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A Study on Model Reference Adaptive Control In Economic Development (VII)

—Model Reference Adaptive Processes in Japan's Nine Regional Economies—

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Abstract

This article aims at applying Model Reference Adaptive Theory to Japan's nine regional economies. The main findings of this study are that MRAS theory is applicable enough to corroborative study based on actual data, and the approaches used in this study give a very effective and meaningful method in making a regional planning.

Key Words: Model reference adaptive system, Japan's nine regional economices, Regional turnpike, Dynamic interregional input-output model.

1. Introduction:

We have applied the theory of Model Reference Adaptive System (MRAS) to the input-output analysis and derived the several theoritical findings in 3), 5), 6), 8) and 9). Among these results, the most applicable one is the theorem relating to MRAS and whose reference model is a turnpike.

In this study we built an interregional dynamic I-O model for the nine economic regions of Japan in order to demonstrate the empirical applicability of these theoritical findings.

2. Model Reference Adaptive System in Regional Dynamic Input-Output Analysis:

In this chapter we shall briefly review the interregional input-output analysis, then offer an explanation for the theoritical background for applying MRAS to interregional dynamic I-O system.

2.1 *Interregional Input-Output Analysis*

As it is well known Interregional Input-Output Analysis (IRIO) was developed by Isard 19), Chenerry 18) and Moses 20) etc. In this section we shall simply explain the Isard and Moses-Chenerry type IRIO, which are the representative methods, to help clarify issues in the subsequent chapters.

(1) Isard type IRIO

The Isard type IRIO is based on the assumption that there is no competition among the sale of the commodities which are produced in different regions. Let us represent the IRIO with m regions and n sectors in mathematical formulation.

$$\begin{pmatrix} x_1^1 \\ \vdots \\ x_n^1 \\ \vdots \\ x_1^m \\ \vdots \\ x_n^m \end{pmatrix} = \begin{pmatrix} A_{11}^{11} \\ \vdots \\ A_{n1}^{11} \\ \vdots \\ A_{11}^{m1} \\ \vdots \\ A_{n1}^{m1} \end{pmatrix} + \cdots + \begin{pmatrix} A_{1n}^{1m} \\ \vdots \\ A_{nn}^{1m} \\ \vdots \\ A_{n1}^{mm} \\ \vdots \\ A_{nn}^{mm} \end{pmatrix} + \begin{pmatrix} f_1^{11} \\ \vdots \\ f_n^{11} \\ \vdots \\ f_1^{m1} \\ \vdots \\ f_n^{m1} \end{pmatrix} + \cdots + \begin{pmatrix} f_1^{1m} \\ \vdots \\ f_n^{1m} \\ \vdots \\ f_1^{mm} \\ \vdots \\ f_n^{mm} \end{pmatrix} + \begin{pmatrix} e_1^1 \\ \vdots \\ e_n^1 \\ \vdots \\ e_1^m \\ \vdots \\ e_n^m \end{pmatrix} - \begin{pmatrix} m_1^1 \\ \vdots \\ m_n^1 \\ \vdots \\ m_1^m \\ \vdots \\ m_n^m \end{pmatrix} \tag{1}$$

The above equation can be written in vector representation as follows.

$$X = \sum_{s=1}^{m} \sum_{j=1}^{n} A_j^s + \sum_{s=1}^{m} f^s + E - M \tag{2}$$

where, x_i^r : output of sector i in region r

A_{ij}^{rs} : intermediate commodity flow sector i in region r to sector j in region s

f_i^{rs} : final demand in region s for the commodity produced by sector i in region r

e_i^r : export of sector i in region r

m_i^r : import of sector i in region r

$X = (x_i^r),\ A_j^s = (A_{ij}^{rs}),\ F^s = (f_i^{rs}),\ E = (e_i^s),\ M = (m_i^s)$

The left hand of (2) stands for the supply in each region and sector while the right hand depicts the demand in each region and sector. Now we introduce the familiar notations of interregional input coefficient and regional import coefficient as,

$$A = (a_{ij}^{rs}) = \left(\frac{A_{ij}^{rs}}{x_j^s}\right), \qquad \hat{M} = (\hat{m}_j^s) = \left(\frac{m_j^s}{x_j^s}\right) \tag{3}$$

then (2) is written as follows.

$$X = AX + \sum_{s=1}^{m} F^s + E - \hat{M}X \tag{4}$$

$$\therefore X = (I - A + \hat{M})^{-1} \left(\sum_{s=1}^{m} F^s + E\right) \tag{5}$$

As similar to national level I-O analysis, when the final demand is derived exogenously the output of each sector in each region, which is the source of supply for the final demand, can be estimated. But because this analysis involves the shipment of final demand goods in each region, the projection of repercusive effect using Isard type IRIO is fraught with difficulties.

(2) Moses-Chenerry type IRIO

The Moses-Chenerry type IRIO is based on the next assumptions.

1) Each region has its own input coefficients which may be different from the other regions.

2) There exists a specific pattern in trade flow among regions. The coefficients

which stand for that pattern are called interregional trade coefficients.

Now m region n sector Moses-Chenery type IRIO is represented by the next equation.

$$\begin{pmatrix} x_1^1 \\ \vdots \\ x_n^1 \\ \vdots \\ x_1^m \\ \vdots \\ x_n^m \end{pmatrix} = \begin{pmatrix} t_1^{11}\ A_{11}^1 \\ \vdots \\ t_n^{11}\ A_{n1}^1 \\ \vdots \\ t_1^{m1} A_{11}^1 \\ \vdots \\ t_n^{m1}\ A_{n1}^1 \end{pmatrix} + \cdots + \begin{pmatrix} t_1^{1m}\ A_{1n}^m \\ \vdots \\ t_n^{1m}\ A_{nn}^m \\ \vdots \\ t_1^{mm} A_{1n}^m \\ \vdots \\ t_n^{mm} A_{nn}^m \end{pmatrix} + \begin{pmatrix} t_1^{11}\ f_1^1 \\ \vdots \\ t_n^{11}\ f_n^1 \\ \vdots \\ t_1^{m1} f_1^1 \\ \vdots \\ t_n^{m1} f_n^1 \end{pmatrix} + \cdots + \begin{pmatrix} t_1^{1m}\ f_1^m \\ \vdots \\ t_n^{1m}\ f_n^m \\ \vdots \\ t_1^{mm}\ f_1^m \\ \vdots \\ t_n^{mm}\ f_n^m \end{pmatrix} + \begin{pmatrix} e_1^1 \\ \vdots \\ e_n^1 \\ \vdots \\ e_1^m \\ \vdots \\ e_n^m \end{pmatrix} - \begin{pmatrix} m_1^1 \\ \vdots \\ m_n^1 \\ \vdots \\ m_1^m \\ \vdots \\ m_n^m \end{pmatrix} \tag{6}$$

When we put $X=(x_i^r)$, $TA_j^s=(t_j^{rs}\ A_{ij}^s)$, $TF^s=(t_i^{rs} f_i^s)$, $E=(e_i^r)$, $M=(m_i^r)$, then (6) is represented as,

$$X=\sum_{s=1}^{m}\sum_{j=1}^{n} TA_j^s+\sum_{s=1}^{m} TF^s+E-M \tag{7}$$

where, x_i^r : output of sector i in region r

A_{ij}^s : demand of intermediate goods of sector j in region s from sector i in all regions

f_i^s : final demand in regions s from sector i

t_i^{rs} : trade coefficient of sector i products from region r to s which is defined as

$$t_i^{rs}=\frac{\sum_{j=1}^{n} A_{ij}^{rs}+F_i^{rs}}{\sum_{j=1}^{n} A_{ij}^{s}+F_i^{s}}$$

$$=\frac{\text{products of sector } i \text{ in region } r \text{ purchased by region } s}{\text{products of sector } i \text{ in all regions purchased by region } s}$$

e_i^r : export of sector i in region r

m_i^r : import of sector i in region r

We introduce the regional input coefficient matrix $A^*=(a_{ij}^s)$, the interregional trade coefficient matrix $T^*=(t_i^{rs})$ and the import coefficient matrix $\hat{M}=(\hat{m}_j^s)$ which are defined as,

$$A^*=\left(\begin{array}{c:c:c} \begin{matrix} a_{11}^1 \cdots a_{1n}^1 \\ \vdots \quad\ \vdots \\ a_{n1}^1 \cdots a_{nn}^1 \end{matrix} & 0 & 0 \\ \hdashline 0 & \ddots & 0 \\ \hdashline 0 & 0 & \begin{matrix} a_{11}^m \cdots a_{1n}^m \\ \vdots \quad\ \vdots \\ a_{n1}^m \cdots a_{nn}^m \end{matrix} \end{array}\right) \qquad a_{ij}^s=\frac{A_{ij}^s}{x_j^s} \tag{8}$$

$$T^* = \left(\begin{array}{cc:c:cc} t_1^{11} & 0 & & t_1^{1m} & 0 \\ & \ddots & \cdots & & \ddots \\ 0 & t_n^{11} & & 0 & t_n^{1m} \\ \hdashline & \vdots & \ddots & & \vdots \\ \hdashline t_1^{m1} & 0 & & t_1^{mm} & 0 \\ & \ddots & \cdots & & \ddots \\ 0 & t_n^{11} & & 0 & t_n^{mm} \end{array}\right) \tag{9}$$

$$\hat{M} = \begin{pmatrix} \hat{m}_1^1 & & & & \\ & \ddots & & 0 & \\ & & \hat{m}_n^1 & & \\ & & & \ddots & \\ & & & & \hat{m}_1^m & \\ & 0 & & & & \ddots \\ & & & & & & \hat{m}_n^m \end{pmatrix} \qquad \hat{m}_j^s = \frac{m_j^s}{x_j^s} \tag{10}$$

then (7) is transformed into

$$X = T^* A^* X + T^* F^* + E - \hat{M}X \tag{11}$$

$$\therefore X = (I - T^* A^* + \hat{M})^{-1}(T^* F^* + E) \tag{12}$$

If final goods are demanded in each region, the regional demanded outputs are derived from (12).

Moses-Chenery type IRIO is realized with the precondition that the trade coefficients are independent of sectors in which commodities are consumed, however, it has the advantage that it is sufficient to estimate only shipped final demands.

2.2 *Dynamic Interregional Input-Output Analysis*

The previous section simply described the IRIO analysis and a proof was established for the projection of a demanded output corresponding to an exogenously derived regional final demand. But those methods may be comparative static ones. In this section we shall internalize private investment and other final demands in IRIO, and then transform that system into a dynamic one.

Now we denote private capital goods of sector i in region r invested in region s as $I^s = (I_i^{rs})$ and a consumption commodity demanded for sector i in region r as $H = (h_i^r)$, then (2) is rewritten as,

$$X = AX + \sum_{s=1}^{m} I^s + H \tag{13}$$

where, $H = \sum_{s=1}^{m} F^s - \sum_{s=1}^{m} I^s - E + M$

Furthermore we represent private capital goods produced by sector i in region r in the possession of sector j in region s as $K = (k_{ij}^{rs})$ and its capital coefficient as $B = (b_{ij}^{rs})$ which is defined by $b_{ij}^{rs} = k_{ij}^{rs}/x_j^s$, then the capital stock owned by sector j in region s $\sum_{r=1}^{m}\sum_{i=1}^{n} k_{ij}^{rs}$ is as follows.

$$\sum_{r=1}^{m}\sum_{i=1}^{n} k_{ij}^{rs} = \sum_{r=1}^{m}\sum_{i=1}^{n} b_{ij}^{rs} x_j^s \tag{14}$$

If a regional private investment is induced by the acceleration principle, it can be represented as,

$$\begin{aligned}\sum_{s=1}^{m} I^s(t) &= \sum_{s=1}^{m}\sum_{j=1}^{n}\left(k_{ij}^{rs}(t+1)-k_{ij}^{rs}(t)\right)\\ &= \sum_{s=1}^{m}\sum_{j=1}^{n} b_{ij}^{rs}\left(x_j^s(t+1)-x_j^s(t)\right) = B\left(X(t+1)-X(t)\right)\end{aligned} \tag{15}$$

And if regional consumption is supposed to be expanded through the regional distributed income which is derived from the regional value added produced by each industry, the regional consumption is written as,

$$H = CSV + \bar{H} \tag{16}$$

where, CS: matrix of regional marginal propensities to consume
V: regional value added coefficient matrix
$\bar{H}$: regional basic consumption vector which is independent of income variation

Accordingly (13) is transformed as,

$$X(t) = AX(t) + B\left(X(t+1)-X(t)\right) + CSVX(t) + \bar{H} \tag{17}$$

(17) denotes that the increment of the output from the t th to the $t+1$ th period can be determined when the regional outputs, the interregional input coefficients, the interregional capital coefficients, the rates of regional value added and regional marginal propensities to consume at the t th period are known. If we assume that B is non-singular i. e. det $B \neq 0$, (17) is transformed as,

$$X(t+1) = B^{-1}(I-A-CSV+B)\,X(t) - B^{-1}\bar{H} \tag{18}$$

(18) stands for a dynamic interregional growth equation.

