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A Location-Allocation Model for Health Care Services Planning

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Abstract

Health care services is an indispensable basic living condition for community welfare as well as personal well-being. In these days, people's demand for health care services is more and more increasing and diversifying. However, actually, health care services are not always equally allocated, socially and spatially, and that inequality in the access opportunity to health care services tends to expand. In this peper, a location-allocation model for regional health care services planning is presented which can be used to analyze the problems of the equity-revenue trade-off relation in the spatial provision of health care services in a free-entry and free-for-service market system. This modelling approach can also be available for regional provision planning of other various social services provided by private sector.

Key Words: Location-allocation analysis, Regional health care service planning, Efficiency-equity trade-off, Equity-revinue maximizing conflict, Spatial interaction, Attractiveness, Accessibility, Health care resources, Location behaviour.

1. Introduction

One of the most important issues that the present-day society is now confronted with is how to manage health care services provision. Here, the expression of 'how to manage' implies two meanings in a broad way. One is to provide the services efficiently by the request from supply-side and another is to secure the equity of access opportunity to the services by the request from demand-side. The relation of these two requests is known well as efficiency-equity trade-off problem.

However, when the society have ample resources for the services or consumers are not sensitive to the services, the efficiency-equity trade-off relation does not become manifest. The emergence of the 'Welfare State' or 'Big Government' is against the social background like the former, and the latter case can be seen in the early years of industrial society.

It goes without saying that social situations of today are quite different from those days. Monetary, physical and human resources to provide the services cannot be adequately prepared because of slowdown in economic growth on the one hand

and people's demand for the services is more and more increasing and diversifying on the other hand. As a result, efficiency-equity trade-off relation now become so obvious for everybody as to be a prime political issue for local governments as well as national governments in the advanced industrial countries such as Japan, U. S. A. and other western countries.

In recent years, many researchers in various fields have turned to their interests to the above-mentioned problems. Geographers and regional scientists are devoting themselves to location-allocation analysis as a planning task for solving the combinatorial problem of the efficiency-equity trade-off in the services provision. The location-allocation analysis is generally defined as an analysis of optimally selecting the locations for a set of service points so as to best serve a set of demand areas, and concurrently finding the allocation of each demand area to that service points which will serve it best (e. g. Ostresh 1975, Smith et al. 1977, Hodgson 1978, Beaumont & Sixsmith 1984).

Location-allocation analysis itself have been studied since 1960's, which has been developed from location theories presented by von Thünen, A. Weber, Lösch, Alonso, and so on, and from allocation problems developed in operations research. Therefore, location-allocation analysis is sometimes done under the title of location analysis or allocation analysis (Francis and Goldstein 1974). Various location-allocation models have been developed and applied to a wide variety of planning activities both in private and public sectors. For example, siting warehouse (Geoffrion 1975), corporate receivable collection systems (Cornuejols et al. 1977), emergency facilities (Toregas and ReVelle 1972, ReVelle et al. 1977), rural services (Fisher and Rushton 1977, Askew 1983), retail facilities (Coelho and Wilson 1976, Zeller et al. 1980), urban public facilities (ReVelle and Church 1977, Hodge and Gatrell 1976), day care facilities (Holmes et al. 1972). Among these many subjects of the application of location-allocation models, public services, especially health care services, are now the most fascinating for regional scientists (Church & Stimson 1983, Clarke 1984).

The purpose of this paper, taking into account the above-mentioned social situation and the lalest scholarship, is to discuss a modelling approach to location-allocation analysis for regional health care services planning in a free-entry and fee-for-service market framework.

2. Fundamental Framework of a Model System

2-1. *Social and spatial inequality in health care services*

Health care services is necessary in particular for human well-being among many public services. Because the quality and quantity of health care services have a strong influence on not only one's life but also people's living ability. Nowadays nobody denies the importance of health care services for human well-being and, health and health care are given sense of fundamental human right (Oda 1980, Daniels 1985). However, actually, health care services are not always equally allocated, socially as well as spatially.

In Japan, the problems of equal allocation of health care services had been discussed under the expression of socialization of health care services since 1910's (Saguchi 1982). The term 'socialization' in this case was used to indicate the diffusion of personal health care services in a broad meaning, and not necessarily used to imply the so-called 'socialized medicine'. The modes of provision of health care services in modern Japan after the Meiji Restoration in 1868 was characterized as the noninterfering free consultation of private practitioners, that is, fee-for-service practitioner system (Kawakami 1965). The social movement of socialization of health care services aimed to reform the socio-economic and spatial inequalities in the opportunity of enjoying health care services which was the harmful effects caused by the fee-for-service practitioner system.

