(i=1,2)$, and λ. Then, if there is some slight change in them by means of social overhead investment, we may observe a drastic change of the optimum solution.

This model was extented into n-region model by Ohtsuki, Y.[7]

3. 3 *Yamamura Model*[8,9]

Rahman model was extended into the regional economic growth model under the condition that any wide of the regional income disparities did not result at the end of the planned period by Yamamura model.

The mathematical formation of this model holds the following conditions.

(1) The allocation of regional investment is aimed at maximizing the national income under the condition that the terminal regional incomes at the planned period are equalized.

(2) The supply of funds for investment will be limited to the sum of savings in each region.

(3) The productivity of investment, saving ratio and local autonomy rate are given through the central government.

(4) The investment for the dissolution of the maximum income disparity is given by the mutual consents of all regions.

The performance equations from condition (2) are as follows:

$$\sum_{j=1}^{M}(X_j^i - X_j^{i-1})/P_j^i = \sum_{j=1}^{M} S_j^{i-1} X_j^{i-1} \tag{3.17}$$

$$(i=1,2,\cdots,N)$$

where

$$X_j^i - X_j^{i-1} = P_j^i U_j^i (1-r)\left(\sum_{j=1}^{M} S_j^{i-1} X_j^{i-1}\right) + P_j^i r S_j^{i-1} X_j^{i-1} \tag{3.18}$$

The boundary condition are as follows:

$$C_j = X_j^0 \tag{3.19}$$

$$D_j^i \leq U_j^i \leq 1 - \sum_{k \neq j} D_k \tag{3.20}$$

The performance equations from condition (1) are as follows:

$$X_1^N = \cdots\cdots\cdots\cdots\cdots X_M^N \tag{3.21}$$

$$J = Z^N \to \text{Max}\left(Z^N = \sum_{j=1}^{M} X_j^N\right) \tag{3.22}$$

Notations

P_j^i: the productivity of investment of region j at time i.

S_j^i: the saving ratio of region j at time i.

U_j^i: the proportion of investment shared to the region j at time i.

$$\left(\sum_{j=1}^{M} U_j^i = 1, \qquad i = 1, 2, \cdots, N\right) \tag{3.23}$$

r: the local autonomy rate.

M: the number of regions.

N: the plan period time.

X_j^i: the regional income of region j at time i.

$$(X_j^i - X_j^{i-1}) \geq 0, \qquad (i = 1, 2, \cdots, N, \ j = 1, 2, \cdots, M) \tag{3.24}$$

$C_j = X_j^0$: the regional income of region j at initial time.

D_j^i: the minimum porportion of investment of region j at time i.

$$(0 \leq D_j^i \leq 1/M) \tag{3.25}$$

z^i: the national income at time i.

$$\left(z^i = \sum_{i=1}^{M} X^i, \qquad i = 1, 2, \cdots, N\right) \tag{3.26}$$

$$\text{Min } X_j^1 : \left\{(1 + rP_j^1 S_j^0)\, X_j^0 + P_j^1 (1-r) \left(\sum_{j=1}^{M} S_j^0 X_j^0\right) D_j^1\right\} \tag{3.27}$$

$$\text{Max } X_j^1 : \left\{(1 + rP_j^1 S_j^0)\, X_j^0 + P_j^1 (1-r) \left(\sum_{j=1}^{M} S_j^0 X_j^0\right) (1 - \sum_{k \neq j} D_k)\right\} \tag{3.28}$$

D_r: the limit point of controllability.

$[0, D_r]$: the feasible region of controllability.

The main results are as follows.

(1) Assume that $S_j^i = S_j$, $D_j^i = D_j$ and $P_j^i = P_j (i = 1, 2, \cdots, N, \ j = 1, 2, \cdots, M)$. The controllability of the minimum proportion of investment (D_j) is not realized if at least one of the following two cases such that $X_j^1 \geq \text{Min } X_j^1$ and $X_j^1 \leq \text{Max } X_j^1$ does not hold.

(2) Assume $P_1 > P_2$, $S_1 = S_2$, $C_1 = C_2$, when the disparity of productivity of investment between P_1 and P_2 increases on the local autonomy rate increases, the limit point of controllability (D_r) decreases and also the feasible region of controllability of D_r decreases.

(3) Assume $P_1 > P_2$, $S_1 = S_2$, $C_1 > C_2$, when the disparity of the regional income at

initial time between C_1 and C_2 increases the plann period time N decreases, the limit point of controllability (D_r) decreases and also the feasible region of controllability of D_r decreases.