Now if $(I-A-CSV)^{-1}B$ is assumed to be a positive and indecomposable matrix, due to the Frobenius theorem, there exists a positive eigen value λ with maximum radius among other eigen values and its associated eigen vector η is positive. When an initial output $X(0)$ are set as $X(0)=\alpha\eta+(I-A-CSV)^{-1}\bar{H}$, a solution of (18) is as follows.

$$X(t) = \left(1+\frac{1}{\lambda}\right)^t\left(X(0)-(I-A-CSV)^{-1}\bar{H}\right)+(I-A-CSV)^{-1}\bar{H} \tag{19}$$

The paths of regional outputs represented by (19) are called the regional turnpike. (see Figure 1.)

2. 3 *Model Reference Adaptive Regional Turnpike Theorem*

We have obtained some results about the relationship between the turnpike and MRAS in our previous papers 5), 6), 8) and 9). From a mathematical point of view, (19) is in the same mathematical formula as those in 5), 6), 8) and 9). Therefore we can obtain two theorems as follows.

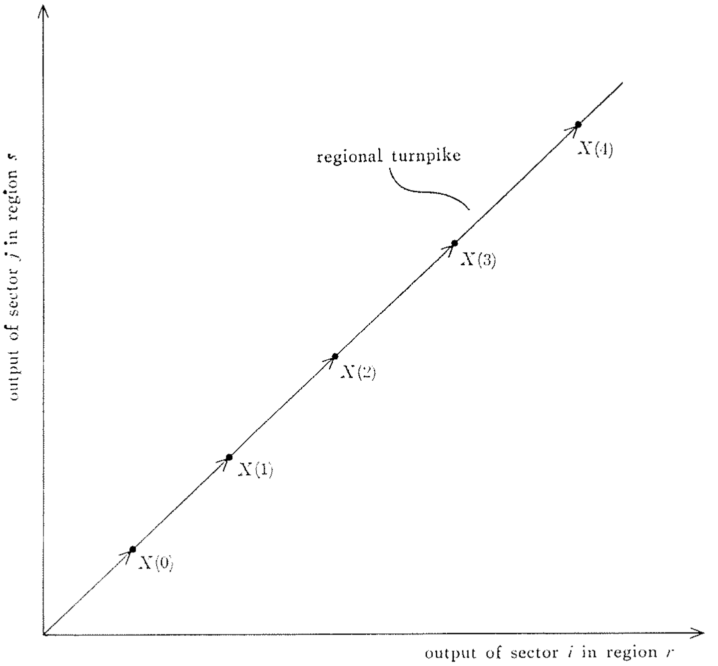


Figure 1. Path of Regional Turnpike.

Theorem 1

reference model

$$Xm(t+1) = CmXm(t) + DmH(t) \tag{20}$$

adaptive model

$$X(t+1) = C(t+1)\,X(t) + D(t+1)\,H(t) \tag{21}$$

disparity equation

$$\varepsilon(t+1) = Cm\varepsilon(t) + \Big(Cm - C(t+1)\Big)\,X(t) + \Big(Dm - D(t+1)\Big)\,H(t) \tag{22}$$

where, $Cm = Bm^{-1}(I - Am - CSmVm + Bm)$

$Dm = -Bm^{-1}$

$C(t+1) = B^{-1}(t+1)\,(I - A(t) - CSV(t) + B(t))$

$D(t+1) = -B^{-1}(t+1)$

$\varepsilon(t) = Xm(t) - X(t)$

Am : reference interregional input coefficient matrix

Bm : reference interregional capital coefficient matrix

CSm : matrix of reference regional marginal propensities to consume

Vm : reference regional value added coefficient matrix

$H(t)$: exogerously derived regional basic consumption vector

$A(t)$: adaptive interregional input coefficient matrix

$B(t)$: adaptive interregional capital coefficient matrix

CS : matrix of adaptive regional marginal propensities to consume

$V(t)$: adaptive regional value added coefficient matrix

If $\sum_{s=1}^{m}\sum_{j=1}^{n}\left(kc_{ij}^{rs}x_j^s(t)^2+kd_{ij}^{rs}h_j^s(t)^2\right)$ is getting sufficiently large, the following adaptation laws make the disparity between the reference and the adaptive outputs asymptotically zero. i. e. $\lim_{t\to\infty}||Xm(t)-X(t)||=0$

$$C(t+1)=C(t)+\left(I+\Gamma(t)\right)^{-1}Kc\otimes\hat{\varepsilon}(t+1)\ X^T(t) \tag{23}$$

$$D(t+1)=D(t)+\left(I+\Gamma(t)\right)^{-1}Kd\otimes\hat{\varepsilon}(t+1)\ H^T(t) \tag{24}$$

where, $Kc=(kc_{ij}^{rs}) \qquad kc_{ij}^{rs}>0$

$Kd=(kd_{ij}^{rs}) \qquad kd_{ij}^{rs}>0$

$\hat{\varepsilon}(t+1)=Xm(t+1)-C(t)\ X(t)-D(t)\ H(t)$

$$\Gamma(t)=\begin{bmatrix}\sum_{s=1}^{m}\sum_{j=1}^{n}\left(kc_{1j}^{1s}x_j^s(t)^2+kd_{1j}^{1s}h_j^s(t)^2\right) & & 0\\ & \ddots & \\ 0 & & \sum_{s=1}^{m}\sum_{j=1}^{n}\left(kc_{nj}^{ms}x_j^s(t)^2+kd_{nj}^{ms}h_j^s(t)^2\right)\end{bmatrix}$$

$\otimes$ stands for the next matrix operation.

$$\begin{bmatrix}a_{11}^{11}\cdots a_{1n}^{1m}\\ \vdots \quad \vdots\\ a_{n1}^{m1}\cdots a_{nn}^{mm}\end{bmatrix}\otimes\begin{bmatrix}b_{11}^{11}\cdots b_{1n}^{1m}\\ \vdots \quad \vdots\\ b_{n1}^{m1}\cdots b_{nn}^{mm}\end{bmatrix}=\begin{bmatrix}a_{11}^{11}\,b_{11}^{11}\cdots a_{1n}^{1m}\,b_{nn}^{mm}\\ \vdots \quad \vdots\\ a_{n1}^{m1}\,b_{n1}^{m1}\cdots a_{nn}^{mm}\,b_{nn}^{mm}\end{bmatrix}$$

Furthermore $A(t)=(a_{ij}^{rs}(t))$, $B(t)=(b_{ij}^{rs}(t))$ and $V(t)=(v_j^s(t))$ are solved as follows.

$$a_{ij}^{rs}(t)=I-B(t)-B(t+1)\ C(t+1) \qquad (r\neq s,\ i\neq j) \tag{25}$$

$$a_{jj}^{ss}(t)=\left\{\left(I-B(t)-B(t+1)\ C(t+1)\right)_{jj}^{ss}-cs_j^s+\sum_{r\neq s}\sum_{i\neq j}a_{ij}^{rs}(t)\right\}\Big/(1-cs_j^s) \tag{26}$$

$$B(t+1)=-D^{-1}(t+1) \tag{27}$$

$$v_j^s(t)=1-\sum_{r=1}^{m}\sum_{i=1}^{n}a_{ij}^{rs}(t) \tag{28}$$

When we set the regional basic consumption to be constant and choose a regional turnpike from the stationary equilibrium point $(I-Am-CSmVm)^{-1}H$ as a reference model, Theorem 2 is obtained as follows.

Theorem 2

reference model

$$Xm(t+1)=\left(1+\frac{1}{\lambda}\right)^{t+1}\left(Xm(0)-(I-Am-CSmVm)^{-1}H\right)$$
$$+(I-Am-CSmVm)^{-1}H \tag{29}$$

adaptive model

$$X(t+1)=C(t+1)\ X(t)+D(t+1)\ H \tag{30}$$

disparity equation

$$\hat{\varepsilon}(t+1)=Cm\hat{\varepsilon}(t)+\left(Cm-C(t+1)\right)X(t)+\left(Dm-D(t+1)\right)H \tag{31}$$

When adaptation laws of $C(t)$ and $D(t)$ are set as

$$C(t+1)=C(t)+\left(I+\Gamma(t)\right)^{-1}Kc\otimes\hat{\varepsilon}(t+1)\ X^T(t) \tag{32}$$

$$D(t+1)=D(t)+\left(I+\Gamma(t)\right)^{-1}Kd\otimes\hat{\varepsilon}(t+1)\ H^T \tag{33}$$

then the adaptive output will approach the turnpike.

where, λ: Frobenius root of $(I-Am-CSmVm)^{-1}Bm$

$Xm(0)$: initial outputs of the reference model and their directions from the stationary equilibrium point are same as ones of the turnpike.

Other notations are the same as those in Theorem 1.

In the above-stated two theorems we can observe the adaptation processes from two points of view, that is, adaptation processes in regional technology and interregional trading with resolving $A(t)$ and $B(t)$ into regional input coefficients, generalized interregional intermediate trade coefficients, regional capital coefficients and generalized interregional capital trade coefficients. These are denoted as follows.

$$A(t)=T^*(t)\ \hat{A}(t) \tag{34}$$

$$B(t)=S^*(t)\ \hat{B}(t) \tag{35}$$

$$\hat{A}(t)=\begin{bmatrix} \begin{matrix}\hat{a}_{11}^1(t)\cdots\hat{a}_{1n}^1(t)\\ \vdots \qquad \vdots\\ \hat{a}_{n1}^1(t)\cdots\hat{a}_{nn}^1(t)\end{matrix} & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \begin{matrix}\hat{a}_{11}^m(t)\cdots\hat{a}_{1n}^m(t)\\ \vdots \qquad \vdots\\ \hat{a}_{n1}^m(t)\cdots\hat{a}_{nn}^m(t)\end{matrix}\end{bmatrix} \tag{36}$$

$$\hat{B}(t)=\begin{bmatrix} \begin{matrix}\hat{b}_{11}^1(t)\cdots\hat{b}_{1n}^1(t)\\ \vdots \qquad \vdots\\ \hat{b}_{n1}^1(t)\cdots\hat{b}_{nn}^1(t)\end{matrix} & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \begin{matrix}\hat{b}_{11}^m(t)\cdots\hat{b}_{1n}^m(t)\\ \vdots \qquad \vdots\\ \hat{b}_{n1}^m(t)\cdots\hat{b}_{nn}^m(t)\end{matrix}\end{bmatrix} \tag{37}$$

$$T^*(t)=A(t)\ \hat{A}^{-1}(t) \tag{38}$$

$$S^*(t)=B(t)\ \hat{B}^{-1}(t) \tag{39}$$

$$\hat{a}_{ij}^s(t)=\sum_{r=1}^{m}a_{ij}^{rs}(t) \tag{40}$$

$$\hat{b}_{ij}^s(t)=\sum_{r=1}^{m}b_{ij}^{rs}(t) \tag{41}$$

For example, a decreasing $\hat{A}(t)$ implies technological progress in the region, while a change in $T^*(t)$ expresses either a deconcentration or a concentration of commodity flow.