The first public intervention to the service system was the enactment of Health Insurence Law in 1922 (enforced in 1927). It was the first social insurence in Japan and covered only three percent of total population (Sugaya 1982). After that, various health insurence were enacted and realized universal joining in any health insurence in 1961. At present, though not perfect, the equalization of the opportunity of using health care services between socio-economic strata is secured by means of various health security systems.

However, there is no system for securing spatial equality in access opportunity to health care services. Accordingly, even if stratum equality in the use of health care services are secured, substantial equality is not realized under the circumstance of the existence of spatial inequality. The typical case is found in physicianless areas. When people living in those areas visit physicians, they are obliged to pay more time and mony costs. Patients transport cars/ships and travelling clinics for remote areas and islands are alternative measures to compensate such inequality. However, in the usual personal health care services, patients move to the locational points of health care facilities in order to be served. Therefore, the problem of spatial inequality in health care services results in a question of locational pattern of health care resources such as physicians as human resources and hospitals as physical resources.

The same arguments are found in some preceeding works (e. g. Hodg and Gatrel 1976, Lea 1979). Lea notes from the viewpoint of spatial welfare theory,

> "The prices vary over individuals in accordance with their location relative to public facilities and the prices are not part of any institutional exclusionary device. The good may still be entirely nonexclusive (as this expression is normally used), however consumers differentially reject the good because of differentially borne private costs. Such goods are not pure public goods, despite their properties of nonexclusion and full jointless ; they are impure public goods because of their spatial context."

Bigman and ReVelle (1979) categorize the public objectives for performing spatial welfare, in other words, spatial equality in the access opportunity to public services, into the following three groups and introduce some works ;

(1) minimize the average time or distance of travelling to the facility (Hakim

1964, ReVelle et al. 1970, ReVelle and Swain 1970), (2) maximize the performance of the public facility as measured by minimizing the maximal travel distance (ReVelle et al. 1970, Teitz 1968), (3) maximize the population covered within a certain distance (Church and ReVelle 1974).

Leonardi (1981) states, after reviewing the discussion on the definition of 'equity', 'welfare' and 'efficiency', that the problem behind these terms is quite simple, and then he points out, apart from technical details, that all measures of equity or welfare used in location problems are measures of nearness, ease of access, and fair distribution of service to users. The transport cost minimization, the maximum coverage and the consumer-surplus maximization are presented from his arguments as the criteria of the public objectives.

Consequently, one of the important task of regional health care services planning of today is to decide some optimal locational patterns of health care service resources which enable to disolve the spatial inequality in health care services, in other words, to realize spatially equitable locational patterns of health care service resources.

2-2. *Health care service provision in a free-entry and fee-for-service system*

Almost all the existing location-allocation models on public service are relating to 'pure' public services. Therefore, efficiency criteria of supply-side in such models are mainly minimizing of construction cost and/or operation cost of public facilities paid by public sector, or often those criteria are not adopted as objective function, or are introduced in the models as budget constraints, or they are entirely neglected. Because the main objective of public sector is regarded to be not maximization of a profit but to maximize a benefit of the general public, that is, some of social welfare.

In contrast, private sector which actually provides public services should ensure a certain amount of revenue (income) to maintain its buisiness activity and wants to make a profit as high as possible. Thus, it can be said that efficiency criterion for private sector is maximization of a profit. The typical case can be found in health care services. This cannot be neglected when the problems on regional health care services planning are examined in a free-entry and fee-for-service market system, especially in these days when private sector is expected to be an important provider of various public services.

Needless to say, if only income maximization is adopted as a efficiency criterion when a location-allocation model is build, the model is not available to solve the problems of spatial provision of public services. Church and Stimson (1983) present a modelling approach for pursuing such task within the realistic constraint of ensuring a minimum level of gross revenue (income) for all practitioners while decreasing any maldistribution of practitioner services and increasing number of people that are provided accessible services. Their idea, that is, equity-revenue maximizing conflct, is quite similar to the author's awareness of the current problems of the spatial provision of public services.

