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Synergetic Location Theory (I)

—focus on Hotelling model (I)—

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Abstract

Hotelling Model was to have an enormous impact. The debate around it became wider in scope and soon encompassed situations which moved further away from those initially covered by the author's hypotheses. In these researches, it remains one discussion of Location model that each entrepreneur's behaviors are effected by these interactions with these surroundings.

In this paper, we shall consider a quantitative description of interacting seller's groups in Hotelling model using the Synergetic approach by Weidlich's Model.

First, the formations of an entrepreneur's preference of location is influenced by the presence of groups of entrepreneurs with the same or the opposite location from the center in line. Second, the formation of two kind entrepreneur's collective location is influenced by the internal sympathy and mutual sympathy in a market area.

Key Words: Synergetics, Synergetic Location Model, Hotelling Model, Master equation, Langevin equation, Fokker-Plank equation, Weidlich's Model.

1. Introduction:

The article which Hotelling (1929) wrote was to have the law of locational agglomeration in a duopoly situation and for a rectilinear, uniform, founded market with perfectly inelastic demand. Smithies (1941) had shown the law that the locations are optimal between the quartiles and the center with a linear demand function and variable transport costs.

Apart from these two cases no progress was made for twenty years. Stevens (1961) was to initiate renewal to the theory of spatial duopoly with game theory introduced into the field. The Hotelling-chamberlin model is assimilated to a two-person zero-sum game. Demand is assumed to be perfectly inelastic. This assumtion was replaced by Lerner and Singer (1937), Smithies (1941) and Jacot (1963).

Linear programming which had been used extensively in game theory was then introduced into spatial theory by Lefeber (1958), Goldman (1958), Isard (1958) and Isard and Schooler (1959), Teitz (1968) introduced the possibility of multiple location for each duopolist, without any cost, and showed that locational equilibrium is only possible if each firm adopts a maximum location strategy. Vickery (1964) adopted another topology to describe the market space and examined the locational competition of two identical firms on the circumference of a circle. Gannon (1972) considered conjunctural variations that each entreprenour's predictions regard the degree of response from his competitor to an initiative taken by him. He uses an individual, general but well-behaved demand function, and a non-restrictive set of assumptions as regards behavioral response predicted by each firm viz-a-viz the other, and arrives at original results. Beckman (1972) re-examined Cournot's oligopoly in a spatially homogeneous market where mill pricing is assumed with a rectilinear demand function. In a discrete market space Kuenne (1977) generalizes the Hotelling-Smithies model to cover any number of entrepreneurs with specified functions.

Brikin and Wilson (1986) extended a new framework of Hotelling model in the industrial location model.

Finally, in these researches mentioned above there remains one discussion on location model that each entrepreneur's behavior is affected by these interactions with the surroundings.

In this paper, we shall consider a quantitative description of interacting entrepreneur's groups in Hotelling model using the Synergetic approach by Weidlich's model.

2. Introduction to Synergetic Location Model

The word "Synergetics" has been introduced into the natural sciences by Haken (1977). Haken defined as Synergetics is the science of collective static or dynamic phenomena in closed or open multi-component systems with cooperative interactions occurring between the units of the system.

In physics, chemistry and biology, synergetics concentrates on the structural self-organizing space-time features of systems on a macroscopic level. On this level, there exist close analogies between various fields, although they are composed of different units with completely different elementary interactions. Due to this fact, the concepts of synergetics are of interdisciplinary universality.

Over the past decade it has been shown that there is a large class of phenomena in a variety of fields to which unifying concepts can be applied. The concepts and methods originally used in physics can be applied to sociological phenomena. One approach is based on detailed mathematical models as initiated by W. Weidlich and his co-workers (1983).

The aim of Synergetic Location Model is to dear the following two concepts.
(1) the dynamic location phenomena with cooperative interactions occurring between the units of the system.
(2) the dynamic location phenomena between micro-location and macro- location.

Before trying to transfer such concepts to the Hotelling model it is worthwhile to appraise and summarize their meaning for master equation's systems.

The simplest type of such equations for macrovariables is that of the Langevin equations, a set of first-order differential equations in the time variable t.

$$\frac{dx_i(t)}{dt}=F_i(x_1,\cdots,x_n)+f_i \qquad \text{with } i\ 1,2,\cdots,n \tag{1}$$

where

f_i: the fluctuation random forces.
$F_i(x_1,\cdots,x_n)$: non-linear functions of the macrovariables $x_j(t)$.