3. Regional Turnpike in Japan's Nine Regional Economies:

In this chapter we shall attempt to calculate a regional turnpike in Japan's nine regional economies by using empirically derived data.

3.1 *Classification of Japan's Nine Regions*

On the basis of economic activities, Japan was classfied into nine regions.

Figure 2. Classification of Nine Regions in Japan.

Table 1. Classification of Nine Regions in Japan

region	prefectures
Hokkaido	Hokkaido
Tohoku	Aomori, Iwate, Miyagi, Akita, Yamagata, Fukushima, Niigata
Kanto	Ibaragi, Tochigi, Gunma, Saitama, Chiba, Tokyo, Kanagawa, Yamanashi Nagano
Tokai	Shizuoka, Gifu, Aichi, Mie
Hokuriku	Toyama, Ishikawa, Fukui
Kinki	Shiga, Kyoto, Osaka, Hyogo, Nara, Wakayama
Chugoku	Tottori, Shimane, Okayama, Hiroshima, Yamaguchi
Shikoku	Tokushima, Kagawa, Ehime, Kochi
Kyushu	Fukuoka, Saga, Nagasaki, Kumamoto, Ooita, Miyazaki, Kagoshima, Okinawa

These are Hokkaido, Tohoku, Kanto, Tokai, Hokuriku, Kinki, Chugoku, Shikoku and Kyushu. These areas are depicted in Figure 2, and the prefectures included in each region are shown in Table 1.

3.2 *The Data*

1980 Interregional Input-Output Tables 25), which is published by Minister of International Trade and Industry, Government of Japan (MITI), was served as our main source of data. Some data, however, had to be estimated, and the estimation procedures are explained below.

(1) Interregional Input Coefficient

The interregional input-output table published by MITI is constituted by nine regions and twenty five sectors. However we reformed the table due to the below-mentioned reasons. First of all, sectoral divisions were aggregated to only one sector for lack of information on interregional sectoral capital flow. Secondly, we recomposed the regional classifications by MITI shown in Table 2 into ones of Table 1 because Okinawa was deemed to be too small in our interregional analysis. This rearrangement was made by the following methods.

1) The initial commodity flows among Tohoku, Kanto, Hokuriku, Kinki and other related regions were given by the ones of 1970 23).

2) The regional supply and demand of intermediate products were estimated from the regional gross products in Annual Report of Prefectural Accounts 1987 26).

3) Final intermediate commodity flows were estimated by the Flator's method with the marginal distribution obtained in the above 2). The interregional input coefficients were obtained by dividing the interregional trade flows by the regional outputs which were estimated from the regional gross products 26).

(2) Interregional Capital Coefficient

The level of capital stock used in this study were derived from stocks owned

Table 2. Classification of Nine Regions in Japan by MITI

region	prefectures
Hokkaido	Hokkaido
Tohou	Aomori, Iwate, Miyagi, Akita, Yamagata, Fukushima
Kanto	Niigata, Ibaragi, Tochigi, Gunma, Saitama, Chiba, Tokyo, Kanagawa Yamanashi, Nagano, Shizuoka
Chubu	Toyama, Ishikawa, Gifu, Aichi, Mie
Kinki	Fukui, Shiga, Kyoto, Osaka, Hyogo, Nara, Wakayama
Chugoku	Tottori, Shimane, Okayama, Hiroshima, Yamaguchi
Shikoku	Tokushima, Kagawa, Ehime, Kochi
Kyushu	Fukuoka, Saga, Nagasaki, Kumamoto, Ooita, Miyazaki, Kagoshima
Okinawa	Okinawa

by both public and private firms or agencies. Though inventory is usually regarded as part of capital stock, we considered the net increment of inventory as consumption because of instability of the inventory data.

The method for the estimations of the interregional capital coefficients is as follows.

1) The regional divisions of the interregional private and public capital formation matrix was rearranged in the same way as in (1).

2) The interregional capital stock matrix was estimated by the equation.

$$k^{rs}(1980)=\left(\sum_{t=1975}^{1980} I^{rs}(t)\right)\bigg/\left(x^s(1981)-x^s(1975)\right) \tag{42}$$

where, $k^{rs}(t)$: capital stock owned in region s produced by region r

$I^{rs}(t)$: capital formation from region r to region s

$x^s(t)$: output of region s

3) The interregional capital coefficient was calculated by

$$b^{rs}(1980)=k^{rs}(1980)/x^s(1980) \tag{43}$$

(3) Regional Value Added Matrix

Regional value added was calculated by the regional output minus the regional intermediate input, however, this in actual sense represents the gross value added. The regional value added matrix is a diagonal one whose element was obtained by dividing the regional value added by the regional output.

(4) Matrix of Regional Marginal Propensity to Consume

As aforementioned, consumption is defined as,
consumption = total amount of final demands − (capital formation of private firm + capital formation of public firm)
We assume here that consumption is a linear function of the regional value added. That is to say,

$$h^r = \alpha^r v^r + \bar{h}^r \tag{44}$$

where, h^r : consumption of region r

α^r : regional marginal propensity to consume in region r

v^r : value added in region r

$\bar{h}^r$: basic consumption of region r which is independent of the level of the value added

We attempted to estimate the parameter α^r and $\bar{h}^r$ in (44) by using the regional cross-section data in 1980 or both 1975 and 1980, however, we could not obtain a good result, as negative results were obtained for some regions. So we estimated next equation

$$\sum_{r=1}^{9} h^r = \alpha \sum_{r=1}^{9} v^r + \bar{h} \tag{45}$$

using the time series data of national level from 1975 to 1980. Therefore the estimated marginal propensities to consume are same in each region, however, this way is, of course, a simplification.

3. 3 *Regional Turnpike in Japan's Nine Regional Economies*

According to the above-mentioned estimation of the data, the regional outputs, the interregional input-output matrix, the interregional input coefficient matrix, the regional value added rates, the interregional intermediate trade coefficient matrix, the interregional capital stock matrix, the interregional capital coefficient matrix, the regional capital productivities, the interregional capital trade coefficient matrix, and the regional basic consumption vector in 1980 are shown in Figures 3～5 and Tables 3～9.

(1) Interregional Input Coefficient

We observe in Figure 4 that Tohoku has the largest value added rate and Hokkaido, Kyushu and Kanto follow in that oder. This depicts that the Japanese local regions have rather larger value added than highly urbanized Kanto, Tokai

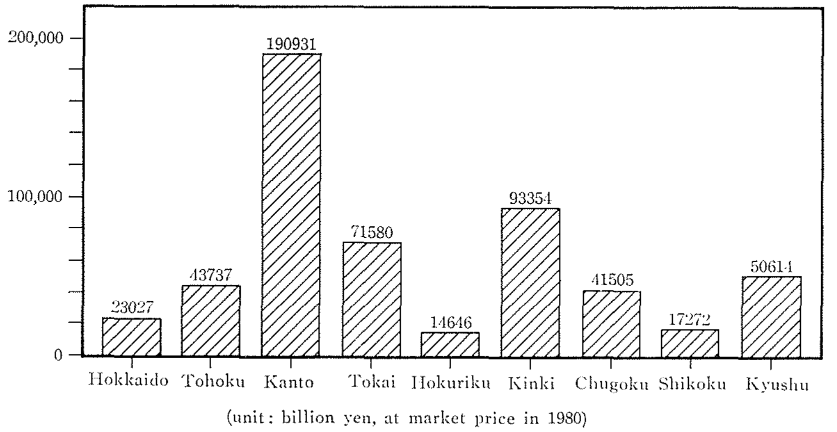


Figure 3. Regional Outputs in 1980.

Table 3. Interregional Input-Output Matrix in 1980

(unit: billion yen, at market price in 1980)

region \ region	Hok-kaido	Tohoku	Kanto	Tokai	Hoku-riku	Kinki	Chugoku	Shikoku	Kyushu
Hokkaido	9,147.	406.	996.	314.	66.	431.	81.	38.	81.
Tohoku	279.	16,300.	3,189.	514.	115.	803.	258.	102.	209.
Kanto	1,251.	3,466.	81,805.	4,206.	1,010.	4,736.	1,751.	721.	1,922.
Tokai	384.	515.	5,157.	29,385.	514.	2,977.	878.	327.	933.
Hokuriku	72.	92.	1,184.	535.	5,156.	898.	215.	94.	219.
Kinki	390.	767.	5,728.	3,007.	806.	36,476.	1,746.	1,061.	1,646.
Chugoku	97.	291.	1,536.	1,088.	279.	1,943.	18,558.	452.	1,145.
Shikoku	44.	161.	892.	392.	102.	884.	369.	6,384.	430.
Kyushu	99.	319.	1,737.	867.	202.	1,269.	784.	271.	19,956.

Table 4. Interregional Input Coefficients in 1980

region \ region	Hok-kaido	Tohoku	Kanto	Tokai	Hoku-riku	Kinki	Chugoku	Shikoku	Kyushu
Hokkaido	0.39722	0.00928	0.00522	0.00439	0.00451	0.00462	0.00194	0.00220	0.00159
Tohoku	0.01210	0.37269	0.01670	0.00718	0.00787	0.00860	0.00623	0.00592	0.00414
Kanto	0.05431	0.07924	0.42846	0.05877	0.06899	0.05073	0.04220	0.04176	0.03797
Tokai	0.01667	0.01178	0.02701	0.41052	0.03508	0.03189	0.02116	0.01891	0.01844
Hokuriku	0.00314	0.00210	0.00620	0.00747	0.35206	0.00962	0.00518	0.00546	0.00434
Kinki	0.01693	0.01754	0.03000	0.04201	0.05503	0.39073	0.04208	0.06141	0.03253
Chugoku	0.00420	0.00665	0.00804	0.01519	0.01902	0.02081	0.44713	0.02616	0.02262
Shikoku	0.00193	0.00369	0.00467	0.00548	0.00697	0.00947	0.00889	0.36963	0.00850
Kyushu	0.00432	0.00729	0.00910	0.01211	0.01381	0.01359	0.01889	0.01569	0.39427

Table 5. Interregional Intermediate Trade Coefficients in 1980

region \ region	Hok-kaido	Tohoku	Kanto	Tokai	Hoku-riku	Kinki	Chugoku	Shikoku	Kyushu
Hokkaido	0.77762	0.01818	0.00975	0.00780	0.00800	0.00855	0.00327	0.00403	0.00304
Tohoku	0.02369	0.73041	0.03119	0.01276	0.01398	0.01593	0.01049	0.01082	0.00789
Kanto	0.10632	0.15529	0.80026	0.10436	0.12247	0.09394	0.07107	0.07632	0.07241
Tokai	0.03263	0.02308	0.05044	0.72901	0.06227	0.05905	0.03564	0.03457	0.03516
Hokuriku	0.00615	0.00411	0.01158	0.01326	0.62497	0.01781	0.00872	0.00998	0.00827
Kinki	0.03314	0.03437	0.05603	0.07461	0.09769	0.72349	0.07087	0.11224	0.06203
Chugoku	0.00823	0.01303	0.01502	0.02698	0.03376	0.03853	0.75314	0.04781	0.04314
Shikoku	0.00378	0.00723	0.00872	0.00973	0.01237	0.01754	0.01497	0.67556	0.01621
Kyushu	0.00845	0.01429	0.01699	0.02150	0.02451	0.02516	0.03183	0.02876	0.75185

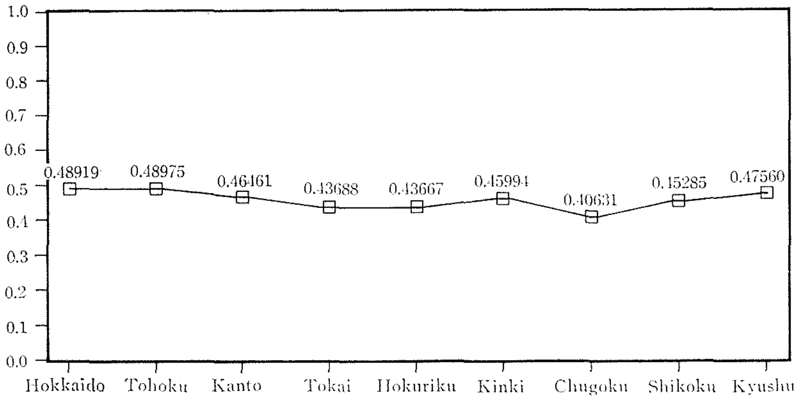


Figure 4. Regional Value Added Rates in 1980.

and Kinki, where three major metropolitan areas are located. The reasons seem to be that labour oriented industries such as agriculture foresty and fisheries, commerce, finance and real estates, service and transportation occupy larger share rates in the local regions, however, the basis of those industries may be weak.