3. The Model

3-1. *Full Problem*

From the above-mentioned framework, the full problem of location-allocation analysis for the equity-revenue conflict problem in regional health care services can be written as ;

$$\text{maximize} \quad \sum_j \sum_t U_{jt}(DR_{jt}, D_{jt}) , \qquad (1)$$

subject to

$$\sqrt{H_j/\pi \cdot DR_{jt}} \leqq d, \qquad (2)$$

$$L_{jt}^{\min} \leqq L_{jt} = D_{jt}/DR_{jt} \leqq L_{jt}^{\max}, \qquad (3)$$

$$\sum_j \sum_t DR_{jt} = T, \qquad (4)$$

where,

DR_{jt} : the number of physicians of consultation subject t in zone j,
D_{jt} : total demand (the total number of patients) for t attracted in j,
H_j : total habitable land areas of j,
r : maximum travel time of patient to visit a doctor,
L_{jt} : work load of a physician of t in j,
$L_{jt}^{\min}$: minimum work load enabling the physician to continue his/her medical profession,
$L_{jt}^{\max}$: maximum work load of a physician,
T : total number of physicians.

3-2. *Allocation of demand*

The general form of a spatial interaction model of public services in a users-attracting systems can be written as follows (Leonardi 1981).

$$S_{ij} = G_i \frac{f(C_{ij})\, W_j}{\sum_j f(C_{ij})\, W_j} \qquad (5)$$

where,

i, j : subscripts labeling the locations of demand and facilities, respectively,
S_{ij} : the predicted number of customers living in location i and using the facilities in location j,
G_i : the total demand for services generated in i per unit time,
W_j : a measure of attractiveness of facilities in location j,
$f(\cdot)$: a space discount function.

Therefore, total attracted demand for facilities in location j, D_j, is given by ;

$$D_j = \sum_i S_{ij} . \qquad (6)$$

As may be understood easily, it is possible to replace the term 'facility' with 'resource' in more general. In that case, the attractiveness should be redefined in accordance with the type of resources.

3-3. *Estimation of total generated demand*

In order to estimate the amount of demand for health care services by practice type (consultation subject) in a given community, first of all, potential amount of demand by type of disease should be estimated. Because, in Japan, there is no institutionalized system of specialties of medical doctors, and that it is impossible to obtain the acurate data on the amount of demand by practice type directly.

The potential amount of disease by type of disease in a given community can be given ;

$$G^l_{id} = \sum_k P_{ik} \cdot J^l_{dk} , \qquad (7)$$

where,

- i : subscript labeling the demand location,
- d : subscript labeling the type of disease,
- k : subscript labeling the age-sex group,
- l : superscript labeling the type of services, inpatient service ($l=1$) or outpatient service ($l=2$),
- P_{ik} : population of age-sex group, k, in the community i,
- J^l_{dk} : inpatient (outpatient) consultation rate of type of disease, d, of age-sex group, k, found by the Patient Survey,
- G^l_{id} : potential amount of disease, d, in the community i.

The number of types of disease, d, is classified into eighteen based on the international classification of disease ($d=1, 2, \cdots, 18$). G^l_{id} is transformed as ;

$$\hat{G}^l_{it} = \sum_d^n G^l_{id} \cdot M_{dt} , \qquad (8)$$

where,

- t : subscript labeling the practice type (the type of consultation subjects),
- M_{dt} : matrix for transforming G^l_{id} into $\hat{G}^l_{it}$.

The number of practice type is ten ($t=1, 2, \cdots, 10$). They are internal, pediatrics, psychiatrics, surgical, obstetrics and gynecology, ophthalmology, otolaryngology, dermatology and urology, others, and dentistry. M_{dt} is a matrix of the percentage by the type of disease-practice type published in the report of the Patient Survey, which is as follows.

	t_1	t_2	………………	t_n	………………	t_{10}	100%
d_1	d_1t_1	d_1t_2	………………	d_1t_n	………………	d_1t_{10}	100%
d_2	d_2t_1	d_2t_2	………………	d_2t_n	………………	d_2t_{10}	100%
……	……	……	……	……	……	……	……
d_m	d_mt_1	d_mt_2	………………	d_mt_n	………………	d_nt_{10}	100%
……	……	……	……	……	……	……	……
d_{10}	$d_{18}t_1$	$d_{18}t_2$	………………	$d_{18}t_n$	………………	$d_{18}t_{10}$	100%

3-4. *Attractiveness in health care services*

Attractiveness, W, can be defined as follows.

$$W_{jt}=aW^1_{jt}+bW^2_{jt}\,, \tag{9}$$

$$0\leqq a\leqq 1\,, \tag{10}$$

$$0\leqq b\leqq 1\,, \tag{11}$$

$$a+b=1\,, \tag{12}$$

where,

W_{jt}: a weight measuring attractiveness of a health care resource t in j,
W^1_{jt}: the attractiveness measure expressed as resource density,
W^2_{jt}: the attractiveness measure expressed as supply capacity,
a, b: given constants, distributive proportion between W^1_{jt} and W^2_{jt}.