This description is illustrated typically by writing down the Langevin equations for Brownian motion (Kubo, Matsuo, Kitahara, 1973).

$$\frac{dx_i}{dt}=\frac{1}{m}P_i,\qquad \frac{dp_i}{dt}=-rp_i+f_i \qquad \text{with}=1,2,3 \tag{2}$$

We denote the ensemble average or ensemble mean value of a quantity $x^\alpha(t)$ by $\langle x(t)\rangle$.

If the ensemble consists of samples $\alpha=1,2,\cdots,w$ with paths $x_i^\alpha(t)$, the ensemble mean value is defined by

$$\langle x_i(t)\rangle=\frac{1}{\omega}\sum_{\alpha=1}^{w}x_i^\alpha(t) \tag{3}$$

For a wide class of systems these random forces obey the following equations (4).

$$f_i(t)=\sum_{j=1}^{n}g_{ij}\Big(x_1(t),\cdots,x_n(t)\Big)\phi_j(t) \tag{4}$$

Equation (4) implies that $f_i(t)$ consists of a sum of random forces $\phi_i(t)$ with strengths factors $g_{ij}(x_1,\cdots,x_n)$ in general dependent on the macrovariables. The random forces $\phi_j(t)$ have vanishing ensemble meanvalues.

$$\langle\phi_j(t)\rangle=0 \tag{5}$$

Further, they have no correlation with the macrovariables at the same or at previous points in time.

$$\Big\langle\phi_j(t+\tau)\,G\Big(x_1(t),\cdots,x_n(t)\Big)\Big\rangle=0 \qquad \text{for } \tau\geq 0 \tag{6}$$

from which it follows that

$$\langle f_i(t)\rangle=0 \tag{7}$$

The random forces $\phi_j(t)$ are, however, correlated with themselves over very short time periods.

$$\langle\phi_j(t+\tau)\,\phi_i(t)\rangle=\delta_{ij}\delta(\tau) \tag{8}$$

Where $\delta(\tau)$ is the so-called delta function.

A probabilistic description of systems on a macroscopic level is now introduced which takes into account from the very beginning the fact that the exact values of macrovariables are unknown because of the fluctuating random influence of microvariables (Kitahara, 1975).

Starting from the ensemble standpoint, the probability

$$P(x_1, \cdots, x_n, t)\, d\tau(x) \qquad \text{where } d\tau(x) = dx_1 \cdot dx_2 \cdots dx_n \tag{9}$$

The Langevin equations with fiuctuating forces is completely equivalent to the following Fokker-Planck equation of motion for the probability distribution function (Risken, 1984)

$$\frac{\partial P(x\,;\,t)}{\partial t} = -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left[K_i(x)\,P(x\,;\,t)\right] + \frac{1}{2}\sum_{i,j=1}^{n} \frac{\partial^2}{\partial x_i \partial x_j}\left[Q_{ij}(x)\,P(x\,;\,t)\right] \tag{10}$$

where

$$K_i(x) = F_i(x) + \frac{1}{2}\sum_{l,j=1}^{n} \frac{\partial g_{ij}(x)}{\partial x_l}\, g_{lj}(x) \tag{11}$$

and

$$Q_{ij}(x) = \sum_{l=1}^{n} g_{il}(x)\, g_{jl}(x) \tag{12}$$

On the other hand, the Fokker-Plank equation may be derived using some approximation procedures from a so-called master equation for $P(x\,;\,t)$ which has a particularly simple intuitive interpretation. The master equation has the form.

$$\frac{\partial P(y\,;\,t)}{\partial t} = \int_x d\tau(x)\left[w(y \leftarrow x)\,P(x\,;\,t) - w(x \rightarrow y)\,P(x\,;\,t)\right] \tag{13}$$

Where, $d\tau(x)\,w(x \leftarrow y)$ is the transition probability per unit time from values $\{y_k\}$ of the macrovariables into the cell $d\tau(x)$ of the configuration space.

The meaning of the master equation (13) becomes clear from probability balance considerations: The change in time $[\partial P(y\,;\,t)/\partial t]\, d\tau(y)$ of the probability in the interval $d\tau(y)$ is due to the probability flux

$$\int_x J\left[d\tau(y) \leftarrow d\tau(x)\right] = \int_x \left[d\tau(y)\; w(y \leftarrow x)\right] d\tau(x)\, P(x\,;\,t) \tag{14}$$

from all points $\{x_k\}$ into the cell $d\tau(y)$ as well as to the probability flux.

$$\int_x J\left[d\tau(x) \leftarrow d\tau(y)\right] = \int_x \left[d\tau(x)\; w(x \leftarrow y)\right] d\tau(y)\, P(y\,;\,t) \tag{15}$$

The master equation and the Fokker-Plank equation have one important property: An arbitrary initial distribution finally develops into a stationary equilibrium distribution $P_{st}(x)$.