The rate of value added for Shikoku is smallest among the nine regions, the reason being that the heavy industries which consume more intermediate goods are sited in this region.

It is observed from the interregional intermediate trade coefficient matrix that the self-sufficiency rates of Kanto, Hokkaido, Chugoku and Kyushu are higher because of the contributions of the regional areas and/or the share rates of the tertiary industries. With the exception of Shikoku, the rates of purchasing from Kanto are larger, and the rates of supply to neighbouring regions are higher.

(2) Interregional Capital Coefficient

It is observed in Figure 5 that the capital productivity of Tokai is the highest and Kanto, Hokuriku and Shikoku follow in that order. On the contrary, the regions with lower capital productivity are the localized ones such as Hokkaido, Tohoku and Kyushu.

From the matrix of the interregional capital trade coefficients, the self-sufficiency rates of capital goods are almost 80% except Hokuriku, Chugoku and Shikoku, and higher than those of the intermediate commodities. The purchasing rate from Kanto in each region, of course except Kanto, is 8～9%, and this shows the high level dependence on Kanto by the other regions.

3.4 *Regional Turnpike in Japan's Nine Regional Economies*

Let us show a regional turnpike in Japan's nine regional economies based on above data. Capital accumulation turnpike is given by a solution of the next programming problem.

programming of regional turnpike (PRT)

$$\max \ p(I-A-CSV+B)X(T) \tag{46}$$

Table 6. Interregional Capital Stock Matrix in 1980

(unit: billion yen, at market price in 1980)

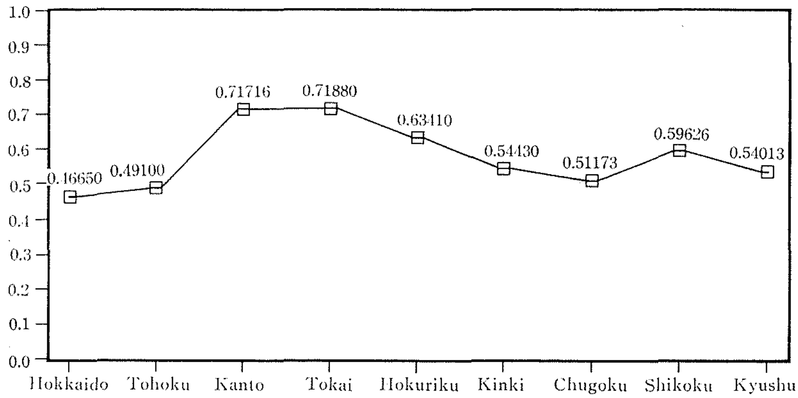
region \ region	Hokkaido	Tohoku	Kanto	Tokai	Hokuriku	Kinki	Chugoku	Shikoku	Kyushu
Hokkaido	40,205.	168.	397.	62.	17.	106.	116.	11.	48.
Tohoku	343.	73,035.	2,878.	491.	131.	928.	231.	64.	203.
Kanto	4,291.	7,913.	239,063.	8,317.	2,299.	16,109.	6,813.	2,652.	8,250.
Tokai	1,796.	2,895.	8,955.	83,013.	1,620.	9,274.	3,015.	1,065.	3,324.
Hokuriku	325.	585.	1,807.	996.	17,190.	1,904.	610.	215.	672.
Kinki	1,395.	3,000.	8,409.	4,619.	1,265.	136,861.	5,246.	2,103.	4,162.
Chugoku	639.	706.	2,219.	1,225.	338.	3,515.	59,446.	677.	1,617.
Shikoku	173.	453.	816.	405.	112.	1,220.	482.	21,886.	2,087.
Kyushu	194.	322.	1,688.	455.	126.	1,595.	5,147.	294.	73,344.

Table 7. Interregional Capital Coefficients in 1980

region \ region	Hokkaido	Tohoku	Kanto	Tokai	Hokuriku	Kinki	Chugoku	Shikoku	Kyushu
Hokkaido	1.74601	0.00384	0.00208	0.00086	0.00117	0.00114	0.00279	0.00065	0.00095
Tohoku	0.01489	1.66987	0.01507	0.00686	0.00892	0.00994	0.00557	0.00372	0.00401
Kanto	0.18633	0.18093	1.25209	0.11619	0.15698	0.17256	0.16415	0.15352	0.16299
Tokai	0.07801	0.06619	0.04690	1.15973	0.11061	0.09935	0.07264	0.06164	0.06567
Hokuriku	0.01413	0.01338	0.00946	0.01391	1.17369	0.02039	0.01470	0.01247	0.01328
Kinki	0.06058	0.06859	0.04404	0.06453	0.08637	1.46604	0.12640	0.12176	0.08224
Chugoku	0.02777	0.01613	0.01162	0.01711	0.02307	0.03765	1.43227	0.03921	0.03195
Shikoku	0.00751	0.01036	0.00428	0.00566	0.00762	0.01307	0.01163	1.26712	0.04123
Kyushu	0.00842	0.00737	0.00884	0.00636	0.00861	0.01708	0.12400	0.01703	1.44907

Table 8. Interregional Capital Trade Coefficients in 1980

region \ region	Hokkaido	Tohoku	Kanto	Tokai	Hokuriku	Kinki	Chugoku	Shikoku	Kyushu
Hokkaido	0.81451	0.00189	0.00149	0.00062	0.00074	0.00062	0.00143	0.00039	0.00051
Tohoku	0.00695	0.81990	0.01081	0.00493	0.00565	0.00541	0.00285	0.00222	0.00217
Kanto	0.08692	0.08884	0.89795	0.08352	0.09954	0.09392	0.08400	0.09154	0.08804
Tokai	0.03639	0.03250	0.03364	0.83362	0.07014	0.05407	0.03717	0.03675	0.03547
Hokuriku	0.00659	0.00657	0.00679	0.01000	0.74424	0.01110	0.00752	0.00744	0.00717
Kinki	0.02826	0.03368	0.03158	0.04638	0.05477	0.79796	0.06468	0.07260	0.04442
Chugoku	0.01295	0.00792	0.00833	0.01230	0.01463	0.02049	0.73294	0.02338	0.01726
Shikoku	0.00350	0.00509	0.00307	0.00407	0.00483	0.00711	0.00595	0.75554	0.02227
Kyushu	0.00393	0.00362	0.00634	0.00457	0.00546	0.00930	0.06346	0.01015	0.78269



(unit: billion yen/billion yen, at market price in 1980)

Figure 5. Regional Capital Productivities in 1980

Table 9. Regional Basic Consumptions

(unit: billion yen, at market price in 1980)

region	basic consumption	region	basic consumption	region	basic consumption
Hokkaido	1,016.	Tokai	753.	Chugoku	421.
Tohoku	1,490.	Hokuriku	146.	Shikoku	290.
Kanto	5,727.	Kinki	2,060.	Kyushu	2,653.

$$\text{subject to} \quad (I-A-CSV+B)\,X(t) \geqq BX(t+1) \tag{47}$$

$$X(t) \geqq 0 \qquad (t=0, 1, \cdots, T-1) \tag{48}$$

where, p: $n-$dim. price vector for evaluation of stocks at T
T: planned period

A solution of the PRT depends on both p and T, however, it has been proved by Tsukui, J. 12) that the solution can be approximated as shown below by the programming technique when T is sufficiently large.

approximation programming of regional turnpike (APRT)

$$\max \ \alpha \tag{49}$$

$$\text{subject to} \quad (I-A-CSV+B)\,X(t) \geqq BX(t+1) \tag{50}$$

$$(I-A-CSV+B)\,X(T'-1) \geqq \alpha B\mu \tag{51}$$

$$X(t) \geqq 0 \qquad (t=0, 1, \cdots, T-1) \tag{52}$$

where, μ: an eigen vector corresponding to $1+\frac{1}{\lambda}\Big(\lambda$ is a Frobenius root of $(I-A-CSV)^{-1}B\Big)$

T': planned period

This programming problem ensures that regional growth paths approach the turnpike at period T. The previous studies have shown that as the APRT is solved, it approaches the turnpike in the initial periods. Therefore we obtained a regional turnpike by solving APRT, but assuming $T'=1$ since it has been clarified in some corroborative studies e. g. 12), 14) and 15) that the solution would rapidly approach the turnpike in initial stages.

Now the eigen values and the associated eigen vectors of $(I-A-CSV)^{-1}B$, which were solved by the power method, are shown in Table 10. Because $(I-A-CSV)^{-1}B$ is positive and indecomposable, $(I-A-CSV)^{-1}B$ has a positive eigen value with the maximum absolute value among the eigen values of $(I-A-CSV)^{-1}B$ by the Frobenius theorem, and that value λ has been calculated as $\lambda=15.76330$. From the result we see that the annual growth rate of the balanced growth paths on the turnpike is $r=1+\frac{1}{\lambda}=1.0634$ i. e. 6.34% annually. Comparing the actual growth rates of the regional outputs from 1975 to 1980 (at constant price in 1980),

Table 10. Eigenvalues and Eigenvectors of $(I-A-CSV)^{-1}B$

	1st.	2nd.	3rd.	4th.	5th.	6th.	7th.	8th.	9th.
eig. val.	15.76330	7.59953	6.60870	5.49354	5.24588	4.58381	4.30014	3.95840	3.61782
Hokkaido	0.09042	0.87612	0.33846	-0.11401	-0.07882	-0.07117	-0.14872	-0.26163	-0.29127
Tohoku	0.16876	0.17479	-0.69488	0.43844	0.34280	0.29323	-0.25087	-0.09908	-0.34025
Kanto	0.74877	0.35183	-0.01815	0.38112	0.19122	-0.11569	0.20914	-0.04724	0.08298
Tokai	0.32747	0.04783	0.18754	0.03286	0.04800	0.14554	-0.32460	0.81612	-0.21374
Hokuriku	0.06905	0.00187	0.04347	0.00763	0.01365	0.04938	-0.03094	0.04454	0.74814
Kinki	0.44911	-0.10780	0.36051	0.02199	0.17758	0.76023	0.05979	-0.36998	-0.17730
Chugoku	0.21755	-0.16934	0.34140	0.08608	0.82824	-0.44725	0.02440	-0.08129	-0.17241
Shikoku	0.07890	-0.03719	0.07425	0.10987	-0.03520	0.12958	0.84654	0.26658	-0.30551
Kyushu	0.18821	-0.18467	0.33640	0.79273	-0.34420	-0.27780	-0.20996	-0.19211	-0.18059

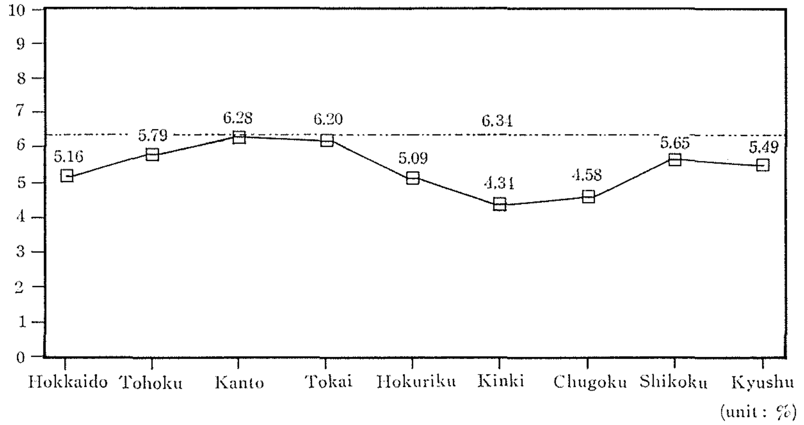


Figure 6. Actual Average and Balanced Growth Rates from 1975 to 1980.