W^1_{jt} is the ratio of the density of health care resource t in location j, which is defined as;

$$W^1_{jt}=w^1/\sum_j w^1_{jt}\,, \tag{13}$$

where w^1_{jt} is the number of health care resources t located within distance which a patient can travel within a given time, that is,

$$w^1_{jt}=v/r\,, \tag{14}$$

$$=v/\sqrt{H_j/\pi\cdot R_{jt}}, \tag{15}$$

where,

v: travel speed an hour of a patient,
r: distance between demand point and supply point,
H_j: total habitable land areas of j,
R_{jt}: the number of health care resources k in j.

Substituting equation (15) into (13), both of travel speed of patient an hour, v, and π are eliminated, that is,

$$W^1_{jt}=w^1_{jt}/\sum_j w^1_{jt}=(v/\sqrt{H_j/\pi\cdot R_{jt}})/\sum_j(v/\sqrt{H_j/\pi\cdot R_{jt}})$$
$$=v\sqrt{\pi}\,(\sqrt{R/H})/v\sqrt{\pi}\sum_j(\sqrt{R_{jt}/H_j})=\sqrt{(R_{jt}/H_j)}/\sum_j\sqrt{(R_{jt}/H_j)}\,.$$

Thus, w^1_{jt} becomes rather simple form as;

$$w^1_{jt}=\sqrt{R_{jt}/H_j}\ . \tag{16}$$

Equation (16) indicates that W^1_{jt} is given by two factors; total area of habitable land areas and the number of health care resources, and that it is a simple increasing function of the number of health care resources as well as a simple decreasing function of total area of habitable land areas.

R_{jt} is defined here as the number of hospitals in the case of the attractiveness for in-patient care services, and total number of clinics of type t and hospitals in the case of out-patient care services, that is;

$$ {}_{I}w^{1}_{jt} = \sqrt{HOS_j / H_j} \,, \tag{17} $$

$$ {}_{O}w^{1}_{jt} = \sqrt{(CLI_{jt} + HOS_j)/H_j} \,, \tag{18} $$

where,

I, O: subscripts labeling in-patient care services and out-patient care services, respectively,

HOS_j: the number of hospitals in j,

CLI_{jt}: the number of clinics of type (consultation subject) t in j.

W^2_{jt} is a measure of congestion, which indicates that the greater is W^2_{jt}, the fewer is the opportunity of being kept waiting long to consult a doctor. W^2_{jt} is defined as;

$$ W^2_{jt} = w^2_{jt} / \sum_j w^2_{jt} \,, \tag{19} $$

and w^2_{jt} is defined as;

$$ W^2_{jt} = (Z_{jt} - D_{jt})/D_{jt} = Z_{jt}/D_{jt} - 1 \,, \tag{20} $$

where,

w^2_{jt}: the measure of the degree of capacity of services provision for the demand for the service t in location j,

Z_{jt}: the maximum amount of health care service which location j can provide (the capacity capable of accepting patients in j),

D_{jt}: the total demand attracted in location j.

Equation (20) indicates that W^2_{jt} is given by two factors; the maximum amount of health care services and the total health care demand. Then, the equation indicates that w^2_{jt} is a simple increasing function of the maximum amount of health care services as well as a simple decreasing function of the total health care demand. Z_{jt} is given by;

$$ Z_{jt} = L^{\max}_t \cdot DR_{jt} \,, \tag{21} $$

where,

$L^{\max}_t$: the maximum work load of a physician,

DR_{jt}: the number of physicians of consultation subject t in j.

The value of $L^{\max}_t$ is estimated by means of the results of the Patient Survey. According to the Survey, almost all the physicians of clinics examine 15 to 75 out-patients a day. Those numbers of out-patients can be regarded as the minimum work load of a physician of clinic, which can be regarded as the minimum number of out-patients enabling him/her to continue his/her medical profession, and the maximum work load of him/her, respectively.

In the case of hospital physicians, work load of a physician is varies with the number of beds of the hospital. For example, according to the same Survey, each hospital with 50 to 99 beds accepts about 40 to 200 out-patients a day, the number of beds per hospital physician in Hokkaido is about 34. Using those data, the minimum and maximum work load of a hospital physician by the number of beds can be estimated as shown in Table 1.