The discrete formation of this regional economic growth model was reformed to a continous case formation without essential difficulty,[10] and to a more generalized model for the redistribution policy of population.[11]

3.4 *Fujita Model*[12]

Fujita model was developed a generalization of the Rahman model by introducing scale economies and diseconomies in the production process.

The mathematical formations are as follows;

$$\text{Maximize} \quad \sum_{i=1}^{n} F\Big(K_i(T)\Big) \tag{3.29}$$

subject to

$$\dot{K}_i(t) = \theta_i(t)\, s \sum_{j=1}^{n} F\Big(K_j(t)\Big) \qquad i=1, 2, \cdots, n \tag{3.30}$$

$$\sum_{i=1}^{n} \theta_i(t) = 1\,, \qquad \theta_i(t) \geq 0\,, \qquad i=1, 2, \cdots, n,\ 0 \leq t \leq T \tag{3.31}$$

$$K_i(0) = K_i^0\,, \qquad i=1, 2, \cdots, n \tag{3.32}$$

Notations

$Y_i(t)=F(K_i(t))$: the regional production function of region i.

$K_i(t)$: the amount of capital in region i at time t.

K_i^0 : the amount of capital in region i at the initial time, $t=0$,

$\theta_i(t)$: the values of investment ratio of region i at time t.

s : saving ratio.

Chose values of investment ratio. $\theta_i(t)$, $i=1, 2, \cdots, n$, at each time so as to maximize the total income in the final period.

The alternative assumption on the form of regional production function F are as follows:

(i) decreasing return to scale, namely,
$F'(K)>0$ and $F''(K)<0$ for all $K \geq 0$.

(ii) constant returns to scale, namely,
$F(K)=aK$ for all K where a is a positive constant.

(iii) increasing returns to scale, namely,
$F'(K)>0$ and $F''(K)>0$ for all K.

(iv) variable return to scale, namely,
$F'(K)>0$ for all $K \geq 0$, and $F''(K)>0$ for $o \leq K \leq K^*$,
$F''(K)=0$ for $K=K^*$, $F''(K)<0$ for $K>K^*$.

In addition, assume in all cases that $F(0) \geq 0$ and $F(K)$ is twice continously differentiable at each $K>0$.

To obtain the optimality conditions, the following Hamiltonian function H is defined by applying the maximum principle.

$$H\Big((P_i)_1^n, (K_i)_1^n, (\theta_i)_1^n\Big) \equiv \sum_{i=1}^{n} P_i K_i = \Big(\sum_{i=1}^{n} \theta_i P_i\Big)\Big(s \sum_{i=1}^{n} F(K_i)\Big) \tag{3.33}$$

Suppose that $\{(K_i(t))_1^n\}_0^T$ and $\{(\theta_i(t)_1^n\}_0^T$ are respectively, an optimal growth path and an optimal allocation path. Then there exits a price path $\{(P_i(t))_1^n\}_0^T$ which satisfied the next three conditions.

a) $H((P_i(t)_1^n,(K_i(t)_1^n,(\theta_i(t))_1^n)=\max\{H(P_i(t))_1^n,(K_i(t))_1^n,(\theta_i)_1^n/\sum_{i=1}^{n}\theta_i=1,\theta_i\geq 0,i=1,2,\cdots,n\}$

for $0\leq t\leq T$ (3. 34)

(b) $\dot{P}_i(t)=-\partial H((P_i(t))_1^n,(K_i(t))_1^n,(\theta_i(t))_1^n)/\partial K_i$

for $i=1,2,\cdots,n$ and $0\leq t\leq T$ (3. 35)

(c) $P_i(T)=F'(K_i(T))$ for $i=1,2,\cdots,n$

where $\dot{P}_i(t)=dP(t)\ dt$ (3. 36)

represents the incremental amount of the final income which is obtained by exogenously increasing the stock of capital in region i by one unit at time t on the optimal path. Namely, $P_i(t)$ represents the accounting price (shadow price) of a unit of capital in region i at time t in terms of the final income.

According to (3. 34), the optimal rule for investment allocation is given by

$$\sum_{i\in\mathscr{L}(t)}\theta_i(t)=1 \tag{3.37}$$

where

$$\left.\begin{aligned} &P(t)=\max_i P_i(t)\\ &\mathscr{L}(t)=\Big\{i|P_i(t)=P(t)\Big\}\end{aligned}\right\} \tag{3.38}$$

That is, at each time, all of the investment fund of the economy should be allocated among regions which have the highest price of capital at that time.