Table 11. Actual in 1980 and Optimal Share Rates of Regional Industrial Locations (unit: %)

region	actual	optimal	region	actual	optimal	region	actual	optimal
Hokkaido	4.21	3.87	Tokai	13.09	14.00	Chugoku	7.59	9.30
Tohoku	8.00	7.22	Hokuriku	2.68	2.96	Shikoku	3.16	3.37
Kanto	34.93	32.02	Kinki	17.08	19.21	Kyushu	9.26	8.05

Table 12. Regional Turnpike from 1980 to the year 2000
(unit: billion yen, at market price in 1980)

region / year	Hokkaido	Tohoku	Kanto	Tokai	Hokuriku	Kinki	Chugoku	Shikoku	Kyushu
1980	23,027.	43,737.	190,931.	71,580.	14,646.	93,354.	41,505.	17,272.	50,614.
1981	21,606.	38,467.	168,644.	67,158.	14,146.	94,612.	43,803.	16,580.	46,714.
1982	22,525.	40,182.	176,253.	70,485.	14,848.	99,176.	46,013.	17,382.	48,627.
1983	23,502.	42,005.	184,344.	74,024.	15,594.	104,029.	48,364.	18,235.	50,660.
1984	24,541.	43,945.	192,949.	77,787.	16,387.	109,190.	50,864.	19,141.	52,823.
1985	25,646.	46,007.	202,100.	81,789.	17,231.	114,679.	53,523.	20,106.	55,123.
1986	26,821.	48,200.	211,831.	86,045.	18,129.	120,515.	56,350.	21,131.	57,569.
1987	28,071.	50,533.	222,179.	90,570.	19,083.	126,722.	59,357.	22,221.	60,171.
1988	29,400.	53,013.	233,184.	95,383.	20,098.	133,323.	62,554.	23,381.	62,937.
1989	30,813.	55,651.	244,887.	100,501.	21,177.	140,342.	65,955.	24,614.	65,879.
1990	32,316.	58,456.	257,332.	105,944.	22,325.	147,807.	69,571.	25,926.	69,007.
1991	33,914.	61,439.	270,567.	111,732.	23,545.	155,745.	73,416.	27,320.	72,334.
1992	35,613.	64,611.	284,642.	117,887.	24,843.	164,187.	77,505.	28,804.	75,871.
1993	37,421.	67,984.	299,609.	124,432.	26,224.	173,164.	81,854.	30,381.	79,634.
1994	39,343.	71,572.	315,526.	131,393.	27,692.	182,711.	86,479.	32,058.	83,635.
1995	41,387.	75,387.	332,452.	138,795.	29,253.	192,864.	91,397.	33,842.	87,889.
1996	43,561.	79,444.	350,453.	146,667.	30,913.	203,660.	96,626.	35,738.	92,414.
1997	45,872.	83,758.	369,595.	155,038.	32,678.	215,142.	102,188.	37,756.	97,226.
1998	48,330.	88,437.	389,951.	163,940.	34,556.	227,352.	108,103.	39,901.	102,343.
1999	50,944.	93,226.	411,599.	173,407.	36,553.	240,336.	114,393.	42,182.	107,784.
2000	53,725.	98,415.	434,621.	183,474.	38,676.	254,145.	121,081.	44,608.	113,571.

Table 13. Wasted Goods in Adjustment to Regional Turnpike
(unit: billion yen, at market price in 1980)

region	wasted goods	region	wasted goods	region	wasted goods
Hokkaido	4,329.	Tokai	12,381.	Chugoku	0.
Tohoku	13,173.	Hokuriku	2,063.	Shikoku	2,486.
Kanto	45,915.	Kinki	6,644.	Kyushu	9,566.

with the balanced growth rate it was found that the latter was 0.06∼2% larger than the former. (see Figure 6). This fact illustrates the existance of some unused resources in each region.

As for the eigen vector corresponding to the Frobenius root, the largest element is Kanto, and Kinki, Chugoku, Kyushu, Tohoku, Hokkaido, Shikoku and Hokuriku follow in that order, and the share rates among the elements of the eigen vector denote the optimal rates of regional industrial locations. (see Table 11.)

The regional turnpike in Japan's nine regional economies was solved as depicted in Table 12. It is shown that the regional outputs "get on" the turnpike at 1981 with the initial outputs in 1980 and then there is balanced growth from the stationary equilibrium point $(I-A-CSV)^{-1}H$. It is also observed that the regional outputs fall off from 1980 to 1981 except Kinki and Chugoku because of adjustment to the turnpike. The regional unused commodities in the adjustment are presented in Table 13. This may be attributed perhaps to the lack of resources in Chugoku, Hokuriku, Shikoku and some other regions.

4. Model Reference Adaptive Processes in Japan's Nine Regional Economies:

In this chapter we shall appraise the future economic development potentialities of the nine regions of Japan by using the theorems mentioned in Chapter 2. However, this analysis is linked to the following assumptions.

Assumption 1. The analysises below aim at introducing changes in regional technology and trading which would lead to the realization of planned regional growth paths.

Assumption 2. Planned growth paths are defined as ones which can be realized only when resources in regions are fully utilized. That is to say, the planned growth paths are Leontief trajectories under some regional technological systems and interregional trade conditions.

4. 1 *Model Reference Adaptive Processes with Reference to Regional Turnpike (case 1)*

In this section we regard the turnpike, which is obtained from the regional technological system and trade pattern in 1980, as a reference model, and simulate the adaptation processes that asymptotically converge to the turnpike. The reference model is defined as the turnpike which is the nearest to the actual outputs in 1980 from the view point of analyzing a long term plan. Furthermore because the adaptive outputs will rapidly approach the reference ones by MRAS, it may be expected that there would be fluctuations in the regional technology system and/or trade pattern of the adaptive model in their initial stages when the differences between the actual and the reference outputs are large. Therefore we shall introduce the target model whose outputs are defined as the paths that are obtained by linking smmothly the initial outputs of the adaptive model to the reference paths, and consider the adaptation processes which converges to the target paths. Consequently, the reference, the target and the adaptive models are represented as follows.

reference model

$$Xm(t+1)=\left(1+\frac{1}{\lambda}\right)^{t+1}\left(Xm(0)-(I-Am-CSmVm)^{-1}H\right)+(I-Am-CSmVm)^{-1}H \tag{53}$$

target model

$$Xg(t+1)=\left(Xm(T')/X(0)\right)^{\frac{t+1}{T'}}X(0) \qquad (0<t+1<T') \tag{54}$$

$$Xg(t+1)=Xm(t+1) \qquad (T'+1<t+1<T) \tag{55}$$

adaptive model

$$X(t+1)=C(t+1)\,X(t)+D(t+1)\,H \tag{56}$$

where, $Xm(t)$: vector of outputs of the reference model
Am : reference interregional input coefficient matrix
Bm : reference interregional capital coefficient matrix
CSm : matrix of reference marginal propensities to consume
Vm : reference regional value added coefficient matrix
H : regional basic consumption vector
λ : Frobenius root of $(I-Am-CSmVm)^{-1}Bm$
$Xm(0)$: initial vector of reference output which satisfies that
$Xm(0)=(I-Am-CSmVm)^{-1}H+\alpha\eta$
and minimizing $||Xm(0)-X(0)||$ with respect to α
η : an eigen vector corresponding to λ
$Xg(t)$: vector of outputs of the target model
T, T' : planned periods that satisfy $0<T'<T$
$X(t)$: vector of outputs of adaptive model
$X(0)$: vector of actual outputs in 1980
$A(t)$: adaptive interregional input coefficient matrix
$B(t)$: adaptive interregional capital coefficient matrix
CS : matrix of adaptive marginal propensities to consume (assumed to be constant)
$V(t)$: adaptive regional value added coefficient matrix

$$C(t+1)=B^{-1}(t+1)\left(I-A(t)-CSV(t)+B(t)\right)$$

$$D(t+1)=-B^{-1}(t+1)$$

$$A(0)=Am,\ B(0)=Bm,\ CS=CSm$$

By Theorem 2 in Chapter 2 the adaptation processes which approach the target paths will be realized by the following adaptation algorithms.

$$C(t+1)=C(t)+\left(I+\Gamma(t)\right)^{-1}Kc\otimes\hat{\varepsilon}(t+1)\,X^{T}(t) \tag{57}$$

$$D(t+1)=D(t)+\left(I+\Gamma(t)\right)^{-1}Kd\otimes\hat{\varepsilon}(t+1)\,H^{T} \tag{58}$$

$$\hat{\varepsilon}(t+1)=Xg(t+1)-C(t)\,X(t)-D(t)\,H \tag{59}$$

Moreover it seems that there exist some constraints in technological progress and changes of trade pattern in general. So let us express those constraints as follows.

$$\underline{a}^{rs}(t+1) \leqq a^{rs}(t+1) \leqq \bar{a}^{rs}(t+1) \qquad (60)$$

$$\underline{b}^{rs}(t+1) \leqq b^{rs}(t+1) \leqq \bar{b}^{rs}(t+1) \qquad (61)$$

The under (upper) bar stands for a lower (upper) bound of changes. Those constraints should be determined normally by characteristics of actual system in the regions, however, it is not easy to clarify explicitly the constraints due to complexities of interregional systems. So we reduce (60) and (61) as follows, assuring the lowest economic implications, that is, non-negativity conditions.

$$\text{if } a^{rs}(t+1)<0 \text{ then } a^{rs}(t+1)=a^{rs}(t) \qquad (62)$$

$$\text{if } b^{rs}(t+1)<0 \text{ then } b^{rs}(t+1)=b^{rs}(t) \qquad (63)$$

Under these preconditions the simulation results of the adaptation processes from 1980 to the year 2000 are presented in Tables 14~16. And now, adaptive changes of input and trade coefficients will be explained briefly in the subsequent sections, because the simulation results of input and trade coefficients are too massive

Table 14. Reference Paths from 1980 to the year 2000

(unit: billion yen, at market prices in 1980)

region / year	Hok-kaido	Tohoku	Kanto	Tokai	Hoku-riku	Kinki	Chugoku	Shikoku	Kyushu
1980	23,316.	41,658.	182,805.	73,351.	15,452.	103,106.	47,917.	18,072.	50,274.
1981	24,343.	43,576.	191,312.	77,071.	16,236.	108,208.	50,389.	18,969.	52,412.
1982	25,436.	45,615.	200,359.	81,028.	17,071.	113,634.	53,017.	19,922.	54,686.
1983	26,598.	47,783.	209,979.	85,235.	17,958.	119,405.	55,812.	20,936.	57,104.
1984	27,833.	50,089.	220,210.	89,709.	18,901.	125,541.	58,785.	22,014.	59,676.
1985	29,147.	52,541.	231,090.	95,467.	19,905.	132,067.	61,946.	23,160.	62,411.
1986	30,544.	55,149.	242,660.	99,527.	20,972.	139,007.	65,308.	24,380.	65,319.
1987	32,030.	57,922.	254,964.	104,908.	22,106.	146,387.	68,883.	25,676.	68,412.
1988	33,610.	60,871.	268,049.	110,631.	23,313.	154,235.	72,684.	27,055.	71,701.
1989	35,290.	64,007.	281,964.	116,716.	24,596.	162,581.	76,727.	28,521.	75,198.
1990	37,077.	67,342.	296,761.	123,187.	25,961.	171,456.	81,027.	30,081.	78,918.
1991	38,977.	70,889.	312,497.	130,069.	27,412.	180,895.	85,599.	31,739.	82,873.
1992	40,998.	74,661.	329,232.	137,387.	28,956.	190,932.	90,461.	33,502.	87,080.
1993	43,147.	78,672.	347,028.	145,170.	30,597.	201,606.	95,631.	35,378.	91,553.
1994	45,432.	82,937.	365,953.	153,446.	32,342.	212,958.	101,130.	37,372.	96,310.
1995	47,863.	87,473.	386,078.	162,247.	34,198.	225,029.	106,978.	39,493.	101,369.
1996	50,447.	92,297.	407,480.	171,607.	36,172.	237,866.	113,196.	41,748.	106,749.
1997	53,196.	97,427.	430,240.	181,560.	38,271.	251,518.	119,809.	44,146.	112,470.
1998	56.118.	102,882.	454,444.	192,144.	40,504.	266,035.	126,841.	46,697.	118,554.
1999	59,227.	108,684.	480,183.	203,400.	42,878.	281,474.	134,320.	49,409.	125,024.
2000	62,532.	114,853.	507,555.	215,369.	45,402.	297,892.	142,273.	52,294.	131,904.