Table 1. The Minimum and Maximum Work Load of a Hospital Physician

no. of beds	L^{min}		L^{max}	
	in	out	in	out
20～29	2.5	20.0	12.5	87.5
30～ 49	3.6	32.0	20.0	70.0
50～ 99	6.0	30.0	25.5	56.5
100～199	9.0	15.0	25.5	37.5
200～299	7.5	12.5	25.5	37.5
300～499	8.0	10.0	25.5	32.0
500～699	12.0	15.0	25.5	39.0
700～899	16.0	20.0	25.5	24.0
900～	20.0	25.0	25.5	30.0

Note ; in : in-patient, out : out-patient

From the above-mentioned discussion, w^2_{jt} is re-defined in detail as ;

$$_Ow^2_{jt}=(HDR_{jt}\cdot {_M}L^{max}_{tOB}+CDR_{jt}\cdot {_C}L^{max}_t)/{_O}D_{jt}-1\,, \tag{22}$$

$$_Iw^2_{jt}=(HDR_{jt}\cdot {_M}L^{max}_{tBI})/{_I}D_{JT}-1\,, \tag{23}$$

$$_Bw^2_{jt}=B_{jt}/{_I}D_{jt}-1\,, \tag{24}$$

where,

O, I, B : subscripts labeling out-patient service, in-patient service, bed, respectively,

$_Ow^2_{jt}$: the measure of the capacity of out-patient service of type t in j,

$_Iw^2_{jt}$: the measure of the capacity of in-patient service of type t in j,

$_Bw^2_{jt}$: the measure of the capacity of beds for the service of type t in j,

HDR_{jt} : total number of hospital physicians of type t in j,

CDR_{jt} : total number of clinic physicians of type t in j,

$_ML^{max}_{tBO}$: the maximum work load of a physician of type t for out-patient service in the hospital of size B (the number of beds is B),

$_CL^{max}_t$: the maximum work load of a clinic physician for out-patient service t,

$_HL^{max}_{tBI}$: the maximum work load of a physician of type t for in-patient service in the hospital of size B,

$_OD_{jt}$: total amount of demand for out-patient service of type t in j,

$_ID_{jt}$: total amount of demand for in-patient services of type t in j,

The distributive proportion between W^1_{jt}, and W^2_{jt}, a and b in equation (9) are gained out of the results of opinion survey about the reason for not visiting a doctor in the National Health Survey. According to the Survey, the distributive proportion of "I did not visit a doctor because there is no medical facility near here." to "because I dislike to wait long to be examined by reason of congestion." is 0.31. Thus, a and b are estimated 0.31 and 0.69, respectively, and equation (9) is re-written as;

$$W_{jt}=0.31\ W^1_{jt}+0.69\ W^2_{jt}\,. \tag{9'}$$

3-5. *Accessibility to supply point of health care services*

Accessibility is given by a space discount function, $f(\cdot)$, and a weighted attractiveness, W_j. Accessibility of demand point i to supply point j, A_i, can be defined as;

$$A_i=\sum_j f(C_{ij})\ W_j\,. \tag{25}$$

A space discount function, $f(C_{ij})$, is defined as;

$$f(C_{ij})=\exp\left(-\beta_i\cdot C_{ij}\right), \tag{26}$$

where,

β_i: a space discount parameter,
C_{ij}: travel time between i and j.

Table 2. Distribution Ratio of Travel Time (unit: minute)

		~15	15~30	30~60	60~90	90~120	120~
Inpatient	Big 11 Cities	17.8%	34.4	26.7	11.1	5.6	4.4
	Urban Areas	24.7	33.9	23.7	9.2	3.4	5.1
	Rural Areas	15.2	25.3	30.9	15.7	6.2	6.7
Outpatient	Big 11 Cities	43.7	33.7	15.7	4.7	1.6	0.6
	Urban Areas	40.7	37.2	14.9	4.6	1.4	1.2
	Rural Areas	35.1	30.3	19.6	7.8	2.9	4.4
Dental in- and out-patient	Big 11 Cities	57.6	33.3	3.0	6.2	—	—
	Urban Areas	43.8	41.1	12.3	2.7	—	—
	Rural Areas	48.1	25.9	14.8	3.7	3.7	3.7

Table 3. Space Discount Parameters

		space discount parameters	coefficients of correlation
Inpatient	Big 11 Cities	0.0200	0.960
	Urban Areas	0.0204	0.882
	Rural Areas	0.0140	0.936
Outpatient	Big 11 Cities	0.0371	0.976
	Urban Areas	0.0324	0.933
	Rural Areas	0.0217	0.872
Dental in- and out-Patient	Big 11 Cities	0.0959	0.899
	Urban Areas	0.1042	0.961
	Rural Areas	0.0366	0.920

Thus, accessibility of demand point i to supply point j, A_i, is given by;

$$A_i = \sum_j \exp(-\beta_i \cdot C_{ij}) W_j . \tag{27}$$

β_i is obtained by computing the distribution ratio of patients' travel time to the medical facilities found by the National Health Survey (Table 2) by the use of exponential regression analysis. The results of the analysis is shown in Table 3.