According to (3. 35),

$$\dot{P}_i(t)=-\Big(\sum_{j=1}^{n}\theta_j(t)\,P_j(t)\Big)sF\ \Big(K_i(t)\Big) \tag{3.39}$$

$$i=1,2,\cdots,n,\text{ at each } t,\ 0\leq t\leq T$$

From (3. 38) to (3. 39),

$$\dot{P}_i(t)=-P(t)\,sF'(K_i(t)),\qquad i=1,2,\cdots,n \tag{3.40}$$

In any of the cases (i) to (iv), $P_i(t)>0$ and $\dot{P}_i(t)<0$ for each i, $i=1,2,\cdots,n$, and all $t\geq 0$.

Furthermore, Fujita tried to analyze the case of n regions and variable returns to scale, and concluded the following results. If the investment allocation is free in the competitive economy, the allocation of investment will be switched from the more developed region to the less developed region only after the more developed region becomes so congested that agglomeration economies accumulated in the initial stages are completely outweighed by agglomeration diseconomics accumulated in the later stages. To achieve the optimal growth path in the competitive economy the government should improve a sufficiently high congestion tax on investment in the more developed region so as to discourage the over-accumulation of capital.

4. Model Reference Adaptive Model[13),14),15),16),17)]

Economic development, a branch of economics, may be defined as the quantitative analysis in its relation to control science and the scope of applications of it has broadened greatly to the economic development.

However, only a relatively small amount of research has been made on the use of adaptive control techniques. It was R. E. Murphy, Jr. who first oriented the study on adaptive process in economic systems. And he treated one of the adaptive techniques such as stochastic information control techniques.

Among various altternative method for adaptive control technique, the use of the technique known as Model reference adaptive technique was oriental one of the most feasible approaches possible for the regional economic growth analysis by Yamamura, E.

The basic scheme of a Model reference adaptive system is called a parallel model reference adaptive system in order to differentiate it from other Model reference adaptive model configurations.

The adaptation of this type is due to the fast that a measure of the difference between the given index of performance specified by the reference model and the index of performance of the adjustable system is obtained directly by the comparison of states of the model with those of the adjustable model.

Model reference adaptive system :

Reference model

$$y_M = f_M(u, P_M, x_M, t)$$

Where P_M are parameters of the reference model.

$$y_s = f_s(u, P_s, x_s, t)$$

Where P_s are parameters of the adjustable model.

Index of performance which expresses the difference between the given index of performance specified by the model and that of the adjustable model.

$$J_{IP} = F(\varepsilon, P_M - P_s, e, t)$$

Where

$$\varepsilon = y_M - y_s$$
$$e = x_M - x_s$$

Model reference adaptive system minimizes the J_{IP} defined above by parameter adaptation or by signal-synthesis adaptation via the adaptation mechanism that his the generalized error as one of its inputs.

The typical economic development system is such as the reference model specified by the developed region economy and the developing region economy specified by the adjustable model.

The aim of this economic development system is how to develop the economic level form the developing region economy to the developed region economy. Namely, the economic development system minimizes the J_{IP} defined above by parameter adaptation or by signal-synthesis adaptation via the adaptation mechanism.

The patameter adaptive is represented in an industrial structure change to develop the developing region industrial structure such as the developed region economy.

The signal-synthesis adaptation is represented in an allocation changes of investments between industrial sectors to develop the output of developing region economy to the same level of developed region economy.

The another typical economic policy system is such as the real economic system specified by the reference model and the economic planning model specified by the adjustable model and also the reverse specification.

The aim of this economic policy system is how to control the index of performance which expresses the difference between the given index of performance specified by real economic system and that of the economic planning model. Namely, the economic policy system minimizes the J_{IP} defined above by parameter adaptation or by signal-synthesis adaptation via the adaptation mechanism.

The parameter adaptation is represented in an minimization of any norm of the difference between the state sector of real economic system and the economic planning model.

The signal-synthesis adaptation is represented in an input control that minimizes the difference between the output of the real economic-system and the economic planning model.

There are also many economic control studies for various applications of Model reference adaptive techniques.

5. Conclusion

We considered the Neoclassical-type model, the Rahman-type model and the Model reference adaptive model.

Among three models, the Model reference adaptive technique is conceived to be a useful method in specifying the quantitative analysis of the adaptive process of regional economic development. And this model was extended the adaptive process of the regional economic development by using the Model reference adaptive technique with the uncertainty concerning the values of the adjustable model's parameters, and also the parametric identification and adaptive state observation.[15,16]

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