Table 15. Target Paths from 1980 to the year 2000

(unit: billion yen, at market prices in 1980)

region / year	Hok-kaido	Tohoku	Kanto	Tokai	Hoku-riku	Kinki	Chugoku	Shikoku	Kyushu
1980	23,027.	43,737.	190,931.	71,580.	14,646.	93,354.	41,505.	17,272.	50,614.
1981	24,150.	47,666.	199,540.	75,573.	15,509.	99,206.	44,376.	18,258.	52,913.
1982	25,329.	47,680.	208,537.	79,789.	16,423.	105,424.	47,446.	19,299.	55,316.
1983	26,564.	49,783.	217,939.	84,241.	17,390.	112,031.	50,729.	20,400.	57,829.
1984	27,860.	51,978.	227,766.	88,941.	18,415.	119,053.	54,239.	21,564.	60,455.
1985	29,219.	54,271.	238,035.	93,903.	19,500.	126,516.	57,991.	22,794.	63,201.
1986	30,645.	56,664.	248,768.	99,141.	20,648.	134,445.	62,003.	24,094.	66,072.
1987	32,140.	59,164.	259,985.	104,673.	21,865.	142,872.	66,293.	25,469.	69,072.
1988	33,708.	61,773.	271,707.	110,512.	23,153.	151,827.	70,880.	26,922.	72,210.
1989	35,352.	64,498.	283,958.	116,678.	24,517.	161,344.	75,783.	28,457.	75,489.
1990	37,077.	67,342.	296,761.	123,187.	25,961.	171,456.	81,027.	30,081.	78,918.
1991	38,977.	70,889.	312,497.	130,069.	27,412.	180,895.	85,599.	31,739.	82,873.
1992	40,998.	74,661.	329,232.	137,387.	28,956.	190,932.	90,461.	33,502.	87,080.
1993	43,147.	78,672.	347,028.	145,170.	30,597.	201,606.	95,631.	35,378.	91,553.
1994	45,432.	82,937.	365,953.	153,446.	32,342.	212,958.	101,130.	37,372.	96,310.
1995	47,863.	87,473.	386,078.	162,247.	34,198.	225,029.	106,978.	39,493.	101,369.
1996	50,447.	92,297.	407,480.	171,607.	36,172.	237,866.	113,196.	41,748.	106,749.
1997	53,196.	97,427.	430,240.	181,560.	38,271.	251,518.	119,809.	44,146.	112,470.
1998	56,118.	102,882.	454,444.	192,144.	40,504.	266,035.	126,841.	46,697.	118,554.
1999	59,227.	108,684.	480,183.	203,400.	42,878.	281,474.	134,320.	49,409.	125,024.
2000	62,532.	114,853.	507,555,	215,369.	45,402.	289,892.	142,273.	52,294.	131,904.

to be presented in this article. Here we applied Kc and Kd as $kc^{rs}=1.0$ and $kd^{rs}=1.0$. We observe in Table 14 that the annual growth rates of the reference paths from the equilibrium point are same and constant because the initial values of the reference paths are on the turnpike. Initially, the output of Kyushu is greater than that of Chugoku, however, in the year 2000, Chugoku's output exceeds that of Kyushu. This tendency may be attributed to the differences in the elements of the turnpike direction vector. We also observe in Table 15 that the average annual growth rates from 1980 to 1990 of the traget paths of Hokuriku, Kinki, Chugoku and Kyushu are more than those of their reference paths. In the other regions, however, it is the reverse. After 1990 all the target paths assume the same level as those under the reference model. Now the brief explanations for the simulation results of the regional outputs, interregional input coefficients and capital coefficients are presented below.

(1) Adaptation Processes in Regional Outputs

It is shown in Tables 15 and 16 that there are few differences between the adaptive and the target paths and after 1985 the adaptive paths approach the target paths with less than 1% in error rate.

Table 16. Adaptive Paths from 1980 to the year 2000

(nuit: billion yen, at market prices in 1980)

region / year	Hok-kaido	Tohoku	Kanto	Tokai	Hoku-riku	Kinki	Chugoku	Shikoku	Kyushu
1980	23,027.	43,737.	190,931.	71,580.	14,646.	93,354.	41,505.	17,272.	50,614.
1981	24,067.	45,858.	200,963.	75,340.	15,381.	97,703.	43,524.	18,150.	52,994.
1982	25,234.	47,946.	210,534.	79,518.	16,259.	103,487.	46,347.	19,168.	55,463.
1983	26,515.	49,995.	219,505.	84,083.	17,279.	110,692.	49,956.	20,317.	57,980.
1984	27,877.	52,066.	228,337.	88,948.	18,398.	118,820.	53,996.	21,557.	60,553.
1985	29,283.	54,243.	237,578.	94,030.	19,564.	127,227.	58,248.	22,847.	63,217.
1986	30,719.	56,584.	247,788.	99,299.	20,743.	135,512.	62,489.	24,166.	66,026.
1987	32,194.	59,098.	259,121.	104,791.	21,940.	143,711.	66,713.	25,523.	69,015.
1988	33,733.	61,752.	271,326.	110,571.	23,187.	152,185.	71,086.	26,946.	72,181.
1989	35,359.	64,513.	284,010.	116,698.	24,519.	161,337.	75,808.	28,461.	76,479.
1990	37,079.	67,366.	296,951.	123,199.	25,955.	171,362.	80,993.	30,080.	78,940.
1991	38,949.	70,673.	311,638.	130,087.	27,446.	181,428.	86,013.	31,767.	82,739.
1992	40,975.	74,476.	328,428.	137,397.	28,987.	191,421.	90,817.	33,525.	86,966.
1993	43,141.	78,622.	346,771.	145,170.	30,607.	201,756.	95,729.	35,384.	91,515.
1994	45,437.	82,972.	366,100.	153,443.	32,336.	212,867.	101.059.	37,367.	96,335.
1995	47,867.	87,514.	386,276.	162,245.	34,190.	224,912.	106.894.	39,487.	101,401.
1996	60,449.	92,310.	407,552.	171,607.	36,169.	237,824,	113,169.	41,746.	106,760.
1997	53,195.	97,422.	430,217.	181,561.	38,272.	251,532.	119,820.	44,147.	112,466.
1998	56,118.	102,876.	454,410.	192,145.	40,505.	266,055.	126,855.	46,698.	118,548.
1999	59,227.	108,682.	480,172.	203,400.	42,878.	281,480.	134,324.	49,409.	125,022.
2000	62,532.	114,854.	507,559.	215,369.	45,402.	297,890.	142,271.	52,294.	131,905.

(2) Adaptation Processes in Interregional Input Coefficients

The adaptive changes of the interregional input-output system which result in the adaptive processes of regional outputs are summarized as follows. We observe that the intermediate input rates decrease from 1980 to 1982 but increase gradually afterwards. This fact indicates that the actual interregional technological system in 1980 can not catch up with the growth of the target paths and regional technological progress, especially in Kanto, are necessary. It was also shown that after 1983 gentle adjustment was needed to approach the target paths because the technological system in 1982 was so progressive that the adaptive paths exceeded the target ones.

The changes of the interregional trade coefficients of the intermediate commodities signify that while the self-sufficiency rate increases in Kanto, it decreases in Kinki from 1980 to 1983. However, it re-emerges into the 1980 pattern thereafter.

(3) Adaptation Processes in Interregional Capital Coefficient

The adaptive trends of the regional capital coefficients are a little slower than the interregional input coefficients. Those decrease from 1980 to 1982 then gradually increase after that. The reason is probably the same as that of the interregional input coefficients explained previously.

Changes of the capital trade coefficients indicate that the regional self-sufficiency rates are getting a little larger in the period 1980～1982 and then behaving with little variations.

Table 17. Projection of Regional Populations in the year 2000

(unit: thousand persons)

region	1980	2000	region	1980	2000
Hokkaido	5,560.	6,090.	Kinki	19,550.	21,430.
Tohoku	11,990.	13,050.	Chugoku	7,590.	8,010.
Kanto	37,720.	41,700.	Shikoku	4,160.	4,350.
Tokai	13,340.	14,870.	Kyushu	14,040.	15,420.
Hokuriku	2,990.	3,200.	Total	116,940.	128,120.

Table 18. Per Capita Outputs of Regional Turnpike from 1980 to the year 2000

(unit: thousand yen/person, at market price in 1980)

region / year	Hokkaido	Tohoku	Kanto	Tokai	Hokuriku	Kinki	Chugoku	Shikoku	Kyushu
1980	4,194.	3,474.	4,846.	5,499.	5,168.	5,274.	6,313.	4,344.	3,581.
1981	4,358.	3,619.	5,047,	5,746.	6,412.	5,510.	6,621.	4,550.	3,716.
1982	4,533.	3,772.	5,259.	6,008.	5,671.	5,759.	6,948.	4,768.	3,859.
1983	4,719.	3,935.	5,484.	6,286.	5,945.	6,024.	7,294.	4,999.	4,010.
1984	4,916.	4,107.	5,722.	6,580.	6,236.	6,305.	7,662.	5,245.	4,171.
1985	5,124.	4,290.	5,975.	6,892.	6,545.	6,602.	8,052.	5,506.	4,342.
1986	5,345.	4,484.	6,242.	7,222.	6,875.	6,917.	8,467.	5,782.	4,523.
1987	5,580.	4,690.	6,526.	7,571.	7,220.	7,251.	8,906.	6,076.	4,715.
1988	5,829.	4,908.	6,827.	7,941.	7,588.	7,605.	9,372.	6,388.	4,919.
1989	6,092.	5,139.	7,145.	8,332.	7,979.	7,980.	9,867.	6,720.	5,135.
1990	6,472.	5,384.	7,483.	8,746.	8,393.	8,377.	10,392.	7,071.	5,364.
1991	6,668.	5,643.	7,840.	9,185.	8,832.	8,797.	10,949.	7,444.	5,606.
1992	6,982.	5,918.	8,219.	9,649.	9,298.	9,243.	11,539.	7,840.	5,863.
1993	7,314.	6,210.	8,619.	10,141.	9,791.	9,715.	12,166.	8,261.	6,135.
1994	7,667.	6,519.	9,044.	10,661.	10,315.	10,215.	12,831.	8,707.	6,424.
1995	8,040.	6,846.	9,494.	11,211.	10,870.	10,745.	13,537.	9,181.	6,730.
1996	8,436.	7,193.	9,970.	11,794.	11,458.	11,305.	14,285.	9,683.	7,054.
1997	8,855.	7,561.	10,474.	12,410.	12,082.	11,900.	15,079.	10,217.	7,397.
1998	9,299.	7,951.	11,008.	13,063.	12,744.	12,529.	15,921.	10,783.	7,761.
1999	9,770.	8,364.	11,573.	13,753.	13,445.	13,195.	16,814.	11,384.	8,146.
2000	10,268.	8,801.	12,172.	14,484.	14,188.	13,901.	17,762.	12,022.	8,554.