Travel time from demand point i to supply point j, C_{ij}, is given by the following way. First of all, the shortest road distance between city (town, village) offices, d_{ij}, is found. When patient consumes health care services within his/her region, his/her travel distance, d_{ii}, is defined as;

$$d_{ii} = \sqrt{H_i/\pi \cdot R_{it}} , \tag{28}$$

where H_i is total area of habitable land areas of i, and R_{it} is the number of health care resources of type t in i. The actual distance, d_{ij}, is transformed into travel time, C_{ij}, by;

$$C_{ij} = d_{ij}/v , \tag{29}$$

v is assumed as;

$$\left.\begin{array}{ll} v = 40 & \text{for } i = j , \\ v = 60 & \text{for } i \neq j , \end{array}\right\} \quad \text{for in-patient}$$

$$\left.\begin{array}{ll} v = (1 - d_{ii}) \cdot 3 + d_{ii} \cdot 40 & \text{for } i = j \text{ and } 0 < d_{ii} \leqq 1 , \\ v = 40 & \text{for } i = j \text{ and } d_{ii} > 1 , \\ v = 60 & \text{for } i \neq j , \end{array}\right\} \quad \text{for out-patient}$$

3–6. *Locational preference of physicians*

As already stated, in a free-entry and fee-for-service market system, physicians' location behaviour is motivated mainly by income maximization. There seem to be several ways to increase their income, for example, raising a fee for medical examination or insignificantly dense examination. However, those means for increasing income are institutionally restricted by the national government as noted before in this paper. Thus, income maximization is achieved mainly/only by ensuring patients as much as they can examine. Consequently, physicians must prefer to locating in the region with much demand for health care services. Therefore, the general form of utility function for physicians can be written as follows;

$$U_{jt}(L_{jt}) = U_{jt}(D_{jt}/DR_{jt}) , \tag{30}$$

where,

- j, t: subscripts labeling the location of physician and the practice type, respectively,
- $U_{jt}(L_{jt})$: a utility function,
- D_{jt}: total number of patients in j,
- DR_{jt}: total number of physicians in j,
- L_{jt}: work load of a physician (the number of patients per physician).

When work load amounts to the maximum one, $U_{jt}(L_{jt})$ is defined as;

$$U_{jt}(L_{jt})=1 \qquad \text{for } L_{jt}\geqq L_{jt}^{\max}, \tag{31}$$

where $L_{jt}^{\max}$ is the maximum work load explained in the previous section. Even if the number of patients, D_{jt}, increase, physician can not accept more patients than now because of reaching to upper limit of his work load, so that his utility does not increase.

When the current work load is within the interval, $0<L_{jt}<L_{jt}^{\max}$, physician's utility is directly influenced by D_{jt} and DR_{jt}. Thus,

$$\begin{aligned} U_{jt}(L_{jt}) &= \frac{D_{jt}-DR_{jt}\cdot L_{jt}^{\min}}{DR_{jt}(L_{jt}^{\max}-L_{jt}^{\min})} \qquad \text{for } 0<L_{jt}<L_{jt}^{\max}, \\ &= \frac{1}{L_{jt}^{\max}-L_{jt}^{\min}}\cdot\frac{D_{jt}}{DR_{jt}}-\frac{L_{jt}^{\min}}{L_{jt}^{\max}-L_{jt}^{\min}}, \end{aligned} \tag{32}$$

where $L_{jt}^{\min}$ is the minimum work load.

Letting $1/(L_{jt}^{\max}-L_{jt}^{\min})$ and $L_{jt}^{\min}/(L_{jt}^{\max}-L_{jt}^{\min})$ be a_{jt} and b_{jt}, respectively, equation (4-33) is rewritten as;

$$U_{jt}(L_{jt})=a_{jt}\frac{D_{jt}}{DR_{jt}}-b_{jt} \qquad \text{for } 0<L_{jt}<L_{jt}^{\max}. \tag{33}$$

In order to practice there, a new physician will evaluate his location by the total amount of health demand (total number of patients) generated and attracted in the region. However, all the patients in the region are not always his customers. His/her market area is generally restricted by patient's travel time to his/her office. Thus, letting the distance within the maximum travel time of patient be r, his/her market area is defined as a circle with the radius of r, that is, πr^2.