For a very long time interval $T \rightarrow \infty$, every Langevin path spends a fraction of time proportional to $P_{st}(y)\, d\tau(y)$ in each interval $d\tau(y)$. This leads to the conclusion that the ensemble mean value of any function $F(x)$ in the stationary ensemble can be identified with its time mean value taken along a Langevin path $x^{a}(t)$:

$$\langle F(x)\rangle_{st} = \int_x F(x)\, P_{st}(x)\, d\tau(x) = \lim_{T \rightarrow \infty} \frac{1}{T}\int_0^T F\left(x^{a}(t)\right) dt \tag{16}$$

3. Synergatic Hotelling Model

In this chapter, we shall consider two models such as the preference model of location and the collective model of location using the models by weidlich W. and G. Haag.

(1) *The preference model of location*

The number of entrepreneurs is very large, and the buyers of a commodity will supposedly be uniformly distributed along a line which may be main street in a town. Each buyer transports his purchases home at a cost per unit distance. Without effect upon the generality of our conclusions, we shall suppose that the cost of production to locations of entreprenurs is zero, and that unit quantity of the commodity is consumed in each unit of time in each unit of length of line. The demand is thus at the extreme of inelasticity.

As a first step we shall treat the simplest case, that of only two kinds of preferences of locations denoted by right side and left side from the center in a line.

Obviously order parameter is the number of entrepreneurs n_+, n_- with corresponding preferences of locations at the right side and left side, respectively.

The basic concept now introduced is that the formation of the preference, i. e., the change of the numbers n_+, n_- is a cooperative effect : The formation of an entrepreneur's preference of location is influenced by the presence of groups of entrepreneur with the same or the opposite location.

We thus assume that there exists a probability per unit time, for the change of location of an entrepreneur from right side to left side or vice versa. We denote these transition probabilities by

$$P_{+-}(n_+, n_-) \text{ and } P_{-+}(n_+, n_-) \tag{17}$$

We are interested in the probability distribution function $f(n_+, n_-, t)$.

The master equation is as follows (Haken, 1977),

$$\begin{aligned}
&\frac{df\,[n_+, n_-\,;\,t]}{dt} = (n_+ + 1)\,P_{+-}[n_+ + 1, n_- - 1] \\
&f[n_+ + 1, n_- - 1\,;\,t] + (n_- + 1)\,P_{-+}[n_+ - 1,\ n_- + 1] \\
&f[n_+ - 1, n_- + 1\,;\,t] - \left\{n_+ P_{+-}[n_+, n_-] + n_- P_{-+}[n_+, n_-]\right\} \\
&f[n_+, n_-\,;\,t]
\end{aligned} \tag{18}$$

Assume that the rate of change of the location of an entrepreneur is enhanced by the group of entrepreneurs with an opposite preference location and diminished by entrepreneur of his own preference location.

Assume further that there is some sort of social overall climate which facilitates the change of location or make it more difficult to form.

Finally one can think of external influence on each entrepreneur, for example, informations from abroad etc.

It is not too difficult to cast these assumption into a mathematical form, if we think of the Ising model of the ferromagnet.

We are led to put in analogy to the Ising model (Nicolis and Prigogine, 1977)

$$P_{+-}[n_+, n_-] = P_{+-}(q) = \nu \exp\left\{\frac{-(Iq+H)}{\theta}\right\}$$
$$= \nu \exp\left\{-(kq+h)\right\} \tag{19}$$

$$P_{-+}[n_+, n_-] = P_{-+}(q) = \nu \exp\left\{\frac{+(Iq+H)}{\theta}\right\}$$
$$= \nu \exp\left\{+(kq+h)\right\} \tag{20}$$

where I is a measure of the strength of adaptation to neighbours. H is a preference parameter which ($H>0$ means the location of right side is preferred to left side), θ is a collective climate parameter corresponding to $k_B T$ in physics (k_B is the boltzman constant and T the temprerature), ν is the frequency of the flipping processes.