4. 2 *Model Reference Adaptive Processes with Reference to Regional Per Capita Output Equalization (case 2)*

In this section we shall consider the adaptation processes which will make regional per capita outputs equalize. It is necessary for this analysis to clarify a mechanism for interregional population movement. Interregional migration is deemed to be dependent on regional income level, living environment, job opportunity etc., however, in this section regional population will be projected under the assumptions of the closed population systems with consideration for only natural growth. This procedure is, of course, a simplification of the actual system. In addition, the assesment of per capita output is also a simplification though even it would have been better to assess per capita income from the standing point of regional welfare.

Table 17 shows a projection of the regional populations from 1980 to the year 2000 27), and Table 18 denotes the trends of regional per capita outputs which were obtained by dividing the regional turnpike outputs in Tables 14 by the regional populations. Because the growth rates of the regional outputs exceed those of the regional populations, the regional per capita outputs are increasing uniformly.

Now let us consider the case where the regional per capita outputs is to be

Table 19. Target Poths from 1980 to the year 2000

(unit: billion yen, at market price in 1980)

region / year	Hokkaido	Tohoku	Kanto	Tokai	Hokuriku	Kinki	Chugoku	Shikoku	Kyushu
1980	23,027.	43,737.	190,931.	71,580.	14,646.	93,354.	41,505.	17,272.	50,614.
1981	24,421.	46,666.	200,565.	75,003.	15,368.	98,309.	43,330.	18,274.	54,060.
1982	25,900.	49,791.	210,684.	78,591.	15,162.	103,527.	45,236.	19,333.	57,741.
1983	27,469.	53,126.	221,315.	82,350.	16,978.	109,022.	47,225.	20,454.	61,672.
1984	29,132.	56,684.	232,485.	86,289.	17,835.	114,808.	49,302.	21,639.	65,871.
1985	30,896.	60,480.	244,212.	90,416.	18,735.	120,902.	51,470.	22,894.	70,356.
1986	32,767.	64,531.	256,534.	94,741.	19,681.	127,318.	53,734.	24,221,	75,146.
1987	34,752.	68,852.	269,478.	99,272.	20,674.	134,076.	56,097.	25,625.	80,263.
1988	36,856.	73,464.	283,074.	104,020.	21,718.	141,192.	58,564.	27,110.	85,727.
1989	39,088.	78,384.	297,357.	108,996.	22,814.	148,686.	61,140.	28,581.	91,564.
1990	41,455.	83,633.	312,361.	114,209.	23,966.	156,578.	63,829.	30,344.	97,798.
1991	43,966.	89,234.	328,122.	119,672.	25,175.	164,888.	66,636.	32,103.	104,457.
1992	46,682.	95,211.	344,678.	125,396.	26,446.	173,640.	69,566.	33,964.	111,568.
1993	49,452.	101,587.	362,069.	131,394.	27,781.	182,856.	72,626.	35,933.	119,165.
1994	52,446.	108,391.	380,337.	137,678.	29,183.	192,561.	75,826.	38,015.	127,278.
1995	55,622.	115,650.	399,528.	144,264.	30,656.	202,781.	79,154.	40,219.	135,944.
1996	58,991.	123,395.	419,687.	151,164.	32,204.	213,544.	82,636.	42,550.	145,199.
1997	62,563.	131,660.	440,863.	158,394.	33,829.	224,878.	86,270.	45,017.	155,085.
1998	66,352.	140,477.	463,107.	165,970.	35,537.	236,813.	90,064.	47,626.	165,644.
1999	70,370.	149,885.	486,474.	173,909.	37,331.	249,382.	94,025.	50,387.	176,922.
2000	74,631.	159,924.	511,019.	182,227.	39,215.	262,618.	98,160.	53,308.	188,986.

equalized at a planned period T under the condition which national output at T is equal to that of the reference model in 4. 1. Then the target and the adaptive models are defined as,

target model

$$\frac{xg_i(t+1)}{p_i(t+1)}=\left(\frac{\sum_{j=1}^{9} xg_j(2000)}{\sum_{j=1}^{9} p_j(2000)} \bigg/ \frac{xg_j(1980)}{p_j(1980)}\right)^{\frac{t-1}{T-1}} \frac{xg_i(1980)}{p_i(1980)} \tag{64}$$

$$xg_i(t+1)=\frac{xg_i(t+1)}{p_i(t+1)}\,p_i(t+1) \tag{65}$$

adaptive model

$$X(t+1)=C(t+1)\,X(t)+D(t+1)\,H \tag{66}$$

where, $p_i(t)$: population of the i th region

When we adopt the same adaptation laws of $C(t)$ and $D(t)$ as those of 4.1 and apply Kc and Kd as $kc^{rs}=10^{-12}$, $kd^{rs}=10^{-12}$, then the simulation results are presented

Table 20. Adaptive Paths from 1980 to the year 2000

(unit : billion yen, at market price in 1980)

region / year	Hok-kaido	Tohoku	Kanto	Tokai	Hoku-riku	Kinki	Chugoku	Shikoku	Kyushu
1980	23,027.	43,737.	190,931.	71,580.	14,646.	93,354.	41,505.	17,272.	50,614.
1981	24,042.	46,004.	201,696.	75,198.	15,313.	96,939.	43,061.	18,100.	53,094.
1982	25,165.	48,515.	213,337.	79,015.	16,013.	100,736.	44,667.	18,997.	55,911.
1983	26,434.	51,343.	225,786.	83,006.	16,752.	104,864.	46,338.	19,983.	59,176.
1984	27,889.	54,567.	238,893.	87,136.	17,534.	109,473.	48,095.	21,076.	63,008.
1985	29,572.	58,272.	252,421.	91,355.	18,368.	114,739.	49,967.	22,295.	67,523.
1986	31,519.	62,529.	266,058.	95,609.	19,261.	120,852.	51,986.	23,656.	72,813.
1987	33,752.	67,388.	279,456.	99,853.	20,225.	127,982.	54,185.	25,168.	78,924.
1988	36,270.	72,857.	292,308.	104,067.	21,270.	136,241.	56,598.	26,831.	85,827.
1989	39,043.	78,885.	304,438.	108,279.	22,406.	145,637.	59,250.	28,633.	93,404.
1990	42,013.	85,365.	315,398.	112,577.	23,641.	156,036.	62,153.	30,549.	101,447.
1991	45,096.	92,135.	327,128.	117,113.	24,974.	167,151.	65,302.	32,546.	109,685.
1992	48,200.	99,010.	338,797.	122,093.	26,400.	178,559.	68,670.	34,589.	117,828.
1993	51,240.	105,815.	351,908.	127,738.	27,903.	189,769.	72,212.	36,647.	125,444.
1994	54,169.	112,442.	367,532.	134,228.	29,457.	200,325.	75,871.	38,705.	133,030.
1995	56,993.	118,893.	386,564.	141,642.	31,034.	209,939.	79,581.	40,772.	140,080.
1996	59,786.	125,319.	409,412.	149,901.	32,606.	218,623.	83,287.	42,883.	147,121.
1997	62,689.	132,026.	435,728.	158,738.	34,159.	226,790.	86,950.	45,095.	154,688.
1998	65,882.	139,435.	464,310.	167,739.	35,703.	235,243.	90,569.	47,482.	163,422.
1999	69,543.	147,975.	493,325.	176,463.	37,280.	245,016.	94,193.	50,110.	173,891.
2000	73,780.	157,940.	520,932.	184,631.	38,960.	257,029.	97,917.	53,018.	186,366.

in Tables 19～21.

(1) Adaptation Processes in Regional Outputs

When we compare the adaptive paths with the target ones, we observe that the gaps between the adaptive and the target paths are less than 3% at 1990 and decrease within 2.2% in the year 2000. In this simulation the disparities of the regional per capita outputs remain up to the year 2000 because the adaptation processes are a little slower.

(2) Adaptation Processes in Interregional Input Coefficients

The adaptive changes of the interregional input coefficients which are necessities in realizing the equalization of the regional per capita outputs are summarized as follows. The interregional input coefficients decrease from 1980 to 1987 then gradually increase till 1995. Thereafter they fall again. This pattern depicts that the regional technological systems progress till 1987 to catch up with the target paths because the growth in the target paths are more than the actual growth in 1980. The behaviour after 1987 is interpreted to mean the phase for adjustment

Table 21. Per Capita Outputs of Adaptive Paths from 1980 to the year 2000

(unit : thousand yen/person, at market price in 1980)

region / year	Hokkaido	Tohoku	Kanto	Tokai	Hokuriku	Kinki	Chugoku	Shikoku	Kyushu
1980	4,142.	3,648.	5,062.	5,366.	4,898.	4,775.	5,468.	4,152.	3,605.
1981	4,304.	3,821.	5,320.	5,607.	5,104.	4,936.	5,658.	4,321.	3,764.
1982	4,485.	4,012.	5,599.	5,859.	5,319.	5,106.	5,852.	4,546.	3,945.
1983	4,690.	4,228.	5,896.	6,122.	5,546.	5,291.	6,056.	4,772.	4,156.
1984	4,925.	4,475.	6,208.	6,392.	5,785.	5,498.	6,269.	5,021.	4,404.
1985	5,199.	4,758.	6,526.	6,665.	6,040.	5,736.	6,495.	5,300.	4,698.
1986	5,516.	5,084.	6,844.	6,937.	6,312.	6,014.	6,739.	5,611.	5,042.
1987	5,880.	5,456.	7,153.	7,206.	6,605.	6,339.	7,006.	5,956.	5,440.
1988	6,290.	5,874.	7,445.	7,470.	6,923.	6,718.	7,298.	6,336.	5,888.
1989	6,740.	6,333.	7,715.	7,730.	7,268.	7,148.	7,619.	6,748.	6,378.
1990	7,220.	6,824.	7,966.	7,993.	7,643.	7,623.	7,971.	7,181.	6,895.
1991	7,715.	7,335.	8,207.	8,270.	8,047.	8,129.	8,353.	7,634.	7,420.
1992	8,208.	7,848.	8,457.	8,575.	8,477.	8,644.	8,760.	8,095.	7,933.
1993	8,686.	8,352.	8,741.	8,923.	8,929.	9,144.	9,187.	8,557.	8,420.
1994	9,141.	8,838.	9,083.	9,326.	9,395.	9,609.	9,626.	9,018.	8,873.
1995	9,574.	9,306.	9,506.	9,788.	9,864.	10,024.	10,070.	9,478.	9,300.
1996	9,998.	9,767.	10,017.	10,302.	10,329.	10,391.	10,511.	9,947.	9,722.
1997	10,435.	10,264.	10,608.	10,860.	10,784.	10,730.	10,943.	10,436.	10,174.
1998	10,915.	10,776.	11,247.	11,404.	11,233.	11,079.	11,368.	10,964.	10,698.
1999	11,471.	11,387.	11,890.	11,932.	11,690.	11,486.	11,791.	11,545.	11,330.
2000	12,115.	12,103.	12,492.	12,416.	12,175.	11,994.	12,224.	12,188.	12,086.

in the excessive technological progress.

As for the intermediate trade coefficients, the self-sufficiency rate of Kanto assume an increase larger from 1980 to 1989 while for the other regions are decreasing. Up till 1995 the self-sufficiency rate of Kanto gets lower while increases are observed for the other regions. Again after 1996 the self-sufficiency rates of Kanto show increase. These trends indicate that the growth pattern which leads to higher output in Kanto and its subsequent distribution to the other regions would lead to overall higher economic growth, the reason being that the production share rate of Kanto is larger and also the purchasing rate of intermediate commodities from Kanto is large in all regions.