According to the National Health Survey, almost all the patients seldom spent over two hours for travel to visit a physician (almost 100% within two hours for any medical care services). In this paper, the travel time which places at 75% in the cumulative percentage is adopted as the maximum travel time of patients (Table 4).

Table 4. Maximum Travel Time of Patient (unit: minute)

	Big 11 Cities	Urban Areas	Rural Areas
Inpatient	56	50	67
Outpatient	28	28	45
Dental patient	23	25	30

Letting the market area of a physician be DA_{jt}, DA_{jt} can be regarded as part of habitable land areas, H_j, in the region. Thus, the utility of a new physician can be defined as;

$$U_{jt}=a_{jt}\frac{D_{jt}\cdot DA_{jt}/H_j}{1+DR_{jt}\cdot DA_{jt}/H_j}-b_{jt}, \tag{34}$$

Therefore, the locations for new physicians are decided by ;

$$\underset{j \in Q}{\text{maximize}}\ U_{jt}, \tag{35}$$

where Q is a set of regions.

Figure 1 shows the overall process to solve the problem. The authors owe a great deal to Leonardi (1981) for the idea of the structure of this model system shown in the Figure.

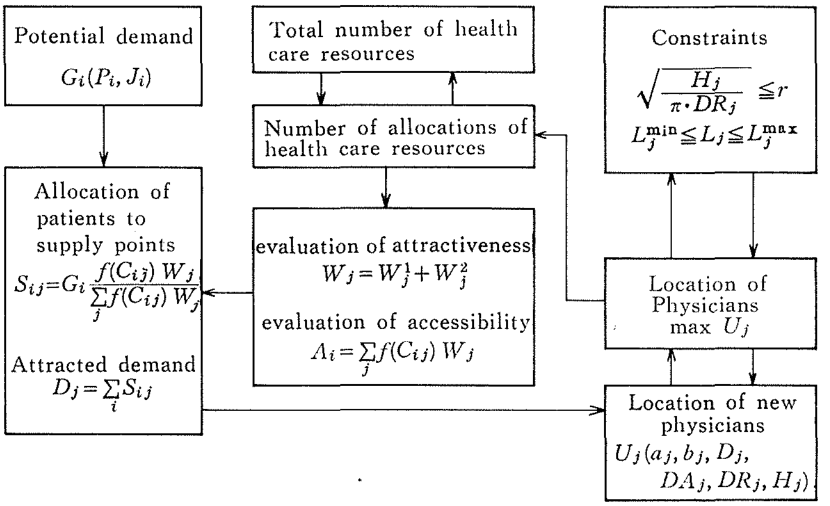

Figure 1. The Overall Location-Allocation Model.

4. Concluding Remarks

In this paper, a modelling approach to the problems of the equity-efficiency trade-off relation in the spatial provision of health care, that is, the relationship between the equalization of access opportunity to health care services and the effective provision of the services in a free-entry and fee-for-service market system, was attempted from the perspective of location-allocation analysis.

The location-allocation models developed in this paper are composed of a spatial interaction model of demand allocation and another is a resource location model when service provider is private sector. In the resource location model, the amount of potential demand in each region and case capacity (work load) of a human as well as a physical resource are internalized.

The demand allocation model consists of a space discount function, attractiveness of the service and potential demand for the service in the region. In the resource location model, revenue maximization was adopted as a criterion of efficiency for private sector as service provider, which is different from the criterion of efficiency usually adopted when public sector is regarded as service provider, that is, minimization of construction and operation costs of public facilities. Therefore, the location-allocation model attempted to developed in this paper will be useful for analyzing equity-revenue trade-off problem in various social services provided by private sector. The next step is to test the model with real data.

References

Askew, Ian (1983): "The location of service facilities in rural areas: A model for generating and evaluating alternative solutions", *Regional studies*, vol. 17, pp. 305-314.

Beaumont, J. R. and A. Sixsmith (1984): "Elderly severely mentally infirm (ESMI) units in Lancashire: An assessment of resource allocation over space", in M. Clarke ed., *Planning and analysis in health care system*, Pion, London.