$$q = (n_+ - n_-)/2n \qquad n = n_+ + n_- \tag{21}$$

For a quantitative treatment of (18) we assume the entrepreneur groups big enough so that q may be treated as a continuous parameter. Transforming (2) to this continous variable and putting;

$$w_{+-}(q) = n_+ P_{+-}[n_+, n_-] = n\left(\frac{1}{2} + q\right) P_{+-}(q)$$
$$w_{-+}(q) = n_- P_{-+}[n_+, n_-] = n\left(\frac{1}{2} - q\right) P_{-+}(q) \tag{22}$$

The master equation (18) is expanded as a Taylor series up to and including terms of the second order and we obtain the following the Fokker-Planck equation (Risken, 1984)

$$\frac{\partial f(q\,;\,t)}{\partial t} = -\frac{\partial}{\partial q}\left[\frac{1}{n}(w_{-+} - w_{+-}) f\right]$$
$$+ \frac{1}{2n^2}\frac{\partial^2}{\partial q^2}\left[(w_{+-} + w_{-+}) f\right] \tag{23}$$

The stational distribution is as follows:

$$f_0(q) = c k_2^{-1}(q) \exp\left\{2\int_{-\frac{1}{2}}^{q} \frac{k_1(y)}{k_2(y)}\, dy\right\} \tag{24}$$

With

$$k_1(q) = \nu\left\{\sin h\,(kq+h) - 2q \cos h\,(kq+h)\right\}$$
$$k_2(q) = (\nu/n)\left\{\cos h\,(kq+h) - 2q \sin h\,(kq+h)\right\} \tag{25}$$

The main results are as follows.

① Corresponding to frequent changes of location with independent decision of entrepreneurs, we find a centered distribution of location.
② If the coupling strength between entrepreneurs is increased, two pronounced groups of locations occur which clearly describe the Polarization phenomenon of groups.

(2) ***The collective model of location***

The numbers of two types of entrepreneurs are very large, and the buyers of two types of commodities will supposedly be uniformly distributed in a market area. The demand is at the extreme of inelasticity.

We specialize this model to the simplest case of two type of entrepreneurs m, n and two regions A, B of the market area. Neglecting immigration or emigration into or from market area, respectively, we have the conservation laws :

$$\begin{aligned} m_A &= \bar{m}+m \qquad m_B = \bar{m}-m \\ n_A &= \bar{n}+n \qquad n_B = \bar{n}-n \end{aligned} \tag{26}$$

where

m_A : the number of one type of entrepreneurs located in region A.
m_B : the number of one type of entrepreneurs located in region B.
n_A : the number of two-types of entrepreneurs located in region A.
n_B : the number of two-types of entrepreneurs located in rrgion B.
$\bar{m}$: the half of the number of one-type of entrepreneurs.
$\bar{n}$: one half of the number of two-types of entrepreneus.

we may introdue the relevant variables m, n.

The following master equations is as follows.

$$\begin{aligned} \frac{df(m, n)}{dt} = & \Big\{(m_A+1)\, P^1_{AB}(m+1, n) f(m+1, n) \\ & +(m_B+1)\, P^1_{BA}(m-1, n) f(m-1, n)+(n_A+1)\, P^2_{AB}(m, n+1) f(m, n+1) \\ & +(n_B+1)\, P^2_{BA}(m, n-1) f(m, n-1)\Big\}-\Big\{m_A P^1_{AB}(m, n)+m_B P^1_{BA}(m, n) \\ & +n_A P^2_{AB}(m, n)+n_B P^2_{BA}(m, n)\Big\} f(m, n) \end{aligned} \tag{27}$$

where

$P^1_{AB}(m+1, n)$: the probability of movement of one-type of entrepreneurs from region A to region B.

The mean values m, n of m, n are as follows :

$$\begin{aligned} \frac{d\tilde{m}}{dt} &= (\bar{m}-\tilde{m})\, P(\tilde{m}, \tilde{n})-(\bar{m}+\tilde{m})\, P^1_{AB}(\tilde{m}, \tilde{n}) \\ \frac{d\tilde{n}}{dt} &= (\bar{n}-\tilde{n})\, P(\tilde{m}, \tilde{n})-(\bar{n}+\tilde{n})\, P^2_{AB}(\tilde{m}, \tilde{n}) \end{aligned} \tag{28}$$

In order to make the model explicit and applicable, we have to make an ansatz for the transition probabilities realistically describing the behaviour of enterpreneurs belonging to the subgroups, and flexible enough to comprise several possibilities.