(3) Adaptation Processes in Interregional Capital Coefficients

We observe that the variations of the interregional capital coefficients are more gentle than the interregional input coefficients. The interregional capital coefficients passingly decrease form 1980 to 1981 then gradually increase till 1988. And after, there is gentle decrease till 1996 when increase starts again.

As for the capital trade coefficients the self-sufficiency rates increase marginally

Table 22. Target Paths from 1980 to the year 2000

(unit : billion yen, at market price in 1980)

year \ region	Hokkaido	Tohoku	Kanto	Tokai	Hokuriku	Kinki	Chugoku	Shikoku	Kyushu
1980	23,027.	43,737.	190,931.	71,580.	14,646.	93,354.	41,505.	17,272.	50,614.
1981	24,431.	45,666.	199,540.	75,573.	1,5509.	99,206.	44,376.	18,258.	52,913.
1982	25,921.	47,680.	208,537.	79,789.	16,423.	105,424.	47,446.	19,299.	55,316.
1983	27,502.	49,783.	217,939.	84,241.	17,390.	112,031.	50,729.	20,400.	57,829.
1984	29,179.	51,978.	227,766.	88,941.	18,415.	119,052.	54,239.	21,564.	60,455.
1985	30,959.	54,271.	238,035.	93,903.	19,500.	126,516.	57,991.	22,794.	63,201.
1986	32,847.	56,664.	248,768.	99,141.	20,548.	134,445.	62,003.	24,094.	66,072.
1987	34,850.	59,164.	259,985.	104,673.	21,865.	142,872.	66,293.	25,469.	69,072.
1988	36,975.	61,773.	271,707.	110,512.	23,153.	151,827.	70,880.	26,922.	72,210.
1989	39,230.	64,498.	283,953.	116,678.	24,517.	161,344.	75,783.	28,457.	75,489.
1990	41,623.	67,342.	296,761.	123,187.	25,961.	171,456.	81,027.	30,081.	78,918.
1991	44,161,	70,889.	312,497.	130,069.	27,412.	180,895.	85,599.	31,739.	82,873.
1992	46,854.	74,661.	329,232.	137,387.	28,956.	190,932.	90,461.	33,502.	87,080.
1993	49,711.	78,672.	347,028.	145,170.	30,597.	201,606.	95,631.	35,378.	91.553.
1994	52,743.	82,937.	365,953.	153,446.	32,342.	212,958.	101,130.	37,372.	96,310.
1995	55,959.	87,473.	386,078.	162,247.	34,198.	225,029.	106,978.	39,493.	101,369.
1996	59,372.	92,297.	407,480.	171,607.	36,172.	237,866.	113,196.	41,748.	106,749.
1997	62,993.	97,427·	430,240.	181,560.	38,271.	251,518.	119,809.	44,146.	112,470.
1998	66,835.	102,882.	454,444.	192,144.	40,504.	266,035.	126,841.	46,697.	118,554.
1999	70,910.	108,684.	480,183.	203,400.	42,878.	281,474.	134,320.	49,409.	125,024.
2000	75,235.	114,853.	607,555.	215,369.	45,402.	297,892.	142,273.	52,294.	131,904.

from 1980 to 1981, then decrease slowly till 1996 and then start to increase.

4. 3 *Model Reference Adaptive Processes with Reference to Higher Growth of Hokkaido (case 3)*

In this section we focus on the adaptation processes that some specific region would be growing higher. This analysis, of course, does not aim at ensuring structural changes of the specific region but total structural changes for all regions. In the subsequent context, Hokkaido is designated as the specific region and its target path equated to the average per capita outputs of the eight regions except Hokkaido in 4. 1. Then the target and the adaptive models are formulated as follows.

target model

$$xg_i(t+1)=\left(xm_i(1990)/x_i(1980)\right)^{\frac{t+1}{T'}}x_i(1980) \quad (1980<t+1<1990) \qquad (67)$$

$$xg_i(t+1)=xm_i(t+1) \quad (1991<t+1<2000) \quad (i=2, 3, \cdots, 9) \qquad (68)$$

Table 23. Adaptive Paths from 1980 to the year 2000

(unit : billion yen, at market price in 1980)

region / year	Hok-kaido	Tohoku	Kanto	Tokai	Hoku-riku	Kinki	Chugoku	Shikoku	Kyushu
1980	23,027.	43,737.	190,931.	71,580.	14,646.	93,354.	41,505.	17,272.	50,614.
1981	24,168.	45,858.	200,963.	75,240.	15,381.	97,703.	43,524.	18,150.	52,994.
1982	25,599.	47,945.	210,531.	79,516.	16,259.	103,488.	46,347.	19,168.	55,463.
1983	27,299.	49,993.	219,493.	84,078.	17,278.	110,690.	49,955.	20,316.	57,980.
1984	29,172.	52,063.	228,311.	88,940.	18,397.	118,815.	53,994.	21,557.	60,552.
1985	31,105.	54,238.	237,542.	94,018.	19,562.	127,219.	68,244.	22,846.	63,215.
1986	33,041.	56,579.	247,749.	99,287.	20,741.	135,503.	62,486.	24,165.	66,024.
1987	34,998.	59,093.	259,090.	104,781.	21,938.	143,703.	66,711.	25,522.	69,014.
1988	37,042.	61,750.	271,306.	110,565.	23,185.	152,180.	71,084.	26,945.	72,180.
1989	39,240.	64,511.	283,998.	116,694.	24,518.	161,333.	75,808.	28,460.	75,496.
1990	41,621.	67,364.	296,940.	123,196.	25,954.	171,359.	80,992.	30,079.	78,939.
1991	44,177.	70,671.	311,624.	130,083.	27,445.	181,424.	86,012.	31,766.	82,739.
1992	46,887.	74,475.	328,406.	137,392.	28,986.	191,417.	90,815.	33,525.	86,955.
1993	49,748.	78,620.	346,759.	145,165.	30,606.	201,752.	95,728.	35,383.	91,515.
1994	52,772.	82,971.	366,090.	153,439.	32,336.	212,865.	101,058.	37,367.	96,334.
1995	55,980.	87,513.	386,267.	162,243.	34,190.	224,910.	106,894.	39,487.	101,400.
1996	59,389.	92,309.	407,544.	171,604.	36,169.	237,823.	113,169.	41,746.	106,759.
1997	63,009.	97,421.	430,210.	131,558.	38,272.	251,530.	119,819.	44,147.	112,466.
1998	66,850.	102,875.	454,403.	192,143.	40,505.	266,054.	126,854.	46,698.	118,548.
1999	70,924.	108,782.	480,167.	203,398.	42,878.	281,478.	134,323.	49,409.	125,022.
2000	75,247,	114,853.	507,554.	215,367.	45,402.	297,889.	142,271.	52,293.	131,905.

$$\frac{xg_1(t+1)}{p_1(t+1)}=\left(\frac{\sum_{i=2}^{9} xg_i(2000)}{\sum_{i=2}^{9} p_i(2000)}\right)^{\frac{t+1}{T}}\left(\frac{xg_1(1980)}{p_1(1980)}\right) \tag{69}$$

$$xg_1(t+1)=\left(\frac{xg_1(t+1)}{p_1(t+1)}\right)p_1(t+1) \tag{70}$$

adaptive model

$$X(t+1)=C(t+1)\,X(t)+D(t+1)\,H \tag{71}$$

Setting $kc^{rs}=10^{-11}$, $kd^{rs}=10^{-11}$, the results of the simulation are presented in Tables 22～24.

(1) Adaptation Processes in Regional Outputs

The adaptive paths converge to the target ones in less than 1% in error rate. The only exception is the initial stages, that is, from 1981 to 1983.

(2) Adaptation Processes in Interregional Input Coefficients

Table 24. Per Capita Outputs of Adaptive Paths from 1980 to the year 2000

(unit : thousand yen/person, at market price in 1980)

region / year	Hokkaido	Tohoku	Kanto	Tokai	Hokuriku	Kinki	Chugoku	Shikoku	Kyushu
1980	4,142.	3,648.	5,062.	5,366.	4,898.	4,775.	5,468.	4,152.	3,605.
1981	4,327.	3,809.	5,301.	5,617.	5,127.	4,975.	5,719.	4,353.	3,757.
1982	4,562.	3,965.	5,526.	5,896.	5,401.	5,245.	6,074.	4,587.	3,913.
1983	4,843.	4,117.	5,732.	6,201.	5,720.	5,584.	6,529.	4,851.	4,072.
1984	5,152.	4,269.	5,933.	6,524.	6,070.	5,967.	7,038.	5,136.	4,233.
1985	5,469.	4,429.	6,142.	6,859.	6,432.	6,360.	7,571.	5,131.	4,398.
1986	5,783.	4,600.	6,373.	7,204.	6,797.	6,743.	8,101.	5,732.	4,572.
1987	6,097.	4,785.	6,632.	7,562.	7,165.	7,118.	8,625.	6,040.	4,757.
1988	6,424.	4,979.	6,910.	7,936.	7,547.	7,503.	9,166.	6,363.	4,952.
1989	6,774.	5,179.	7,197.	8,331.	7,953.	7,918.	9,749.	6,705.	5,155.
1990	7,153.	5,385.	7,487.	8,747.	8,391.	8,372.	10,387.	7,071.	5,365.
1991	7,557.	5,626.	7,818.	9,186.	8,843.	8,823.	11,002.	7,451.	5,597.
1992	7,985.	5,904.	8,198.	9,650.	9,308.	9,266.	11,585.	7,846.	5,855.
1993	8,433.	6,206.	8,613.	10,140.	9,794.	9,722.	12,178.	8,262.	6,133.
1994	8,905.	6,522.	9,047.	10,660.	10,313.	10,210.	12,822.	8,706.	6,426.
1995	9,404.	6,850.	9,498.	11,211.	10,867.	10,739.	13,526.	9,179.	6,732.
1996	9,931.	7,194.	9,971.	11,794.	11,457.	11,303.	14,282.	9,683.	7,054.
1997	10,489.	7,561.	10,473.	12,410.	12,082.	11,900.	15,080.	10,217.	7,397.
1998	11,077.	7,950.	11,007.	13,063.	12,744.	12,530.	15,923.	10,783.	7,760.
1999	11,699.	8,363.	11,573.	13,753.	13,445.	13,195.	16,815.	11,384.	8,146.
2000	12,356.	8,801.	12,172.	14,483.	14,188.	13,901.	17,762.	12,022.	8,554.

The adaptive variations of the interregional input coefficients from which the adaptive outputs are realized are characterized as follows. The trend of the interregional input coefficients resembles a wave motion with a little oscillation from 1980 to 1982 and the increasing sharply till 1990. Thereafter, there is a decrease followed by an increase.

With regard to the interregional trade coefficients of the intermediate commodities, the self-sufficiency rates in all regions show increase from 1980 to 1996, except Kanto. The self-sufficiency rate in Kanto decreases from 1984 to 1988, then after that a little irregular fluctuations are observed. From the fact that the trade flow pattern of Hokkaido is varying uniformly, it is interpreted that the delicate adaptation is the result of changes experienced in Kanto.

(3) Adaptation Processes in Interregional Capital Coefficients

The interregional capital coefficients show a decline from 1980 to 1981, and gradual increase till 1983. After 1983, again a gradual decline is felt before attaining level off after 1990.

With respect to the interregional capital trade coefficients, the self-sufficiency rates experience slight increase from 1980 to 1981, then a little decrease. Invariably, however, they depict relative stability. These characteristics can be interpreted to mean that with adjustments in the capital coefficients in the initial stages, the adaptation in the outputs is the result of the changes in the input coefficients.

5. Conclusion

This study has been one of the pioneering attempts in applying MRAS for a corroborative analysis. From this study results, it has been proved that MRAS approaches are very effective and meaningful in regional planning studies. Areas worth examining for further research and analysis include subdividing the classifications of the sectors and the final demand and making the data estimation more precise.

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