Bigman, D. and C. ReVelle (1979): "The theory of welfare considerations in public facility location problems", *Geographycal analysis*, vol. 10, pp. 229-240.

Church, R. and R. Stimson (1983): "Modelling spatial allocation-location solutions for general partitioner medical services in cities: The equity-revenue maximizing conflict case", *Regional science and urban economics*, no. 13, pp. 161-172.

Church, R. and C. ReVelle (1974): "The maximal covering location problem", *Papers of the regional science association*, no. 32, pp. 101-118.

Clarke, M. ed. (1984): *Planning and analysis in health care system*, Pion, London.

Coelho, J. D. and A. G. Wilson (1976): "The optimum location and size of shopping centers", *Regional studies*, vol. 10, pp. 413-421.

Cornuejols, G. et al. (1977): "Location of bank accounts to optimize float: An analytic study of exact and approximate algorithms", *Management science*, vol. 23, pp. 789-810.

Daniels, N. (1985): *Just Health Care*, Cambridge University Press, London.

Fisher, H. B. and G. Rushton (1977): "Spatial efficiency of service locations and the regional development process", *Papers of the regional science association*, no. 42, pp. 83-97.

Francis, R. L. and J. M. Goldstein (1974): "Location theory: A selected bibliography", *Operations research*, vol. 22, no. 2, pp. 400-410.

Geoffrion, A. M. (1975): "A guide to computer-assisteds methods for distribution systems planning", *Sloan management review*, vol . 2, pp. 17-41.

Hakim, S. (1964): "Optimum location of switching centers and absolute centers and medians of a graph", *Operations research*, vol. 12, pp. 450-459.

Hodge, D. and A. Gatrell (1976): "Spatial constraint and the location of urban public facilities", *Environment and planning A*, vol. 8, pp. 215-230.

Hodgson, M. J. (1978): "Toward more realistic allocation in location-allocation models: An interaction approach", *Environment and planning A*, vol. 10, pp. 1273-1285.

Holmes, J. et al. (1972): "Facility location under a maximum travel restriction: An example using day care facilities, vol. 4, no. 3, pp. 258-266.

Kawakami, T. (1965): *Gendai nihon iryo shi* (A history of medical care in modern Japan), Keisei shobo, Tokyo (in Japanese).

Lea, A. C. (1979): "Welfare theory, public goods, and public facility location", *Geographical analysis*, vol. 11, no. 3, pp. 213-239.

Leonardi, G. (1981): "An unifying framework for public facility location problems": Part 1, Part 2, *Environment and planning A*, vol. 13, no. 3, pp. 1001-1028 and 1085-1108.

Oda, T. (1980): "Kenko-ken to fukusi-ken" (The right of health and welfare), in Hoken-iryo to fukushino togo wo mezashite (Towards integration between health and welfare), *Papers of the Japanese society of health and medical society*, no. 4 (in Japanese), Kakiuchi-shuppan, Tokyo, pp. 65-99.

Ostresh, L. M. Jr (1975): "An efficient algorithm for solving the two center location-allocation problem", *Journal of regional science*, vol. 15, no. 2, pp. 209-216.

ReVelle, C. and R. Church (1977): "A spatial model for locational construct of Teitz", *Papers of regional Science association*, no. 39, pp. 129–135.

ReVelle, C., D. Bigman, D. Schilling, J. Cohen and R. Church (1977): "Facility location: A review of context free and ESM models", *Health services research*, vol. 12, pp. 129–146.

ReVelle C. & R. Swain (1970): "Central facilities location", *Geographical analysis*, 2, pp. 30–42.

Saguchi, T. (1982): *Iryo no shakaika* (Socialization of medical care), Keisoshobo, Tokyo (in Japanese)

Smith, A. P. et al. (1977): "The utilization of services", in A. G. Wilson et al. eds., *Models of cities and regions: Theoretical and empirical developments*, John Wiley & Sons, Chichester.

Sugaya, A. (1982): *Gendai no iryo-mondai* (The recent health care problems in Japan), Yuhikaku, Tokyo (in Japanese).

Teitz, M. B. (1968): "Toward a theory of urban public facility location", *Papers of the regional science association*, vol. 21, pp. 35–51.

Toregas, C. and C. ReVelle (1972): "Optimal location under time or distance constraints", *Papers of regional science association*, vol. 28, pp. 133–143.

Zeller, R. E., D. D. Achabal and L. A. Brown (1980): "Market penetration and locational conflict in franchise systems", *Decision science*, vol. 11, pp. 58–80.

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