A sufficiently flexible ansatz for $P^{\alpha}_{ij}(m, n)$ is given by (Georgii, 1979)

$$
\begin{aligned}
P^{1}_{BA}(m, n) &= \gamma \exp\left\{\hat{\xi}_1 + \hat{\alpha}_1 m + \hat{\beta}_1 n\right\} \\
P^{1}_{AB}(m, n) &= \gamma \exp\left\{-(\hat{\xi}_1 + \hat{\alpha}_1 m + \hat{\beta}_1 n)\right\} \\
P^{2}_{BA}(m, n) &= \gamma \exp\left\{\hat{\xi}_2 + \hat{\beta}_2 m + \hat{\alpha}_2 n\right\} \\
P^{2}_{AB}(m, n) &= \gamma \exp\left\{-(\hat{\xi}_2 + \hat{\beta}_2 m + \hat{\alpha}_2 n)\right\}
\end{aligned}
\tag{29}
$$

The interpretation of parameters $\hat{\xi}_i$, $\hat{\alpha}_i$, $\hat{\beta}_i$ follows from the meaning of (29).

As $\xi_1 > 0$ leads to favouring of region A before region B by entrepreneurs m, we denote $\hat{\xi}_1$ (and analogously $\hat{\xi}_2$) as natural preference parameter. As $\hat{\alpha}_1 > 0$ leads to a clustering trend of entrepreneurs m in the same region of the market area, (trend to live together), we denote $\hat{\alpha}_1$ (and $\hat{\alpha}_2$) as internal oriented parameter.

As $\hat{\beta}_1 > 0$ means, that entreprenurs m prefers to live together with n in the same region of market area, (analogously $\hat{\beta}_2 > 0$ after exchange $m \leftrightarrow n$), we denote $\hat{\beta}_1$, as external oriented parameter.

The main results are as follows.

① We assume internal orientation and week mutual orientation. This leads to stability of the homogeneous distributions of both entrepreneurs over both region of the market area.

② We assume weak internal orientation but strong mutual orientation. This leads to instability of the homogeneous distribution and to sponteneous formation of stable concentration of both entrepreneurs either in region A or region B of the market area.

③ We assume extremely strong internal orientation and strong mutual orientation. Beyond the stable concentrations of both entrepreneurs either in region A or region B of the market area there exists a stable focus corresponding to concentration of both entrepreneurs in different regions of the market area in spite of mutual orientation. This leads to the very strong internal oriented prohibiting disintegration of existing clusters of entrepreneurs in different region of market area.

④ If both entrepreneurs have a preference for region A, the shift of the situation with mutual and internal orientation are described.

⑤ We assume weak internal orientation but strong assynmetric mutual orientation. This leads to the homogeneous distribution of entrepreneurs.

⑥ We assume strong internal orientation and the same strong assymmetric mutual orientation. This leads to the limit cycle case discussed above leading to a permanent afflicting process which nevertheless might be realistic. Although oversimplified, this case of the model may describe the sequential erosion of regions of market area by migration of entrepreneurs of different market standards under some mutual assynmetric orientation.

4. Conclusion

We have explored the Synergetic Hotelling models such as the Preference model and Collective Model with interacting entrepreneur's groups using the models by W. Weidlich and G. Haag.

The main results of the preference model are as follows.

(1) Corresponding to frequent changes of location with of entrepreneurs, we find a centered distribution of location.

(2) If the coupling strength between entrepreneurs is increased, two pronounced groups of locations occur which clearly describe the polarization phenomenon of groups.

The main results of the collective model are as follows.

(1) We assume internal orientation and weak mutual orientation. This leads to stability of the homogeneous distributions of both entrepreneurs over both region of the market area.

(2) We assume weak internal orientation but strong mutual orientation. This leads to instability of the homogeneous destribution and to spontaneous formation of stable concentration of both entrepreneurs either in region A or region B of the market area.

(3) We assume extremely strong internal orientation and strong mutual orientation. This lead to the very strong internal oriented prohibiting disintegration of existing clusters of entrepreneurs in different region of market.

(4) If both entrepreneurs have a preference for region A, the shift of the situation with mutual and internal orientation we described.

(5) We assume weak internal orientation but strong assymmetric mutual orientation. This leads to the homogeneous distribution of population.

(6) We assume strong internal orientation and the same strong assynmetric mutual orientation. This leads to the limit cycle case.

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